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## Syllabi of Semester-VII and VIII based on UGCF - 2022

### DEPARTMENT OF MATHEMATICS

#### Category-I

### **B.Sc. (Hons.) Mathematics, Semester-VII**

#### **DISCIPLINE SPECIFIC CORE COURSE – 19: LINEAR ANALYSIS**

#### **CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Linear Analysis	4	3	1	0	Class XII pass with Mathematics	Metric Spaces, Advanced Linear Algebra

**Learning Objectives:** The objective of the course is to introduce:

- Norm and normed spaces, Banach spaces and Hilbert spaces as complete normed spaces and their properties.
- Various forms of matrix norms with examples.
- Classes of bounded linear operators on normed spaces and Hilbert spaces, respectively.
- Four important theorems: Hahn-Banach, Uniform boundedness, Open mapping, and Closed graph as the cornerstones of the theory of Banach spaces.

**Learning Outcomes:** This course will enable the students to:

- Analyze and demonstrate examples of normed linear spaces with their properties.
- Characterize the bounded linear operators on normed spaces as continuous functions.
- Understand and apply Schwarz and Bessel's inequality, Parseval's identity.
- Illustrate linear operators, self-adjoint, unitary and normal operators on Hilbert spaces.
- Prove and apply fundamental theorems from the theory of normed and Banach spaces.

#### **SYLLABUS OF DSC-19**

#### **UNIT – I: Normed Spaces and Banach Spaces (15 hours)**

Normed spaces, Banach spaces, Properties of normed spaces, Finite dimensional normed spaces and subspaces, Compactness and finite dimension; Matrix norms; Linear operators, Bounded linear operators; Linear functionals, Linear operators and functionals on finite dimensional spaces; Normed spaces of operators, Dual space.

**UNIT – II: Hilbert Spaces****(15 hours)**

Overview of inner product spaces and its properties, Hilbert spaces, Orthogonal complements and direct sums, Orthonormal sets and sequences, Bessel inequality; Total orthonormal sets and sequences; Riesz representations theorem, Hilbert-adjoint operator, Self-adjoint, Unitary and normal operators.

**UNIT – III: Fundamental Theorems for Normed and Banach Spaces****(15 hours)**

Hahn Banach theorems for real and complex vector spaces, Hahn Banach theorem for normed spaces; Reflexive spaces; Uniform boundedness theorem, Open mapping theorem, Closed graph theorem.

**Essential Readings**

1. Kreyszig, Erwin (1989). Introductory Functional Analysis with Applications (1st ed.). John Wiley & Sons. Wiley-India Student Edition. Indian Reprint 2007.
2. Horn, Roger A. and Johnson, Charles R. (2013). Matrix Analysis (2nd ed.). Cambridge University Press.

**Suggestive Readings**

- Bollobás Béla (1999). Linear Analysis: An Introductory Course (2nd ed.). Cambridge University Press.
- Rynne, Bryan P. and Youngson, Martin A. (2008). Linear Functional Analysis (2nd ed.). Springer-Verlag London Limited.

## DSE Courses of B.Sc. (Hons) Mathematics, Semester-VII

### DISCIPLINE SPECIFIC ELECTIVE COURSE – 5(i): ADVANCED DIFFERENTIAL EQUATIONS

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Advanced Differential Equations	4	3	1	0	Class XII pass with Mathematics	Multivariate Calculus, Ordinary and Partial Differential Equations

**Learning Objectives:** The main objective of this course is to:

- Study the existence, uniqueness, and stability of solutions of IVPs, to explore the solution of system of linear equations.
- Study Green's function and its applications in boundary value problems, Eigenvalues and Eigenfunctions of Sturm Liouville systems.
- Investigate the solutions and applications of Laplace, wave, and diffusion equations.

**Learning Outcomes:** This course will enable the students to find the:

- Existence, uniqueness, and continuity of solutions of IVPs.
- Properties of zeros of solutions of linear second order ODE's.
- Green's function of a BVP and its applications.
- Eigenvalues and eigenfunctions of Sturm-Liouville systems.
- Solutions of Laplace, wave, and diffusion equations with their applications.

#### SYLLABUS OF DSE-5(i)

##### UNIT – I: Existence and Uniqueness for Initial-Value Problems (15 hours)

Well posed problems, Picard's existence theorem, uniqueness and continuity theorems for initial value problems of first order; Existence and uniqueness theorems for systems and higher order IVPs; Sturm separation and comparison theorems; Homogeneous linear systems, Nonhomogeneous linear systems, Linear systems with constant coefficients.

##### UNIT – II: Stability Theory and Boundary-Value Problems (10 hours)

Stability of autonomous system of differential equations, Critical point of an autonomous system and their classification, Stability of linear systems with constant coefficients, Linear plane autonomous systems, Perturbed systems; Two-point boundary-value problem, Green's functions and their construction; Sturm-Liouville systems, Eigenvalues and Eigenfunctions.

**UNIT – III: Laplace, Wave and Diffusion Equations with Applications (20 hours)**

Laplace's equation, Boundary value problems, Maximum and minimum principles, Uniqueness of solution and their continuous dependence on boundary data, Solution of the Dirichlet and Neumann problem for a half plane by Fourier transform method, Theory of Green's function for Laplace's equation in three dimension and application in solution of Dirichlet and Neumann problem for semi-infinite spaces; Wave equation, Helmholtz's first and second theorems, Theory of Green's function for wave equation and its applications; Diffusion equation, Solution of initial boundary value problems for diffusion equation, Green's function for diffusion equation and its applications.

**Essential Readings**

1. Myint-U, Tyn (1978). Ordinary Differential Equations. Elsevier, North-Holland, Inc.
2. Ross S. L. (2007). Differential Equations (2nd ed.) John Wiley & Sons. India.
3. Sneddon Ian N. (2006). Elements of Partial Differential Equations. Dover Publications.

**Suggestive Readings**

- Coddington, E. A. (2012). An Introduction to Ordinary Differential Equations. Dover Publications.
- Amaranath T. (2023). An Elementary Course in Partial Differential Equations (3rd ed.). Narosa Publishing House.
- McOwen, Robert C. (2003). Partial Differential Equations, Pearson Education.

**DISCIPLINE SPECIFIC ELECTIVE COURSE – 5(ii): DYNAMICAL SYSTEMS****CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Dynamical Systems	4	3	1	0	Class XII pass with Mathematics	Calculus, Differential Equations, Linear Algebra, Metric spaces

**Learning Objectives:** Primary objective of this course is to introduce:

- The fundamental concepts of dynamical systems and emphasize on its study through several applications.
- The concepts of the periodic points, hyperbolicity and chaos explained through examples.
- Symbolic dynamics which help to represent and understand various dynamical systems.

**Learning Outcomes:** This course will enable the students to:

- Understand and demonstrate the basic concepts of dynamical systems and properties.
- Obtain fixed points and discuss the stability of the dynamical system.
- Understand Sharkovsky's theorem, Schwarzian derivative and Devaney chaos.
- Gain command in understanding subshifts of finite type and Markov chain which eventually leads to various areas of dynamical systems.

## **SYLLABUS OF DSE-5(ii)**

### **UNIT – I: Orbits under Discrete Dynamical Systems (12 hours)**

Dynamical systems: Discrete and continuous, Population Models, Newton's Method; Discrete dynamical system: Definition, examples and orbits, Periodic and eventually periodic points, Stable and unstable sets, Phase portrait, Graphical analysis of one-dimensional maps; Hyperbolicity, A glimpse of bifurcations, Analysis of families of logistic maps.

### **UNIT – II: Introduction to Chaos (15 hours)**

Symbolic dynamics, Sequence space, Shift map, Itinerary map, Subshifts of finite type, Conjugacy and chaos, Sensitive dependence on initial conditions, Topological transitivity, Devaney chaos, Expansive homeomorphisms, Expansivity of interval and circle maps; Structural stability, Sharkovsky's theorem and examples, Schwarzian derivative; Period 3 case.

### **UNIT – III: More on Symbolic Dynamics (18 hours)**

Full shifts, Shift spaces, Languages, Higher block shifts and higher power shifts, Sliding block codes; Finite type constraints, Graphs and their shifts, Graph representations of shifts of finite type, Markov chain; Shadowing property and subshifts of finite type.

### **Essential Readings**

1. Aoki, N. and Hiraide, K. (1994). Topological Theory of Dynamical Systems: Recent Advances. Elsevier Science, North-Holland.
2. Devaney, Robert L. (2022). An Introduction to Chaotic Dynamical Systems (3rd ed.). CRC Press, Taylor & Francis Group.
3. Lind, Douglas and Marcus, Brian (2021). An Introduction to Symbolic Dynamics and Coding (2nd ed.). Cambridge University Press.

### **Suggestive Readings**

- Bruin, Henk (2022). Topological and Ergodic Theory of Symbolic Dynamics. Graduate Studies in Mathematics (228), American Mathematical Society.
- Martelli, Mario (1999). Introduction to Discrete Dynamical Systems and Chaos. John Wiley & Sons, Inc., New York.
- Robinson, Clark (1998). Dynamical Systems: Stability, Symbolic Dynamics, and Chaos (2nd ed.). CRC press.

## DISCIPLINE SPECIFIC ELECTIVE COURSE – 5(iii): FUNDAMENTALS OF TOPOLOGY

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Fundamentals of Topology	4	3	1	0	Class XII pass with Mathematics	Metric Spaces

**Learning Objectives:** The main objective of this course is to:

- Having in depth understanding of metric spaces and realizing strength of notions like path connectedness, countability axioms and theorems due to Tietze and Baire.
- Create Topological spaces fundamentals, naturally abstracting out from metric spaces.
- Study powerful notions like connectedness, compactness, product topology leading to major results like Tychonoff Theorem.

**Learning Outcomes:** This course will enable the students to:

- Realize beautiful transitions of some of the major notions and results from metric spaces to topological spaces wherein we do not have facility of distance.
- Appreciate possibility of continuous deformation of several spaces into known spaces through notions developed during the course work.
- Enhance ability to create examples and counter examples classifying various notions.
- Have better understanding of Euclidean spaces and its subspaces, infinite dimensional spaces, and several non-Euclidean spaces.
- Acquire a detailed elucidation of connectedness and compactness of topological spaces.

### SYLLABUS OF DSE-5(iii)

#### UNIT-I: Countability Axioms, Separability and Lindelöf Spaces (12 hours)

Review of the properties of metric spaces; Spaces of sequences of numbers, their convergence and completeness, Completion of a metric space; Local base and base, First and second axiom of countability, Separable and Lindelöf spaces.

#### UNIT-II: Baire Category Theorem and Localized Versions of Connectedness (12 hours)

Nowhere dense subsets, Category I and category II sets, Baire category theorem; Extension theorems; Tietze's Extension Theorem; Local connectedness, Arcwise connectedness; Totally bounded sets and its connection with completeness and compactness.

#### UNIT-III: Introduction to Topological Spaces (21 hours)

Topology; Basis and subbasis for a topology; Product and subspace topology; Closed sets, Closure, Interior and limit points of a set, Hausdorff spaces; Continuous functions, Homeomorphism; Product topology for an indexed family of spaces; Connectedness and Compactness.



### Essential Readings

1. Munkres James R. (2002). Topology (2nd ed.). Prentice Hall of India Pvt. Ltd.
2. Shirali Satish and Vasudeva, H. L. (2009). Metric Spaces. Springer. Indian Reprint 2019.

### Suggestive Readings

- Kumaresan, S. (2014). Topology of Metric Spaces (2nd ed.). Narosa Publishing House. Delhi.
- Searcoid, Mícheál Ó (2007). Metric Spaces. Springer-Verlag.
- Simmons, G. F. (2017). Introduction to Topology and Modern Analysis. McGraw Hill Education. Delhi.

## DISCIPLINE SPECIFIC ELECTIVE COURSE – 5(iv): INFORMATION THEORY AND CODING

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Information Theory and Coding	4	3	1	0	Class XII pass with Mathematics	Probability and Statistics, Linear Algebra

**Learning Objectives:** The main objective of this course is to:

- Define and comprehend the concepts of information and its relationship with uncertainty and entropy.
- Apply basic principles of probability theory to measure information content.
- Learn basic information inequalities and their applications.
- Introduce error-detecting and error-correcting codes.
- Learn various decoding techniques.
- Get exposure to linear codes and bounds on linear codes.

**Learning Outcomes:** This course will enable the students to:

- Understand information and entropy, and calculate various entropies.
- Apply mutual information, conditional entropy, and information-theoretic measures.
- Know about the detection and correction of errors while transmission.
- Understand and demonstrate encoding and decoding of linear codes, and gain knowledge about some bounds on linear codes.

### SYLLABUS OF DSE-5(iv)

#### UNIT – I: Concepts of Information Theory

(15 hours)

A measure of uncertainty, H function as a measure of uncertainty, Sources and binary sources, Measure of information for two-dimensional discrete finite probability schemes. Entropy,

Joint entropy and conditional entropy, Relative entropy and mutual information, Chain rules for entropy, Conditional relative entropy and conditional mutual information, A measure of mutual information.

## **UNIT – II: Information Inequality and Coding Theory (15 hours)**

Interpretation of Shannon's fundamental inequalities, Redundancy, Efficiency and channel capacity, Jensen's inequality and its characterizations, The log sum inequality and its applications. Introduction to error detecting and correcting codes, Maximum likelihood decoding, Hamming distance, Nearest neighbour/minimum distance decoding, Distance of a code, Main coding theory problems, Equivalence of codes, Sphere-packing bound, Perfect codes, Balanced block designs, Finite fields, The ISBN code.

## **UNIT – III: Linear Codes (15 hours)**

Introduction to vector space over finite fields, Linear codes, Bases for linear codes, Encoding and decoding with a linear code, Dual code, Generator and parity check matrices, Nearest neighbour decoding for linear codes, Syndrome decoding. Binary Hamming codes,  $q$ -ary Hamming codes.

### **Essential Readings**

1. Cover, Thomas M. and Thomas, Joy A. (2006). Elements of Information Theory (2nd ed.). Wiley India. Indian Reprint 2017.
2. Hill, Raymond. (1996). A First Course in Coding Theory. Oxford University Press.
3. Reza, Fazlollah M. (1961). An Introduction to Information Theory. Dover Publications Inc, New York. Reprint July 2022.

### **Suggestive Readings**

- Bose, R. (2016). Information Theory, Coding and Cryptography (3rd ed.). McGraw-Hill.
- Hamming, R. W. (1980). Coding and Information Theory, Prentice Hall, Englewood.
- Ling, S. and Xing, C. (2004). Coding Theory: A First Course. Cambridge University Press.
- Pless, V. (1998). Introduction to the Theory of Error-Correcting Codes. John-Wiley.
- Sloane, N. J. A. and MacWilliams, F. J. (2007). Theory of Error Correcting Codes. North-Holland Mathematical Library 16, North-Holland.

## **DISCIPLINE SPECIFIC ELECTIVE COURSE – 5(v): OPTIMIZATION**

### **CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
<b>Optimization</b>	<b>4</b>	<b>3</b>	<b>1</b>	<b>0</b>	<b>Class XII pass with Mathematics</b>	<b>Multivariate Calculus</b>

**Learning Objectives:** The main objective of this course is to introduce:

- Nonlinear optimization problems.
- Convex and generalized convex functions and their properties.
- Optimality and duality in nonlinear optimization.
- Methods to solve unconstrained optimization problems, quadratic and fractional programming problems with linear constraints.

**Learning Outcomes:** This course will enable the students to:

- Learn about the optimal solutions of nonlinear optimization problems.
- Understand and apply Karush-Kuhn-Tucker (KKT) necessary and sufficient optimality conditions for nonlinear optimization problems.
- Demonstrate and apply Lagrangian duality results, and techniques to solve certain classes of nonlinear optimization problems.

## **SYLLABUS OF DSE-5(v)**

### **UNIT – I: Nonlinear Optimization and Convex Functions (15 hours)**

Problem statement of a nonlinear optimization problem, Example of production inventory, Location facilities, Stochastic resource allocation, Convex sets, Convex functions, Epigraph and hypograph of a function, Differentiable convex function, Twice differentiable convex function, Minima of convex function, Quasiconvex functions, Psuedoconvex functions.

### **UNIT – II: Optimality and Duality Theory in Nonlinear Optimization (15 hours)**

Unconstrained problems: Necessary optimality conditions, Sufficient optimality conditions; Problems having inequality constraints: Fritz John optimality conditions, Karush-Kuhn-Tucker (KKT) necessary optimality conditions; Fritz John conditions, KKT necessary and sufficient optimality conditions for problems with inequality and equality constraints; Lagrangian dual problem, Weak duality theorem, Duality gap, Strong duality theorem.

### **UNIT – III: Numerical Methods to Solve Nonlinear Optimization Problems (15 hours)**

Descent property, Order of convergence, Global convergence, Steepest descent method, Newton's method, Wolfe's method for quadratic programming problem; Linear fractional programming problem and simplex algorithm.

### **Essential Readings**

1. Bazaraa, Mokhtar S., Sherali, Hanif D. & Shetty, C. M. (2006). Nonlinear Programming: Theory and Algorithms (3rd ed.). John Wiley & Sons. Wiley India (2017).
2. Chandra, Suresh, Jayadeva and Mehra, Aparna (2009). Numerical Optimization with Applications. Narosa Publishing House Pvt. Ltd. Delhi. Second Reprint 2016.

### **Suggestive Readings**

- Durea, Marius and Strugariu, Radu. (2014). An Introduction to Nonlinear Optimization Theory. de Gruyter Open.
- Eiselt, H. A. and Sandblom, Carl-Louis. (2019). Nonlinear Optimization: Methods and Applications. Springer Nature Switzerland.
- Luenberger, David, G. and Ye, Yinyu. (2021). Linear and Nonlinear Programming (5th ed.). Springer Nature Switzerland.

## DISCIPLINE SPECIFIC ELECTIVE COURSE – 5(vi): RESEARCH METHODOLOGY

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Research Methodology	4	3	0	1	Class XII pass with Mathematics	NIL

**Learning Objectives:** The main objective of this course is to:

- Prepare the students with skills needed for successful research in mathematics.
- Develop a basic understanding of how to pursue research in mathematics.
- Prepare students for professions other than teaching, that requires independent mathematical research, critical analysis, and advanced mathematical knowledge.
- Introduce some open source softwares to carry out mathematical research.
- Impart the knowledge of journals, their rankings and the disadvantages of rankings.

**Learning Outcomes:** The course will enable the students to:

- Develop researchable questions and to make them inquisitive enough to search and verify new mathematical facts.
- Understand the methods in research and carry out independent study in areas of mathematics.
- Write a basic mathematical article and a research project.
- Gain knowledge about publication of research articles in good journals.
- Communicate mathematical ideas both in oral and written forms effectively.

### SYLLABUS OF DSE - 5(vi)

#### UNIT– I: How to Learn, Write, and Research Mathematics (17 hours)

How to learn mathematics, How to write mathematics: Goals of mathematical writing, general principles of mathematical writing, avoiding errors, writing mathematical solutions and proofs, the revision process, What is mathematical research, finding a research topic, Literature survey, Research Criteria, Format of a research article (including examples of mathematical articles) and a research project (report), publishing research.

#### UNIT- II: Mathematical Typesetting and Presentation using LaTeX (16 hours)

How to present mathematics: Preparing a mathematical talk, Oral presentation, Use of technology which includes LaTeX, PSTricks and Beamer; Poster presentation.

#### UNIT- III: Mathematical Web Resources and Research Ethics (12 hours)

Web resources- MAA, AMS, SIAM, arXiv, ResearchGate; Journal metrics: Impact factor of journal as per JCR, MCQ, SNIP, SJR, Google Scholar metric; Challenges of journal metrics;

Reviews/Databases: MathSciNet, zbMath, Web of Science, Scopus; Ethics with respect to science and research, Plagiarism check using software like URKUND/Ouriginal by Turnitin.

### Essential Readings

1. Bindner, Donald, & Erickson Martin (2011). A Student's Guide to the Study, Practice, and Tools of Modern Mathematics. CRC Press, Taylor & Francis Group.
2. Committee on Publication Ethics- COPE (<https://publicationethics.org/>)
3. Declaration on Research Assessment.  
[https://en.wikipedia.org/wiki/San\\_Francisco\\_Declaration\\_on\\_Research\\_Assessment](https://en.wikipedia.org/wiki/San_Francisco_Declaration_on_Research_Assessment)
4. Evaluating Journals using journal metrics;  
(<https://academicguides.waldenu.edu/library/journalmetrics#s-lg-box-13497874>)
5. Gallian, Joseph A. (2006). Advice on Giving a Good PowerPoint Presentation (<https://www.d.umn.edu/~jgallian/goodPPTalk.pdf>). MATH HORIZONS.
6. Lamport, Leslie (2008). LaTeX, a Document Preparation System, Pearson.
7. Locharoenrat, Kitsakorn (2017). Research Methodologies for Beginners, Pan Stanford Publishing Pte. Ltd., Singapore.
8. Nicholas J. Higham. Handbook for writing for the Mathematical Sciences, SIAM, 1998.
9. Steenrod, Norman E., Halmos, Paul R., Schiffer, M. M., & Dieudonné, Jean A. (1973). How to Write Mathematics, American Mathematical Society.
10. Tantau, Till, Wright, Joseph, & Miletic, Vedran (2023). The BEAMER class, Use Guide for Version 3.69. TeX User Group.  
(<https://tug.ctan.org/macros/latex/contrib/beamer/doc/beameruserguide.pdf>)
11. University Grants Commission (Promotion of Academic Integrity and Prevention of Plagiarism in Higher Educational Institutions) Regulations 2018 (The Gazette of India: Extraordinary, Part-iii-Sec.4)

**Practical (30 hours):** Practical work to be performed in the computer lab of the following using any TeX distribution software:

1. Starting LaTeX, Preparing an input file, Sequences and paragraphs, Quotation marks, Dashes, Space after a period, Special symbols, Simple text- generating commands, Emphasizing text, Preventing line breaks, Footnotes, ignorable input.
2. The document, The document class, The title page, Sectioning, Displayed material, Quotations, Lists, Displayed formulas, Declarations.
3. Running LaTeX, Changing the type style, Accents, Symbols, Subscripts and superscripts, Fractions, Roots, Ellipsis.
4. Mathematical Symbols, Greek letters, Calligraphic letters, Log-like functions, Arrays, The array environment, Vertical alignment, Delimiters, Multiline formulas.
5. Putting one thing above another, Over and underlining, Accents, Stacking symbols, Spacing in math mode, Changing style in math mode, Type style, Math style.
6. Defining commands, Defining environments, Theorems.
7. Figure and tables, Marginal notes, The tabbing environment, The tabular environment.
8. The Table and contents, Cross-references, Bibliography and citation.
9. Beamer: Templates, Frames, Title page frame, Blocks, Simple overlays, Themes.
10. PSTricks
11. Demonstration of web resources.

**B.A. (Prog.) Semester-VII with Mathematics as Major**  
**Category-II**

**DISCIPLINE SPECIFIC CORE COURSE (DSC-7): NUMERICAL METHODS**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Numerical Methods	4	3	0	1	Class XII pass with Mathematics	Calculus, Real Analysis

**Learning Objectives:** The primary objective of this course is to introduce:

- Solutions of nonlinear equations in one variable by various methods.
- Interpolation and approximation, numerical differentiation, and integration.
- Direct methods for solving linear systems, numerical solution of ODE's.

**Learning Outcomes:** This course will enable the students to:

- Find the consequences of finite precision and the inherent limits of numerical methods.
- Appropriate numerical methods to solve algebraic and transcendental equations.
- Solve first order initial value problems of ODE's numerically using Euler methods.

**SYLLABUS OF DSC-7**

**UNIT-I: Errors and Roots of Transcendental and Polynomial Equations (12 hours)**

Errors: Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence, and terminal conditions; Bisection method, Secant method, Regula-Falsi method, Newton-Raphson method.

**UNIT-II: Algebraic Linear Systems and Interpolation (18 hours)**

Gaussian elimination method (with row pivoting); Iterative methods: Jacobi method, Gauss-Seidel method; Interpolation: Lagrange form, Newton form, Finite difference operators.

**UNIT-III: Numerical Differentiation, Integration and ODE (15 hours)**

Numerical differentiation: First and second order derivatives; Numerical integration: Trapezoidal rule, Simpson's rule; Ordinary differential equation: Euler's method.

**Essential Readings**

1. Chapra, Steven C. (2018). Applied Numerical Methods with MATLAB for Engineers and Scientists (4th ed.). McGraw-Hill Education.
2. Fausett, Laurene V. (2009). Applied Numerical Analysis Using MATLAB. Pearson. India.
3. Jain, M. K., Iyengar, S. R. K., & Jain R. K. (2012). Numerical Methods for Scientific and Engineering Computation (6th ed.). New Age International Publishers. Delhi.

### **Suggestive Reading**

- Bradie, Brian (2006). A Friendly Introduction to Numerical Analysis. Pearson Education India. Dorling Kindersley (India) Pvt. Ltd. Third Impression, 2011.

**Note:** Non programmable scientific calculator may be allowed in the University examination.

Practical / Lab work to be performed in Computer Lab: Use of computer algebra software (CAS), for example Mathematica/MATLAB/Maple/ Maxima/Scilab etc., for developing the following numerical programs:

1. Bisection method
2. Secant method and Regula–Falsi method
3. Newton-Raphson method
4. Gauss–Jacobi method and Gauss–Seidel method
5. Lagrange interpolation and Newton interpolation
6. Trapezoidal rule and Simpson’s rule
7. Euler’s method for solving first order initial value problems of ODE’s.

**DSE Courses of B.A. (Prog.) Semester-VII**  
**Category-II**

**DISCIPLINE SPECIFIC ELECTIVE COURSE – 3(i): ADVANCED LINEAR ALGEBRA**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Advanced Linear Algebra	4	3	1	0	Class XII pass with Mathematics	Linear Algebra

**Learning Objectives:** The objective of the course is to introduce:

- Linear functionals, dual basis and the dual (or transpose) of a linear transformation.
- Diagonalization problem and Jordan canonical form for linear operators or matrices using eigenvalues.
- Inner product, norm, Cauchy-Schwarz inequality, and orthogonality on real or complex vector spaces.
- The adjoint of a linear operator with application to least squares approximation and minimal solutions to linear system.
- Characterization of self-adjoint (or normal) operators on real (or complex) spaces in terms of orthonormal bases of eigenvectors and their corresponding eigenvalues.

**Learning Outcomes:** This course will enable the students to:

- Understand the notion of an inner product space in a general setting and how the notion of inner products can be used to define orthogonal vectors, including to the Gram-Schmidt process to generate an orthonormal set of vectors.
- Use eigenvectors and eigenspaces to determine the diagonalizability of a linear operator.
- Find the Jordan canonical form of matrices when they are not diagonalizable.
- Learn about normal, self-adjoint, and unitary operators and their properties, including the spectral decomposition of a linear operator.
- Find the singular value decomposition of a matrix.

**SYLLABUS OF DSE-3(i)**

**UNIT-I: Dual Spaces, Diagonalizable Operators and Canonical Forms (18 hours)**

The change of coordinate matrix; Dual spaces, Double dual, Dual basis, Transpose of a linear transformation and its matrix in the dual basis, Annihilators; Eigenvalues, eigenvectors, eigenspaces and the characteristic polynomial of a linear operator; Diagonalizability, Direct sum of subspaces, Invariant subspaces and the Cayley-Hamilton theorem; The Jordan canonical form and the minimal polynomial of a linear operator.



**UNIT-II: Inner Product Spaces and the Adjoint of a Linear Operator (12 hours)**

Inner products and norms, Orthonormal basis, Gram-Schmidt orthogonalization process, Orthogonal complements, Bessel's inequality; Adjoint of a linear operator with applications to least squares approximation and minimal solutions to systems of linear equations.

**UNIT-III: Class of Operators and Their Properties (15 hours)**

Normal, self-adjoint, unitary and orthogonal operators and their properties; Orthogonal projections and the spectral theorem; Singular value decomposition for matrices.

**Essential Reading**

1. Friedberg, Stephen H., Insel, Arnold J., & Spence, Lawrence E. (2019). Linear Algebra (5th ed.). Pearson Education India Reprint.

**Suggestive Readings**

- Hoffman, Kenneth, & Kunze, Ray Alden (1978). Linear Algebra (2nd ed.). Prentice Hall of India Pvt. Limited. Delhi. Pearson Education India Reprint, 2015.
- Lang, Serge (1987). Linear Algebra (3rd ed.). Springer.

**DISCIPLINE SPECIFIC ELECTIVE COURSE-3(ii): ELEMENTS OF METRIC SPACES****CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Metric Spaces	4	3	1	0	Class XII pass with Mathematics	Calculus, Real Analysis

**Learning Objectives:** The objective of the course is to introduce:

- The usual idea of distance into an abstract form on any set of objects, maintaining its inherent characteristics, and the resulting consequences.
- The two important topological properties, namely connectedness, and compactness of metric spaces with their characterizations.

**Learning Outcomes:** This course will enable the students to:

- Learn various natural and abstract formulations of distance on the sets of usual or unusual entities.
- Analyze how a theory advances from a particular frame to a general frame.
- Appreciate the mathematical understanding of various geometrical concepts, viz. balls or connected sets etc. in an abstract setting.

## SYLLABUS OF DSE-3(ii)

### UNIT-I: Topology of Metric Spaces

(18 hours)

Inequalities, Definition and examples, Sequences and Cauchy sequences, Complete metric space; Open and closed balls, Neighborhood, Open set, Interior of a set, Limit point of a set, Closed set, Closure of a set; Subspaces.

### UNIT-II: Continuity and Uniform Continuity in Metric Spaces

(15 hours)

Continuous mappings, Sequential criterion, and other characterizations of continuity; Uniform continuity; Homeomorphism, isometry, and equivalent metrics.

### UNIT-III: Connected and Compact Spaces

(12 hours)

Connected subsets of  $\mathbb{R}$ , Connectedness and continuous mappings; Compactness and boundedness, Characterizations of compactness, Continuous functions on compact spaces.

### Essential Reading

1. Shirali, Satish & Vasudeva, H. L. (2009). Metric Spaces. Springer. Indian Reprint 2019.

### Suggestive Reading

- Kumaresan, S. (2014). Topology of Metric Spaces (2nd ed.). Narosa Publishing House. New Delhi.

## DISCIPLINE SPECIFIC ELECTIVE COURSE-3(iii): MATHEMATICAL DATA SCIENCE

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mathematical Data Science	4	3	0	1	Class XII pass with Mathematics	Basic knowledge of R/Python, Probability and Statistics

**Learning Objectives:** The main objective of this course is to:

- Introduce various types of data and their sources, along with steps involved in data science case-study, including problems with data and their rectification and creation methods.
- Cover dimensionality reduction techniques, clustering algorithms and classification methods.

**Learning Outcomes:** The course will enable the students to:

- Gain a comprehensive understanding of data science, its mathematical foundations including practical applications of regression, principal component analysis, singular value decomposition, clustering, support vector machines, and  $k$ -NN classifiers.
- Demonstrate data analysis and exploration, linear regression techniques such as simple, multiple explanatory variables, cross-validation and regularization using R/Python.
- Use real-world datasets to practice dimensionality reduction techniques such as PCA, SVD, and multidimensional scaling using R/Python.

## **SYLLABUS OF DSE-3(iii)**

### **UNIT-I: Principles of Data Science (12 hours)**

Types of Data: nominal, ordinal, interval, and ratio; Steps involved in data science case-study: question, procurement, exploration, modeling, and presentation; Structured and unstructured data: streams, frames, series, survey results, scale and source of data – fixed, variable, high velocity, exact and implied/inferred; Overview of problems with data – dirty and missing data in tabular formats – CSV, data frames in R/Pandas, anomaly detection, assessing data quality, rectification and creation methods, data hygiene, meta-data for inline data-description-markups such as XML and JSON; Overview of other data-source formats – SQL, pdf, Yaml, HDF5, and Vaex.

### **Unit-II: Mathematical Foundations (15 hours)**

Model driven data in  $R^n$ , Log-likelihoods and MLE, Chebyshev, and Chernoff-Hoeffding inequalities with examples, Importance sampling; Norms in Vector Spaces– Euclidean, and metric choices; Types of distances: Manhattan, Hamming, Mahalanobis, Cosine and angular distances, KL divergence; Distances applied to sets– Jaccard, and edit distances; Modeling text with distances; Linear Regression: Simple, multiple explanatory variables, polynomial, cross-validation, regularized, Lasso, and matching pursuit; Gradient descent.

### **Unit-III: Dimensionality Reduction, Clustering and Classification (18 hours)**

Problem of dimensionality, Principal component analysis, Singular value decomposition (SVD), Best  $k$ -rank approximation of a matrix, Eigenvector and eigenvalues relation to SVD, Multidimensional scaling, Linear discriminant analysis; Clustering: Voronoi diagrams, Delaunay triangulation, Gonzalez's algorithm for  $k$ -center clustering, Lloyd's algorithm for  $k$ -means clustering, Mixture of Gaussians, Hierarchical clustering, Density-based clustering and outliers, Mean shift clustering; Classification: Linear classifiers, Perceptron algorithm, Kernels, Support vector machines, and  $k$ -nearest neighbors ( $k$ -NN) classifiers.

### **Essential Readings**

1. Mertz, David. (2021). Cleaning Data for Effective Data Science, Packt Publishing.
2. Ozdemir, Sinan. (2016). Principles of Data Science, Packt Publishing.
3. Phillips, Jeff M. (2021). Mathematical Foundations for Data Analysis, Springer. (<https://mathfordata.github.io/>).

### Suggestive Readings

- Frank Emmert-Streib, et al. (2022). Mathematical Foundations of Data Science Using R. (2nd ed.). De Gruyter Oldenbourg.
- Wes McKinney. (2022). Python for Data Analysis (3rd ed.). O'Reilly.
- Wickham, Hadley, et al. (2023). R for Data Science (2nd ed.). O'Reilly.

**Practical (30 hours)-** Practical work to be performed in Computer Lab using R/Python:

1. To explore different types data (nominal, ordinal, interval, ratio) and identify their properties.
2. To deal with dirty and missing data, such as imputation, deletion, and data normalization.
3. Use the real-world datasets (<https://data.gov.in/>) to demonstrate the following:
  - a) Data analysis and exploration, linear regression techniques such as simple, multiple explanatory variables, cross-validation, and regularization.
  - b) Dimensionality reduction techniques such as principal component analysis, singular value decomposition (SVD), and multidimensional scaling.
  - c) Clustering algorithms such as  $k$ -means, hierarchical, and density-based clustering and evaluate the quality of the clustering results.
  - d) Classification methods such as linear classifiers, support vector machines (SVM), and  $k$ -nearest neighbors ( $k$ -NN).

### DISCIPLINE SPECIFIC ELECTIVE COURSE-3(iv): INTEGRAL TRANSFORMS

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
<b>Integral Transforms</b>	<b>4</b>	<b>3</b>	<b>1</b>	<b>0</b>	<b>Class XII pass with Mathematics</b>	<b>Differential Equations, Elementary Mathematical Analysis</b>

**Learning Objectives:** Primary objective of this course is to introduce:

- The basic idea of integral transforms of functions and their applications through an introduction to Fourier series expansion of a periodic function.
- Fourier transform and Laplace transform of functions of a real variable with applications to solve ODE's and PDE's.

**Learning Outcomes:** The course will enable the students to:

- Understand the Fourier series associated with a periodic function, its convergence, and the Gibbs phenomenon.
- Compute Fourier and Laplace transforms of classes of functions.
- Apply techniques of Fourier and Laplace transforms to solve ordinary and partial differential equations and initial and boundary value problems.

## SYLLABUS OF DSE-3(iv)

### UNIT-I: Fourier Series and Integrals

(18 hours)

Piecewise continuous functions and periodic functions, Systems of orthogonal functions, Fourier series: Convergence, examples and applications of Fourier series, Fourier cosine series and Fourier sine series, The Gibbs phenomenon, Complex Fourier series, Fourier series on an arbitrary interval, The Riemann-Lebesgue lemma, Pointwise convergence, uniform convergence, differentiation, and integration of Fourier series; Fourier integrals.

### UNIT-II: Integral Transform Methods

(15 hours)

Fourier transforms, Properties of Fourier transforms, Convolution theorem of the Fourier transform, Fourier transforms of step and impulse functions, Fourier sine and cosine transforms, Convolution properties of Fourier transform; Laplace transforms, Properties of Laplace transforms, Convolution theorem and properties of the Laplace transform, Laplace transforms of the heaviside and Dirac delta functions.

### UNIT-III: Applications of Integral Transforms

(12 hours)

Finite Fourier transforms and applications, Applications of Fourier transform to ordinary and partial differential equations; Applications of Laplace transform to ordinary differential equations, partial differential equations, initial and boundary value problems.

### Essential Readings

1. Tyn Myint-U & Lokenath Debnath (2007). Linear Partial Differential Equations for Scientists and Engineers (4th ed.). Birkhauser. Indian Reprint.
2. Lokenath Debnath & Dambaru Bhatta (2015). Integral Transforms and Their Applications (3rd ed.). CRC Press Taylor & Francis Group.

### Suggestive Readings

- Baidyanath Patra (2018). An Introduction to Integral Transforms. CRC Press.
- Joel L. Schiff (1999). The Laplace Transform-Theory and Applications. Springer.
- Rajendra Bhatia (2003). Fourier Series (2nd ed.). Texts and Readings in Mathematics, Hindustan Book Agency, Delhi.
- Yitzhak Katznelson (2004). An Introduction to Harmonic Analysis (3rd ed.). Cambridge University Press.

## DISCIPLINE SPECIFIC ELECTIVE COURSE-3(v): RESEARCH METHODOLOGY

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Research Methodology	4	3	0	1	Class XII pass with Mathematics	NIL

**Learning Objectives:** The main objective of this course is to:

- Prepare the students with skills needed for successful research in mathematics.
- Develop a basic understanding of how to pursue research in mathematics.
- Prepare students for professions other than teaching, that requires independent mathematical research, critical analysis, and advanced mathematical knowledge.
- Introduce some open source softwares to carry out mathematical research.
- Impart the knowledge of journals, their rankings and the disadvantages of rankings.

**Learning Outcomes:** The course will enable the students to:

- Develop researchable questions and to make them inquisitive enough to search and verify new mathematical facts.
- Understand the methods in research and carry out independent study in areas of mathematics.
- Write a basic mathematical article and a research project.
- Gain knowledge about publication of research articles in good journals.
- Communicate mathematical ideas both in oral and written forms effectively.

### **SYLLABUS OF DSE-3(v)**

#### **UNIT– I: How to Learn, Write, and Research Mathematics (17 hours)**

How to learn mathematics, How to write mathematics: Goals of mathematical writing, general principles of mathematical writing, avoiding errors, writing mathematical solutions and proofs, the revision process, What is mathematical research, finding a research topic, Literature survey, Research Criteria, Format of a research article (including examples of mathematical articles) and a research project (report), publishing research.

#### **UNIT- II: Mathematical Typesetting and Presentation using LaTeX (16 hours)**

How to present mathematics: Preparing a mathematical talk, Oral presentation, Use of technology which includes LaTeX, PSTricks and Beamer; Poster presentation.

#### **UNIT- III: Mathematical Web Resources and Research Ethics (12 hours)**

Web resources- MAA, AMS, SIAM, arXiv, ResearchGate; Journal metrics: Impact factor of journal as per JCR, MCQ, SNIP, SJR, Google Scholar metric; Challenges of journal metrics; Reviews/Databases: MathSciNet, zbMath, Web of Science, Scopus; Ethics with respect to science and research, Plagiarism check using software like URKUND/Ouriginal by Turnitin.

### **Essential Readings**

1. Bindner, Donald, & Erickson Martin (2011). A Student's Guide to the Study, Practice, and Tools of Modern Mathematics. CRC Press, Taylor & Francis Group.
2. Committee on Publication Ethics- COPE (<https://publicationethics.org/>)
3. Declaration on Research Assessment.  
[https://en.wikipedia.org/wiki/San\\_Francisco\\_Declaration\\_on\\_Research\\_Assessment](https://en.wikipedia.org/wiki/San_Francisco_Declaration_on_Research_Assessment)
4. Evaluating Journals using journal metrics;  
(<https://academicguides.waldenu.edu/library/journalmetrics#s-lg-box-13497874>)

5. Gallian, Joseph A. (2006). Advice on Giving a Good PowerPoint Presentation (<https://www.d.umn.edu/~jgallian/goodPPTalk.pdf>). MATH HORIZONS.
6. Lamport, Leslie (2008). LaTeX, a Document Preparation System, Pearson.
7. Locharoenrat, Kitsakorn (2017). Research Methodologies for Beginners, Pan Stanford Publishing Pte. Ltd., Singapore.
8. Nicholas J. Higham. Handbook for writing for the Mathematical Sciences, SIAM, 1998.
9. Steenrod, Norman E., Halmos, Paul R., Schiffer, M. M., & Dieudonné, Jean A. (1973). How to Write Mathematics, American Mathematical Society.
10. Tantau, Till, Wright, Joseph, & Miletić, Vedran (2023). The BEAMER class, Use Guide for Version 3.69. TeX User Group. (<https://tug.ctan.org/macros/latex/contrib/beamer/doc/beameruserguide.pdf>)
11. University Grants Commission (Promotion of Academic Integrity and Prevention of Plagiarism in Higher Educational Institutions) Regulations 2018 (The Gazette of India: Extraordinary, Part-iii-Sec.4)

**Practical (30 hours):** Practical work to be performed in the computer lab of the following using any TeX distribution software:

1. Starting LaTeX, Preparing an input file, Sequences and paragraphs, Quotation marks, Dashes, Space after a period, Special symbols, Simple text- generating commands, Emphasizing text, Preventing line breaks, Footnotes, ignorable input.
2. The document, The document class, The title page, Sectioning, Displayed material, Quotations, Lists, Displayed formulas, Declarations.
3. Running LaTeX, Changing the type style, Accents, Symbols, Subscripts and superscripts, Fractions, Roots, Ellipsis.
4. Mathematical Symbols, Greek letters, Calligraphic letters, Log-like functions, Arrays, The array environment, Vertical alignment, Delimiters, Multiline formulas.
5. Putting one thing above another, Over and underlining, Accents, Stacking symbols, Spacing in math mode, Changing style in math mode, Type style, Math style.
6. Defining commands, Defining environments, Theorems.
7. Figure and tables, Marginal notes, The tabbing environment, The tabular environment.
8. The Table and contents, Cross-references, Bibliography and citation.
9. Beamer: Templates, Frames, Title page frame, Blocks, Simple overlays, Themes.
10. PSTricks
11. Demonstration of web resources.

**B.Sc. (Physical Sciences/Mathematical Sciences) Semester-VII**  
**Category-III**

**DISCIPLINE SPECIFIC CORE COURSE (DSC-7): NUMERICAL METHODS**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Numerical Methods	4	3	0	1	Class XII pass with Mathematics	Calculus, Real Analysis

**Learning Objectives:** The primary objective of this course is to introduce:

- Solutions of nonlinear equations in one variable by various methods.
- Interpolation and approximation, numerical differentiation, and integration.
- Direct methods for solving linear systems, numerical solution of ODE's.

**Learning Outcomes:** This course will enable the students to:

- Find the consequences of finite precision and the inherent limits of numerical methods.
- Appropriate numerical methods to solve algebraic and transcendental equations.
- Solve first order initial value problems of ODE's numerically using Euler methods.

**SYLLABUS OF DSC-7**

**UNIT-I: Errors and Roots of Transcendental and Polynomial Equations (12 hours)**

Errors: Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence, and terminal conditions; Bisection method, Secant method, Regula-Falsi method, Newton-Raphson method.

**UNIT-II: Algebraic Linear Systems and Interpolation (18 hours)**

Gaussian elimination method (with row pivoting); Iterative methods: Jacobi method, Gauss-Seidel method; Interpolation: Lagrange form, Newton form, Finite difference operators.

**UNIT-III: Numerical Differentiation, Integration and ODE (15 hours)**

Numerical differentiation: First and second order derivatives; Numerical integration: Trapezoidal rule, Simpson's rule; Ordinary differential equation: Euler's method.

**Essential Readings**

1. Chapra, Steven C. (2018). Applied Numerical Methods with MATLAB for Engineers and Scientists (4th ed.). McGraw-Hill Education.
2. Fausett, Laurene V. (2009). Applied Numerical Analysis Using MATLAB. Pearson. India.
3. Jain, M. K., Iyengar, S. R. K., & Jain R. K. (2012). Numerical Methods for Scientific and Engineering Computation (6th ed.). New Age International Publishers. Delhi.



### **Suggestive Reading**

- Bradie, Brian (2006). A Friendly Introduction to Numerical Analysis. Pearson Education India. Dorling Kindersley (India) Pvt. Ltd. Third Impression, 2011.

**Note:** Non programmable scientific calculator may be allowed in the University examination.

Practical / Lab work to be performed in Computer Lab: Use of computer algebra software (CAS), for example Mathematica/MATLAB/Maple/ Maxima/Scilab etc., for developing the following numerical programs:

1. Bisection method
2. Secant method and Regula-Falsi method
3. Newton-Raphson method
4. Gauss-Jacobi method and Gauss-Seidel method
5. Lagrange interpolation and Newton interpolation
6. Trapezoidal rule and Simpson's rule
7. Euler's method for solving first order initial value problems of ODE's.

## **DSE Courses of B.Sc. (Physical Sciences/Mathematical Sciences) Sem-VII**

### **Category-III**

#### **DISCIPLINE SPECIFIC ELECTIVE COURSE – 5(i): ADVANCED LINEAR ALGEBRA**

#### **CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Advanced Linear Algebra	4	3	1	0	Class XII pass with Mathematics	Linear Algebra

**Learning Objectives:** The objective of the course is to introduce:

- Linear functionals, dual basis and the dual (or transpose) of a linear transformation.
- Diagonalization problem and Jordan canonical form for linear operators or matrices using eigenvalues.
- Inner product, norm, Cauchy-Schwarz inequality, and orthogonality on real or complex vector spaces.
- The adjoint of a linear operator with application to least squares approximation and minimal solutions to linear system.
- Characterization of self-adjoint (or normal) operators on real (or complex) spaces in terms of orthonormal bases of eigenvectors and their corresponding eigenvalues.

**Learning Outcomes:** This course will enable the students to:

- Understand the notion of an inner product space in a general setting and how the notion of inner products can be used to define orthogonal vectors, including to the Gram-Schmidt process to generate an orthonormal set of vectors.
- Use eigenvectors and eigenspaces to determine the diagonalizability of a linear operator.
- Find the Jordan canonical form of matrices when they are not diagonalizable.
- Learn about normal, self-adjoint, and unitary operators and their properties, including the spectral decomposition of a linear operator.
- Find the singular value decomposition of a matrix.

#### **SYLLABUS OF DSE-5(i)**

#### **UNIT-I: Dual Spaces, Diagonalizable Operators and Canonical Forms (18 hours)**

The change of coordinate matrix; Dual spaces, Double dual, Dual basis, Transpose of a linear transformation and its matrix in the dual basis, Annihilators; Eigenvalues, eigenvectors, eigenspaces and the characteristic polynomial of a linear operator; Diagonalizability, Direct sum of subspaces, Invariant subspaces, and the Cayley-Hamilton theorem; The Jordan canonical form and the minimal polynomial of a linear operator.

**UNIT-II: Inner Product Spaces and the Adjoint of a Linear Operator (12 hours)**

Inner products and norms, Orthonormal basis, Gram-Schmidt orthogonalization process, Orthogonal complements, Bessel's inequality; Adjoint of a linear operator with applications to least squares approximation and minimal solutions to systems of linear equations.

**UNIT-III: Class of Operators and Their Properties (15 hours)**

Normal, self-adjoint, unitary and orthogonal operators and their properties; Orthogonal projections and the spectral theorem; Singular value decomposition for matrices.

**Essential Reading**

1. Friedberg, Stephen H., Insel, Arnold J., & Spence, Lawrence E. (2019). Linear Algebra (5th ed.). Pearson Education India Reprint.

**Suggestive Readings**

- Hoffman, Kenneth, & Kunze, Ray Alden (1978). Linear Algebra (2nd ed.). Prentice Hall of India Pvt. Limited. Delhi. Pearson Education India Reprint, 2015.
- Lang, Serge (1987). Linear Algebra (3rd ed.). Springer.

**DISCIPLINE SPECIFIC ELECTIVE COURSE-5(ii): ELEMENTS OF METRIC SPACES****CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Metric Spaces	4	3	1	0	Class XII pass with Mathematics	Calculus, Real Analysis

**Learning Objectives:** The objective of the course is to introduce:

- The usual idea of distance into an abstract form on any set of objects, maintaining its inherent characteristics, and the resulting consequences.
- The two important topological properties, namely connectedness, and compactness of metric spaces with their characterizations.

**Learning Outcomes:** This course will enable the students to:

- Learn various natural and abstract formulations of distance on the sets of usual or unusual entities.
- Analyse how a theory advances from a particular frame to a general frame.
- Appreciate the mathematical understanding of various geometrical concepts, viz. balls or connected sets etc. in an abstract setting.

## SYLLABUS OF DSE-5(ii)

### UNIT-I: Topology of Metric Spaces

(18 hours)

Inequalities, Definition and examples, Sequences and Cauchy sequences, Complete metric space; Open and closed balls, Neighborhood, Open set, Interior of a set, Limit point of a set, Closed set, Closure of a set; Subspaces.

### UNIT-II: Continuity and Uniform Continuity in Metric Spaces

(15 hours)

Continuous mappings, Sequential criterion, and other characterizations of continuity; Uniform continuity; Homeomorphism, isometry, and equivalent metrics.

### UNIT-III: Connected and Compact Spaces

(12 hours)

Connected subsets of  $\mathbb{R}$ , Connectedness and continuous mappings; Compactness and boundedness, Characterizations of compactness, Continuous functions on compact spaces.

### Essential Reading

1. Shirali, Satish & Vasudeva, H. L. (2009). Metric Spaces. Springer. Indian Reprint 2019.

### Suggestive Reading

- Kumaresan, S. (2014). Topology of Metric Spaces (2nd ed.). Narosa Publishing House. New Delhi.

## DISCIPLINE SPECIFIC ELECTIVE COURSE-5(iii): MATHEMATICAL DATA SCIENCE

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mathematical Data Science	4	3	0	1	Class XII pass with Mathematics	Basic knowledge of R/Python, Probability and Statistics

**Learning Objectives:** The main objective of this course is to:

- Introduce various types of data and their sources, along with steps involved in data science case-study, including problems with data and their rectification and creation methods.
- Cover dimensionality reduction techniques, clustering algorithms and classification methods.

**Learning Outcomes:** The course will enable the students to:

- Gain a comprehensive understanding of data science, its mathematical foundations including practical applications of regression, principal component analysis, singular value decomposition, clustering, support vector machines, and  $k$ -NN classifiers.
- Demonstrate data analysis and exploration, linear regression techniques such as simple, multiple explanatory variables, cross-validation and regularization using R/Python.
- Use real-world datasets to practice dimensionality reduction techniques such as PCA, SVD, and multidimensional scaling using R/Python.

## **SYLLABUS OF DSE-5(iii)**

### **UNIT-I: Principles of Data Science (12 hours)**

Types of Data: nominal, ordinal, interval, and ratio; Steps involved in data science case-study: question, procurement, exploration, modeling, and presentation; Structured and unstructured data: streams, frames, series, survey results, scale and source of data – fixed, variable, high velocity, exact and implied/inferred; Overview of problems with data – dirty and missing data in tabular formats – CSV, data frames in R/Pandas, anomaly detection, assessing data quality, rectification and creation methods, data hygiene, meta-data for inline data-description-markups such as XML and JSON; Overview of other data-source formats – SQL, pdf, Yaml, HDF5, and Vaex.

### **Unit-II: Mathematical Foundations (15 hours)**

Model driven data in  $R^n$ , Log-likelihoods and MLE, Chebyshev, and Chernoff-Hoeffding inequalities with examples, Importance sampling; Norms in Vector Spaces– Euclidean, and metric choices; Types of distances: Manhattan, Hamming, Mahalanobis, Cosine and angular distances, KL divergence; Distances applied to sets– Jaccard, and edit distances; Modeling text with distances; Linear Regression: Simple, multiple explanatory variables, polynomial, cross-validation, regularized, Lasso, and matching pursuit; Gradient descent.

### **Unit-III: Dimensionality Reduction, Clustering and Classification (18 hours)**

Problem of dimensionality, Principal component analysis, Singular value decomposition (SVD), Best  $k$ -rank approximation of a matrix, Eigenvector and eigenvalues relation to SVD, Multidimensional scaling, Linear discriminant analysis; Clustering: Voronoi diagrams, Delaunay triangulation, Gonzalez's algorithm for  $k$ -center clustering, Lloyd's algorithm for  $k$ -means clustering, Mixture of Gaussians, Hierarchical clustering, Density-based clustering and outliers, Mean shift clustering; Classification: Linear classifiers, Perceptron algorithm, Kernels, Support vector machines, and  $k$ -nearest neighbors ( $k$ -NN) classifiers.

### **Essential Readings**

1. Mertz, David. (2021). Cleaning Data for Effective Data Science, Packt Publishing.
2. Ozdemir, Sinan. (2016). Principles of Data Science, Packt Publishing.
3. Phillips, Jeff M. (2021). Mathematical Foundations for Data Analysis, Springer.  
(<https://mathfordata.github.io/>).

### Suggestive Readings

- Frank Emmert-Streib, et al. (2022). Mathematical Foundations of Data Science Using R. (2nd ed.). De Gruyter Oldenbourg.
- Wes McKinney. (2022). Python for Data Analysis (3rd ed.). O'Reilly.
- Wickham, Hadley, et al. (2023). R for Data Science (2nd ed.). O'Reilly.

**Practical (30 hours)-** Practical work to be performed in Computer Lab using R/Python:

1. To explore different types data (nominal, ordinal, interval, ratio) and identify their properties.
2. To deal with dirty and missing data, such as imputation, deletion, and data normalization.
3. Use the real-world datasets (<https://data.gov.in/>) to demonstrate the following:
  - e) Data analysis and exploration, linear regression techniques such as simple, multiple explanatory variables, cross-validation, and regularization.
  - f) Dimensionality reduction techniques such as principal component analysis, singular value decomposition (SVD), and multidimensional scaling.
  - g) Clustering algorithms such as  $k$ -means, hierarchical, and density-based clustering and evaluate the quality of the clustering results.
  - h) Classification methods such as linear classifiers, support vector machines (SVM), and  $k$ -nearest neighbors ( $k$ -NN).

### DISCIPLINE SPECIFIC ELECTIVE COURSE-5(iv): INTEGRAL TRANSFORMS

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Integral Transforms	4	3	1	0	Class XII pass with Mathematics	Differential Equations, Elementary Mathematical Analysis

**Learning Objectives:** Primary objective of this course is to introduce:

- The basic idea of integral transforms of functions and their applications through an introduction to Fourier series expansion of a periodic function.
- Fourier transform and Laplace transform of functions of a real variable with applications to solve ODE's and PDE's.

**Learning Outcomes:** The course will enable the students to:

- Understand the Fourier series associated with a periodic function, its convergence, and the Gibbs phenomenon.
- Compute Fourier and Laplace transforms of classes of functions.
- Apply techniques of Fourier and Laplace transforms to solve ordinary and partial differential equations and initial and boundary value problems.

## **SYLLABUS OF DSE-5(iv)**

### **UNIT-I: Fourier Series and Integrals**

**(18 hours)**

Piecewise continuous functions and periodic functions, Systems of orthogonal functions, Fourier series: Convergence, examples and applications of Fourier series, Fourier cosine series and Fourier sine series, The Gibbs phenomenon, Complex Fourier series, Fourier series on an arbitrary interval, The Riemann-Lebesgue lemma, Pointwise convergence, uniform convergence, differentiation, and integration of Fourier series; Fourier integrals.

### **UNIT-II: Integral Transform Methods**

**(15 hours)**

Fourier transforms, Properties of Fourier transforms, Convolution theorem of the Fourier transform, Fourier transforms of step and impulse functions, Fourier sine and cosine transforms, Convolution properties of Fourier transform; Laplace transforms, Properties of Laplace transforms, Convolution theorem and properties of the Laplace transform, Laplace transforms of the heaviside and Dirac delta functions.

### **UNIT-III: Applications of Integral Transforms**

**(12 hours)**

Finite Fourier transforms and applications, Applications of Fourier transform to ordinary and partial differential equations; Applications of Laplace transform to ordinary differential equations, partial differential equations, initial and boundary value problems.

### **Essential Readings**

1. Tyn Myint-U & Lokenath Debnath (2007). Linear Partial Differential Equations for Scientists and Engineers (4th ed.). Birkhauser. Indian Reprint.
2. Lokenath Debnath & Dambaru Bhatta (2015). Integral Transforms and Their Applications (3rd ed.). CRC Press Taylor & Francis Group.

### **Suggestive Readings**

- Baidyanath Patra (2018). An Introduction to Integral Transforms. CRC Press.
- Joel L. Schiff (1999). The Laplace Transform-Theory and Applications. Springer.
- Rajendra Bhatia (2003). Fourier Series (2nd ed.). Texts and Readings in Mathematics, Hindustan Book Agency, Delhi.
- Yitzhak Katznelson (2004). An Introduction to Harmonic Analysis (3rd ed.). Cambridge University Press.

## DISCIPLINE SPECIFIC ELECTIVE COURSE-5(v): RESEARCH METHODOLOGY

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Research Methodology	4	3	0	1	Class XII pass with Mathematics	NIL

**Learning Objectives:** The main objective of this course is to:

- Prepare the students with skills needed for successful research in mathematics.
- Develop a basic understanding of how to pursue research in mathematics.
- Prepare students for professions other than teaching, that requires independent mathematical research, critical analysis, and advanced mathematical knowledge.
- Introduce some open source softwares to carry out mathematical research.
- Impart the knowledge of journals, their rankings and the disadvantages of rankings.

**Learning Outcomes:** The course will enable the students to:

- Develop researchable questions and to make them inquisitive enough to search and verify new mathematical facts.
- Understand the methods in research and carry out independent study in areas of mathematics.
- Write a basic mathematical article and a research project.
- Gain knowledge about publication of research articles in good journals.
- Communicate mathematical ideas both in oral and written forms effectively.

### SYLLABUS OF DSE-5(v)

#### **UNIT– I: How to Learn, Write, and Research Mathematics (17 hours)**

How to learn mathematics, How to write mathematics: Goals of mathematical writing, general principles of mathematical writing, avoiding errors, writing mathematical solutions and proofs, the revision process, What is mathematical research, finding a research topic, Literature survey, Research Criteria, Format of a research article (including examples of mathematical articles) and a research project (report), publishing research.

#### **UNIT- II: Mathematical Typesetting and Presentation using LaTeX (16 hours)**

How to present mathematics: Preparing a mathematical talk, Oral presentation, Use of technology which includes LaTeX, PSTricks and Beamer; Poster presentation.

#### **UNIT- III: Mathematical Web Resources and Research Ethics (12 hours)**

Web resources- MAA, AMS, SIAM, arXiv, ResearchGate; Journal metrics: Impact factor of journal as per JCR, MCQ, SNIP, SJR, Google Scholar metric; Challenges of journal metrics;



Reviews/Databases: MathSciNet, zbMath, Web of Science, Scopus; Ethics with respect to science and research, Plagiarism check using software like URKUND/Ouriginal by Turnitin.

### Essential Readings

1. Bindner, Donald, & Erickson Martin (2011). A Student's Guide to the Study, Practice, and Tools of Modern Mathematics. CRC Press, Taylor & Francis Group.
2. Committee on Publication Ethics- COPE (<https://publicationethics.org/>)
3. Declaration on Research Assessment.  
[https://en.wikipedia.org/wiki/San\\_Francisco\\_Declaration\\_on\\_Research\\_Assessment](https://en.wikipedia.org/wiki/San_Francisco_Declaration_on_Research_Assessment)
4. Evaluating Journals using journal metrics;  
(<https://academicguides.waldenu.edu/library/journalmetrics#s-lg-box-13497874>)
5. Gallian, Joseph A. (2006). Advice on Giving a Good PowerPoint Presentation (<https://www.d.umn.edu/~jgallian/goodPPTalk.pdf>). MATH HORIZONS.
6. Lamport, Leslie (2008). LaTeX, a Document Preparation System, Pearson.
7. Locharoenrat, Kitsakorn (2017). Research Methodologies for Beginners, Pan Stanford Publishing Pte. Ltd., Singapore.
8. Nicholas J. Higham. Handbook for writing for the Mathematical Sciences, SIAM, 1998.
9. Steenrod, Norman E., Halmos, Paul R., Schiffer, M. M., & Dieudonné, Jean A. (1973). How to Write Mathematics, American Mathematical Society.
10. Tantau, Till, Wright, Joseph, & Miletić, Vedran (2023). The BEAMER class, Use Guide for Version 3.69. TeX User Group.  
(<https://tug.ctan.org/macros/latex/contrib/beamer/doc/beameruserguide.pdf>)
11. University Grants Commission (Promotion of Academic Integrity and Prevention of Plagiarism in Higher Educational Institutions) Regulations 2018 (The Gazette of India: Extraordinary, Part-iii-Sec.4)

**Practical (30 hours):** Practical work to be performed in the computer lab of the following using any TeX distribution software:

1. Starting LaTeX, Preparing an input file, Sequences and paragraphs, Quotation marks, Dashes, Space after a period, Special symbols, Simple text- generating commands, Emphasizing text, Preventing line breaks, Footnotes, ignorable input.
2. The document, The document class, The title page, Sectioning, Displayed material, Quotations, Lists, Displayed formulas, Declarations.
3. Running LaTeX, Changing the type style, Accents, Symbols, Subscripts and superscripts, Fractions, Roots, Ellipsis.
4. Mathematical Symbols, Greek letters, Calligraphic letters, Log-like functions, Arrays, The array environment, Vertical alignment, Delimiters, Multiline formulas.
5. Putting one thing above another, Over and underlining, Accents, Stacking symbols, Spacing in math mode, Changing style in math mode, Type style, Math style.
6. Defining commands, Defining environments, Theorems.
7. Figure and tables, Marginal notes, The tabbing environment, The tabular environment.
8. The Table and contents, Cross-references, Bibliography and citation.
9. Beamer: Templates, Frames, Title page frame, Blocks, Simple overlays, Themes.
10. PSTricks
11. Demonstration of web resources.

**COMMON POOL OF GENERIC ELECTIVES (GE) Semester-VII COURSES OFFERED  
BY DEPARTMENT OF MATHEMATICS**

**Category-IV**

**GENERIC ELECTIVES (GE-7(i)): APPLIED ALGEBRA**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Applied Algebra	4	3	1	0	Class XII pass with Mathematics	Linear Algebra, Abstract Algebra

**Learning Objectives:** The primary objective of this course is to:

- Introduce the applications of linear algebra in the field of science and arts.
- Develop the analytical and numerical skills to apply the algebraic concepts in real-life situations.
- Understand the identification numbers and different check digit schemes that can be used to reduce the errors during their transmission.

**Learning Outcomes:** This course will enable the students to:

- Understand the system of linear equations, matrices, and transformations in the fields of economics, science, engineering, and computer science.
- Apply the combinatorics and graph theory in scheduling and reliability theory.
- Learn about identification numbers and using check digits to check for errors after the identification number has been transmitted.

**SYLLABUS OF GE-7(i)**

**UNIT-I: Applications of Linear Algebra (15 hours)**

Applications of linear systems: Leontief input-output model in economics, Traffic flow, and diet problem; Applications to computer graphics, difference equations and Markov chains; Applications to linear models: Least-squares problems, and least-squares lines.

**UNIT-II: Latin Squares and Graph Models (12 hours)**

Latin squares, Table for a finite group as a Latin square, Latin squares as in design of experiments; Mathematical models for matching jobs, Spelling checker, Network reliability, Street surveillance, Scheduling meetings, Interval graph modeling and Influence model, Pitcher pouring puzzle.

**UNIT-III: Various Check Digit Schemes****(18 hours)**

Developing identification numbers, Types of identification numbers, Transmission errors, Check digits, Integer division, Modular arithmetic, US postal money orders, Airline ticket identification numbers, The Universal Product Code check digit scheme, ISBN check digit scheme, Creating Identification numbers, IBM scheme, Symmetry, Symmetry and Rigid motions, Verhoeff check digit scheme.

**Essential Readings**

1. David C. Lay, Steven R. Lay and Judi J. McDonald (2016). Linear Algebra and Its Applications (5th ed.). Pearson.
2. Tucker, Alan (2012). Applied Combinatorics (6th ed.). John Wiley & Sons, Inc.
3. Kirtland, Joseph (2001). Identification Numbers and Check Digit Schemes. Mathematical Association of America.

**Suggestive Readings**

- Andirilli, Stephen and Hecker, David (2016). Elementary Linear Algebra (5th ed.). Academic Press, Elsevier.
- Lidl, Rudolf and Pilz, Günter (1998). Applied Abstract Algebra (2nd ed.). Springer. Indian Reprint 2014.
- Strang, Gilbert (2016). Introduction to Linear Algebra (5th ed.). Wellesley-Cambridge.

**GENERIC ELECTIVES (GE-7(ii)): ELEMENTS OF METRIC SPACES****CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Metric Spaces	4	3	1	0	Class XII pass with Mathematics	Calculus, Real Analysis

**Learning Objectives:** The objective of the course is to introduce:

- The usual idea of distance into an abstract form on any set of objects, maintaining its inherent characteristics, and the resulting consequences.
- The two important topological properties, namely connectedness, and compactness of metric spaces with their characterizations.

**Learning Outcomes:** This course will enable the students to:

- Learn various natural and abstract formulations of distance on the sets of usual or unusual entities.
- Analyse how a theory advances from a particular frame to a general frame.
- Appreciate the mathematical understanding of various geometrical concepts, viz. balls or connected sets etc. in an abstract setting.

## SYLLABUS OF GE-7(ii)

### UNIT-I: Topology of Metric Spaces

(18 hours)

Inequalities, Definition and examples, Sequences and Cauchy sequences, Complete metric space; Open and closed balls, Neighborhood, Open set, Interior of a set, Limit point of a set, Closed set, Closure of a set; Subspaces.

### UNIT-II: Continuity and Uniform Continuity in Metric Spaces

(15 hours)

Continuous mappings, Sequential criterion, and other characterizations of continuity; Uniform continuity; Homeomorphism, isometry, and equivalent metrics.

### UNIT-III: Connected and Compact Spaces

(12 hours)

Connected subsets of  $\mathbb{R}$ , Connectedness and continuous mappings; Compactness and boundedness, Characterizations of compactness, Continuous functions on compact spaces.

### Essential Reading

1. Shirali, Satish & Vasudeva, H. L. (2009). Metric Spaces. Springer. Indian Reprint 2019.

### Suggestive Reading

- Kumaresan, S. (2014). Topology of Metric Spaces (2nd ed.). Narosa Publishing House.

## GENERIC ELECTIVES (GE-7(iii)): INTRODUCTION TO GRAPH THEORY

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Introduction to Graph Theory	4	3	1	0	Class XII pass with Mathematics	NIL

**Learning Objectives:** The primary objective of this course is to introduce:

- Problem-solving techniques using various concepts of graph theory.
- Various properties like planarity and chromaticity of graphs.
- Several applications of these concepts in solving practical problems.

**Learning Outcomes:** This course will enable the students to:

- Good familiarity with all initial notions of graph theory and related results and seeing them used for some real-life problems.
- Learning notion of trees and their enormous usefulness in various problems.
- Learning various algorithms and their applicability.
- Studying planar graphs, Euler theorem associated to such graphs and some useful applications like coloring of graphs.

## SYLLABUS OF GE-7(iii)

### UNIT-I: Graphs, Types of Graphs and Basic Properties (12 hours)

Graphs and their representation, Pseudographs, Subgraphs, Degree sequence, Euler's theorem, Isomorphism of graphs, Paths and circuits, Connected graphs, Euler trails and circuits, Hamiltonian paths and cycles, Adjacency matrix, Weighted graphs, Travelling salesman problem, Dijkstra's algorithm.

### UNIT-II: Directed Graphs and Applications, Trees (18 hours)

The Chinese postman problem; Digraphs, Bellman-Ford algorithm, Tournaments, Directed network, Scheduling problem; Trees and their properties, Spanning trees, Kruskal's algorithm, Prim's algorithm, Acyclic digraphs and Bellman's algorithm.

### UNIT-III: Planar Graphs, Graph Coloring and Network Flows (15 hours)

Planar graphs, Euler's formula, Kuratowski theorem, Graph coloring, Applications of graph coloring, Circuit testing and facilities design, Flows and cuts, Max flow-min cut theorem, Matchings, Hall's theorem.

#### Essential Reading

1. Goodaire, Edgar G., & Parmenter, Michael M. (2011). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint.

#### Suggestive Readings

- Bondy, J. A. & Murty, U.S.R. (2008), Graph Theory with Applications. Springer.
- Chartrand, Gary, & Zhang, P. (2012). A First Course in Graph Theory. Dover Publications.
- Diestel, R. (1997). Graph Theory (Graduate Texts in Mathematics). Springer Verlag.
- West, Douglas B. (2001). Introduction to graph theory (2nd ed.). Pearson India.

## GENERIC ELECTIVES (GE-7(iv)): TOPICS IN MULTIVARIATE CALCULUS

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Topics in Multivariate Calculus	4	3	1	0	Class XII pass with Mathematics	Calculus

**Learning Objectives:** The primary objective of this course is to introduce the:

- Extension of the studies of single variable differential and integral calculus to functions of two or more independent variables.
- Applications of multivariable calculus tools to physics, economics, and optimization.

- Geometry and visualisation of curves and surfaces in two dimensions (plane) and three dimensions (space).
- Techniques of integration to functions of two and three independent variables.

**Learning Outcomes:** This course will enable the students to:

- Learn the conceptual variations when advancing in calculus from one variable to multivariable discussion.
- Understand the maximization and minimization of multivariable functions subject to the given constraints on variables.
- Learn about inter-relationship amongst the line integral, double and triple integral formulations.
- Familiarize with Green's, Stokes' and Gauss divergence theorems.

## **SYLLABUS OF GE-7(iv)**

### **UNIT-I: Calculus of Functions of Several Variables (18 hours)**

Basic Concepts, Limits and Continuity, Tangent Planes, Partial Derivatives, Total Differential, Differentiability, Chain Rules, Directional Derivatives and the Gradient, Extrema of Functions of Two Variables, Method of Lagrange multipliers with one constraint.

### **UNIT-II: Double and Triple Integrals (15 hours)**

Double integration over rectangular and nonrectangular regions, Double integrals in polar co-ordinates, Triple integral over a parallelepiped and solid regions, Volume by triple integrals, Triple integration in cylindrical and spherical coordinates, Change of variables in double and triple integrals.

### **UNIT-III: Green's, Stokes' and Gauss Divergence Theorem (12 hours)**

Line integrals, Applications of line integrals: Mass and Work, Fundamental theorem for line integrals, Conservative vector fields, Green's theorem, Area as a line integral, Surface integrals, Stokes' theorem, Gauss divergence theorem.

### **Essential Reading**

1. Strauss, Monty J., Bradley, Gerald L., & Smith, Karl J. (2007). Calculus (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. (Pearson Education). Delhi. Indian Reprint 2011.

### **Suggestive Reading**

- Marsden, J. E., Tromba, A., & Weinstein, A. (2004). Basic Multivariable Calculus. Springer (SIE). First Indian Reprint.

## Syllabi of Semester - VIII based on UGCF - 2022

### DEPARTMENT OF MATHEMATICS

#### Category-I

#### **B.Sc. (Hons.) Mathematics, Semester-VIII**

#### **DISCIPLINE SPECIFIC CORE COURSE (DSC)– 20: FIELD THEORY AND GALOIS EXTENSION**

#### **CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Field Theory and Galois Extension	4	3	1	0	Class XII pass with Mathematics	Group Theory, Ring Theory

**Learning Objectives:** The objective of the course is to introduce:

- Tools of field theory such as field extensions, splitting fields, normal extensions, separability, and separable extensions.
- Galois extensions and the Fundamental theorem of Galois theory.
- Link between group theory and the roots of polynomials, developed by Galois, to solve the problem of solvability of polynomial equations by radicals.
- Some applications, such as cyclotomic polynomials, finite fields, and simple extensions.

**Learning Outcomes:** This course will enable the students to:

- Identify and construct examples of fields, distinguish between algebraic and transcendental extensions, and characterize normal extensions in terms of splitting fields.
- Identify and characterize separable extensions, define Galois extensions, construct examples of automorphism groups of a field as well as prove the fundamental theorem of Galois theory.
- Use the Galois theory of equations to prove that a polynomial equation over a field is solvable by radicals if and only if its Galois group is solvable and hence deduce that a general quintic equation is not solvable by radicals.
- Define cyclotomic polynomials and find its Galois group using roots of unity, classify finite fields and prove that every finite separable extension is simple.

#### **SYLLABUS OF DSC-20**

##### **UNIT – I: Field Extensions**

**(15 hours)**

Fields and prime subfields, Field extensions, Degree of field extensions, Tower theorem, Algebraic and transcendental elements, Algebraic and transcendental extensions, Monomorphism of field extensions, Ruler and compass constructions, Splitting fields, Extensions of monomorphisms, Uniqueness of splitting field.

## **UNIT – II: Galois Extensions and the Fundamental Theorem (15 hours)**

Normal extensions, Separability and separable extensions, Monomorphisms and automorphisms of field extension, Galois extensions, Automorphism/Galois groups and fixed fields, Galois theory of polynomials, The fundamental theorem of Galois theory.

## **UNIT – III: Some Applications and Solvability by Radicals (15 hours)**

The Discriminant, Cyclotomic polynomials, extensions and its Galois group, Solution by radicals, Existence and Uniqueness of finite fields, Simple extensions, and the primitive element theorem.

### **Essential Readings**

1. Garling, D. J. H. (2021). Galois Theory and Its Algebraic Background (2nd ed.). Cambridge University Press.
2. Dummit, David S., and Foote, Richard M. (2011). Abstract Algebra (3rd ed.). Wiley.

### **Suggestive Readings**

- Stewart, Ian (2022). Galois Theory (5th ed.). CRC Press. Chapman and Hall.
- Cox, David A. (2012). Galois Theory (2nd ed.). John Wiley & Sons.
- Cohn, P. M. (2003). Basic Algebra, Springer International Edition.



## DSE Courses of B.Sc. (Hons) Mathematics, Semester -VIII

### DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(i): ADVANCED MECHANICS

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Advanced Mechanics	4	3	1	0	Class XII pass with Mathematics	Calculus, Differential Equations, Mechanics

**Learning Objectives:** The main objective of this course is to:

- Provide the students an enriching experience of the basic concepts of mechanics in space and its related concepts.
- Impart quality understanding to the students about Newtonian, Lagrangian and Hamiltonian mechanics along with practical applications of these concepts in real life.
- Understand the concept of fluid, their classifications, model, and approaches to study the fluid flow.

**Learning Outcomes:** This course will equip the students with the:

- Fundamental concepts of force systems, generalized coordinates, kinematics of a particle and a rigid body.
- Thorough and in depth understanding of the classification of dynamical systems, Lagrangian and Hamiltonian's equations.
- Formulation of mass and momentum conservation principle; solution for non-viscous flow, the motion of sphere, cylinder, and two-dimensional flow.
- Understanding of the concepts of stress and strain in viscous flow; derivation of Navier-Stokes equation of motion and related problems.

#### SYLLABUS OF DSE-6(i)

##### UNIT – I: Newtonian Mechanics (15 hours)

General force systems, Equilibrium of a system of particles, Reduction of a force systems, Equilibrium of a rigid body, Generalized coordinates and constraints, Work and potential energy, Kinematics of a particle and a rigid body. Moments and product of inertia. Kinetic energy and angular momentum, Motion of a particle and a system, Moving frame of reference, Motion of a rigid body.

##### UNIT – II: Lagrangian and Hamiltonian Mechanics (12 hours)

Lagrange's equations for a particle in plane, Classification of dynamical systems, Lagrange's equations for any simple dynamical system, general dynamical system and for impulsive motion; Applications of Lagrange's equations, Hamiltonian and the Canonical equations of motion, The passage from the Hamiltonian to the Lagrangian, Conservative systems.

### UNIT – III: Fluid Mechanics

(18 hours)

Classification of fluids, Continuum model, Eulerian and Lagrangian approach of description, Differentiation following the fluid motion, Velocity of a fluid particle, Irrotational flow, Velocity potential, Equipotential surfaces, Streamlines and Pathlines, Mass flux density, Conservation of mass leading to equation of continuity, Boundary surface; Forces in fluid flows, Conservation of linear momentum and its mathematical formulation (Euler's equation of motion), Bernoulli's equation, Axi-symmetric flows and motion of sphere; Two-dimensional flows, Motion of cylinder, Stream function, Complex potential, Line sources and line sinks, Line doublet, Milne-Thomson circle theorem; Viscous flow, Stress components in a real fluid, Stress and strain analysis, Navier-Stokes equations of motion and its applications.

### Essential Readings

1. Chorlton, F. (2005). Textbook of Fluid Dynamics. CBS Publishers, Delhi. Reprint 2018.
2. Synge, J. L. and Griffith, B. A. (2017). Principles of Mechanics (3rd ed.). McGraw-Hill Education. Indian Reprint.

### Suggestive Readings

- Gantmacher, F. (1975). Lectures in Analytic Mechanics. MIR publisher, Moscow.
- Goldstein, H., Poole, C.P. and Safco, J.L. (2002). Classical Mechanics. (3rd ed.). Addison Wesley.
- Kundu, Piyush K. and Cohen, Ira M., Dowling, David R. (2016). Fluid Mechanics (6th ed.). Academic Press.
- Mitchell, John W. (2020). Fox and McDonald's Introduction to Fluid Mechanics. (10th ed.). John Wiley & Sons.
- Taylor, John R. (2005). Classical Mechanics. University Science Books.

### DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(ii): CRYPTOGRAPHY

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Cryptography	4	3	1	0	Class XII pass with Mathematics	Group Theory, Linear Algebra

**Learning Objectives:** Primary objective of this course is to:

- Learn challenges and types of attacks on the security of cryptographic protocols.
- Understand concept of confusion and diffusion, that is central to the security of symmetric key cryptography.
- Learn mathematical hard problems, which can be used to build various public key cryptosystems.
- Gain knowledge of post quantum cryptography that resist quantum attacks.

**Learning Outcomes:** This course will enable the students to:

- Learn classical cryptosystems Caesar cipher, Monoalphabetic cipher, Hill cipher, Vigenère cipher and their security analysis.
- Understand Feistel cipher structure to achieve confusion and diffusion in case of Data Encryption Standard (DES).
- Understand Advanced Encryption Standard (AES) structure and its operations along with key generation.
- Learn key sharing protocol – Diffie Hellman key exchange, Public-key cryptosystems – RSA, Elgamal, and Elliptic curve cryptography.
- Learn Lagrange interpolation secret sharing scheme.
- Learn hash functions and their applications, digital signatures scheme.
- Gain knowledge of code-based cryptography – McEliece cryptosystem.

## **SYLLABUS OF DSE-6(ii)**

### **UNIT–I: Classical Cryptosystems and Review of Finite Fields (15 hours)**

Overview of Cryptography, Symmetric key and Public-key cryptography, Security attacks, Relation between key length and security, Objectives and applications of cryptography primitives, Types of attacks from cryptanalyst view, Kerckhoff's principle; Substitution techniques - Caesar cipher, Monoalphabetic cipher, Hill cipher, Vigenère cipher, One-time pad; Euclidean Algorithm, Modular Arithmetic, Statement of Fermat's, Euler's and Chinese Remainder theorems, Discrete logarithm, Finite fields of the form  $GF(p)$  and  $GF(2^n)$ , Binary and ASCII representation, Pseudo-random bit generation.

### **UNIT – II: Modern Block Ciphers (12 hours)**

Introduction to stream and block ciphers, Diffusion and Confusion, The Feistel cipher Structure, Data Encryption Standard (DES); Advanced Encryption Standard (AES) Structure, AES transformation functions, Key expansion, AES Example.

### **UNIT – III: Public-key Cryptography, Hash Functions, Digital Signatures and Post Quantum Cryptography (18 hours)**

Introduction to Public key cryptography, RSA cryptosystem, Diffie Hellman key exchange, Man in the middle attack, Elgamal cryptosystem, Elliptic curve arithmetic, Elliptic curve cryptography, Secret sharing; Hash functions, Applications of hash functions – MAC and digital signature, Simple Hash functions, Security requirements of Hash functions, Properties of SHA family of hash functions; Digital signatures, Elgamal and Schnorr digital signature scheme; Introduction to post quantum cryptography, Linear codes, Generating matrix, Parity check matrix, McEliece cryptosystem.

## **Essential Readings**

1. Stallings, William (2023). Cryptography and Network Security, Principles and Practice (8th ed.). Pearson Education Limited. Global Edition.
2. Stinson, Douglas R. and Paterson, Maura, B. (2019). Cryptography: Theory and Practice (4th ed.). CRC Press.
3. Trappe, Wade and Washington, Lawrence C. (2020). Introduction to Cryptography with Coding Theory (3rd ed.). Pearson Education International.

### Suggestive Readings

- Hoffstein, Jeffrey. Pipher, Jill & Silverman, Joseph H. (2014). An Introduction to Mathematical Cryptography (2nd ed.). Springer New York.
- Goldreich O. (2005). Foundations of Cryptography: Basic tools - Vol.1, Cambridge University Press.
- Goldreich O. (2009). Foundations of Cryptography: Vol.2, Basic applications, Cambridge University Press.

### DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(iii): INDUSTRIAL MATHEMATICS

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Industrial Mathematics	4	3	0	1	Class XII pass with Mathematics	Calculus, Real Analysis, Linear Algebra, Ordinary and Partial Differential Equations

**Learning Objectives:** The main objective of this course is to:

- Orient the learners to understand nature and working of industrial systems and their models.
- Familiarize the learners with control and maneuvering of industrial processes through sample case-studies and encourage design-thinking and understanding.

**Learning Outcomes:** This course will enable the students to:

- Determine the controllability, stability, and observability of a system from the model description.
- Comprehend the signal processing landscape and analyse signals using real and spatial domain representations.
- Model/analyse an industrial system from its description and use mathematical formulations to investigate and manipulate the system for specific objectives.

### SYLLABUS OF DSE-6(iii)

#### UNIT – I: Understanding Systems from their Mathematical Description (15 hours)

Continuous-time linear systems, Laplace transform, Transfer function and analogous systems, State-space models, Block-diagram algebra, Signal flow graph, Order of a system and reduced-order models; Discrete-time systems, Z-transform and its inverse, Feedback systems, Stability: Routh-Hurwitz criterion, Root locus method, Controllability and Observability.

#### UNIT – II: Mathematical Tools for Signals (15 hours)

Signal-to-noise ratio, Analog and digital messages, Channel bandwidth and rate of communication, Modulation, Randomness and redundancy; Signal energy and power, Period and aperiodic signals, Signal operations, Unit impulse function, Vector representation of signals, Orthogonality, Correlation of signals, Signal representation by orthogonal signal sets.

**UNIT – III: Case Studies****(15 hours)**

Sample Cases: Continuous casting, Water filtration, Factory fires, Irrigation.

**Essential Readings**

1. Fulford, Glenn R., and Broadbridge, Philip (2002). Industrial Mathematics: Case Studies. Cambridge University Press.
2. Kheir, Naim A. (Ed.). (1996). Systems Modeling and Computer Simulation, CRC Press.
3. Lathi, B.P., and Ding, Zhi (2019). Modern Digital and Analog Communication Systems (5th ed.). Oxford University Press.

**Suggestive Readings**

- Friedman A., and Littman W. (1994). Industrial Mathematics: A Course in Solving Real-World Problems. SIAM (Society for Industrial and Applied Mathematics).
- Kreyszig, Erwin (2011). Advance Engineering Mathematics (10th ed.). John Wiley & Sons.
- MacClauer, Charles R. (2000). Industrial Mathematics: Modeling in Industry, Science, and Government. Prentice Hall, Inc.

**Practical (30 hours)- Practical/Lab work using:**

Mathematica/MATLAB/SciLab/C/C++/Python/R/FORTRAN or similar as per availability.

1. Use following methods to study, describe, and evaluate continuous/discrete systems:
  - (a) Root locus method.
  - (b) Routh-Horowitz criterion.
  - (c) Transfer function using Laplace transform.
  - (d) z-transform to convert continuous systems to equivalent discrete systems.
2. To apply controllability and observability analysis on a system description, using corresponding tools/libraries available.
3. To represent a signal/wave as vector data (sampling, choosing basis, and checking orthogonality).
4. To convolve and deconvolve signal/wave functions and represent the result as graphs.

**Case Studies:**

Besides reading the mentioned case-studies, ONE case may be chosen (in consultation with the instructor) as Semester Assignment for a brief similar study and analysis.

**DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(iv):  
GEOMETRY OF CURVES AND SURFACES**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Geometry of Curves and Surfaces	4	3	1	0	Class XII pass with Mathematics	Calculus

**Learning Objectives:** The main objective of this course is to:

- Introduces the concept of curves and surfaces in Euclidean spaces  $\mathbb{R}^n$ .
- Study of the curves and surfaces via the tools of calculus and introduction of concepts like first and second fundamental forms, curvatures, and differential forms.
- Complete the celebrated Gauss-Bonnet theorem that establishes a connection between curvature of a geometric object with its topology.

**Learning Outcomes:** This course will enable the students to:

- Understand the concept of curves and surfaces embedded in the Euclidean spaces  $\mathbb{R}^n$ .
- Compute the curvature and torsion for a curve in the space.
- Understand the concept of differential forms and their integration.
- Make sense of the infinitesimal distance element via the study of the Riemannian metric.
- Get prepared to venture into further study of modern differential geometry of manifolds.

## **SYLLABUS OF DSE-6(iv)**

### **UNIT – I: Geometry of Curves**

**(15 hours)**

Concept of plane and space curves with examples, Parametrized plane and space-curves, Concepts of curvatures for curves, Frenet-Serret's formula for space curves, Global theorems for plane and space curves.

### **UNIT – II: Local Theory of Surfaces in the Space**

**(15 hours)**

Concept of surfaces in the space with examples, Fundamental forms and curvatures with examples, Orthonormal frames, Exterior differential forms in two variables and their uses.

### **UNIT – III: Geometry of Surfaces**

**(15 hours)**

Riemannian metric on a surface, Vector fields, Covariant derivatives, Concept of geodesic, Integration of exterior differential forms, Gauss-Bonnet theorem.

## **Essential Reading**

1. Kobayashi, Shoshichi (2019). Differential Geometry of Curves and Surfaces. Springer Nature Singapore Pte Ltd. <https://doi.org/10.1007/978-981-15-1739-6>

## **Suggestive Readings**

- Abbena Elsa, Salamon Simon, and Gray Alfred (2006). Modern Differential Geometry of Curves and Surfaces with Mathematica (3rd ed.). CRC Press.
- Carmo, Manfredo P. Do (2016). Differential Geometry of Curves and Surfaces (Revised and Updated Second Edition). Dover Publications.
- Pressley, Andrew (2010). Elementary Differential Geometry (2nd ed.). Springer-Verlag.
- Tapp, Kristopher (2016). Differential Geometry of Curves and Surfaces. Springer.

**DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(v): INTEGRAL EQUATIONS AND CALCULUS OF VARIATIONS**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
<b>Integral Equations and Calculus of Variations</b>	<b>4</b>	<b>3</b>	<b>1</b>	<b>0</b>	<b>Class XII pass with Mathematics</b>	<b>Ordinary, and Partial Differential Equations, Multivariate Calculus</b>

**Learning Objectives:** The main objective of this course is to:

- Familiarize the learner with methods for solving Volterra and Fredholm integral equations.
- Know the determination of extremum of functional, necessary condition for an extremum, Euler's equation, and its generalization.

**Learning Outcomes:** This course will enable the students to:

- Compute the solutions to Volterra integral equations by method of resolvent kernel, method of successive approximations, method of Laplace transform, system of Volterra integral equations and integro-differential equation.
- Determine the solutions of Fredholm integral equations and derivation of Hilbert-Schmidt theorem.
- Understand the formulation of variational problems, the variation of a functional and its properties, extremum of functional, necessary condition for an extremum.

**SYLLABUS OF DSE-6(v)**

**UNIT – I: Volterra Integral Equations (12 hours)**

Integral equations, Introduction and relation with linear differential equations; Volterra integral equations and its solutions, Method of resolvent kernel, Method of successive approximations, Convolution type of equation, Method of Laplace transform, System of Volterra integral equations, Integro-differential equation, Abel's integral equation and its generalizations.

**UNIT – II: Fredholm Integral Equations (18 hours)**

Fredholm integral equations and its solutions, Method of resolvent kernels, Method of successive approximations, Integral equations with degenerate kernels, Eigenvalues and eigen functions and their properties, Hilbert-Schmidt theorem, Nonhomogeneous Fredholm integral equation with symmetric kernel, Fredholm alternative.

**UNIT – III: Calculus of Variations****(15 hours)**

Variational problems, Variation of a functional and its properties, Extremum of functional, Necessary condition for an extremum, Euler's equation and its generalization, Variational derivative, General variation of a functional and variable end point problem, Sufficient conditions for the extremum of a functional.

**Essential Readings**

1. Gelfand, I. M. and Fomin, S.V. (2000). Calculus of Variations. Dover Publications, Inc.
2. Krasnov, M., Kiselev, A. and Makarenko, G. (1971). Problems and Exercises Integral Equations, Mir Publication Moscow.
3. Logan, J. David (1987). Applied Mathematics: A Contemporary Approach, John Wiley & Sons, Inc.

**Suggestive Readings**

- Hildebrand, F. B. (1992). Methods of Applied Mathematics (2nd ed.). Dover Publications.
- Zemyan, Stephen M. (2012). The Classical Theory of Integral Equations: A Concise Treatment. Birkhäuser.

**DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(vi):  
MACHINE LEARNING: A MATHEMATICAL APPROACH**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
<b>Machine Learning: A Mathematical Approach</b>	<b>4</b>	<b>3</b>	<b>0</b>	<b>1</b>	<b>Class XII pass with Mathematics</b>	<b>Basic Knowledge of Python</b>

**Learning Objectives:** The main objective of this course is to:

- Gain mathematical insights into the functioning of popular methods of Regression, Classification, Clustering and Dimension reduction.
- Understand the mathematical framework of learning and apply it to assess the performance of a number of regression, classification and density estimation algorithms
- Detect overfitting and employ regularization techniques to control it.

**Learning Outcomes:** This course will enable the students to:

- Learn how to build popular models of regression and classification including Linear regression, Polynomial regression, Logistic classifier, Support vector machine, Decision Tree, Random forests, Naïve Bayes classifier.
- Evaluate the performance of models on test data through analytical techniques (VC bounds and dimension) and Cross-validation to facilitate model selection and feature selection.
- Improve model performance by controlling overfitting through regularization techniques like Ridge and Lasso.



- Understand when to apply dimension reduction and combine it with other supervised learning methods.
- Understand and implement the key principles of Artificial Neural Networks in the context of regression and classification and employ them in function approximation.

## **SYLLABUS OF DSE-6(vi)**

### **UNIT – I: Introduction to Machine Learning and its Applications (18 hours)**

Overview of different tasks: Regression, Classification, and Clustering. Evaluation metrics– Mean Absolute error (MAE), Mean Squared Error (MSE), Root Mean Squared Error (RMSE). Linear Regression, Cost function, Polynomial Regression, Gradient Descent Algorithm (GDA). Logistic Regression: Evaluation metrics - accuracy, precision, recall, confusion matrix, Receiver Operating Characteristic Curve (ROC curve) and Area Under ROC Curve (AUC), Vapnik-Chervonenkis (VC) dimension, VC bounds (only statement).  $k$ -fold validation, Concepts of training set, validation set and test set, Underfitting-Overfitting, Regularization techniques– Ridge, Lasso for Linear Regression and Logistic Regression, Bias-variance tradeoff.

### **UNIT – II: Popular Machine Learning Techniques (18 hours)**

Cross-entropy and Gini Index, Decision Tree, Regression Tree, Random Forest and Bagging, Tree Pruning. Support Vector Machine (SVM), Kernel SVM (Gaussian) Similarity Criterion,  $k$ -Means clustering technique. Naive Bayes classifier- Linear Discriminant Analysis (LDA), Quadratic Discriminant Analysis (QDA). Dimensionality Reduction, Feature Selection, Principal Component Analysis (PCA).

### **UNIT – III: Introduction to Deep Learning (9 hours)**

Artificial neural network (ANN), Activation functions – definition and examples (Sigmoid, ReLU, Tanh), neurons, layers, Cost function, Information passing, Back propagation algorithm, Optimizers, Learning rate, Statement of Universal Approximation Theorem for continuous functions, Regularization with ANN, Normalization.

### **Essential Readings**

1. Abu-Mostafa, Y. S., Magdon-Ismael, M. & Lin, H.-T. (2012), Learning from Data, AML Book.
2. James, Gareth., Witten, D., Hastie, T., Tibshirani, R. and Taylor, J. (2023), An Introduction to Statistical Learning: with Applications in Python, Springer Nature Switzerland.
3. Ovidiu Calin, Springer. (2020). Deep Learning Architectures: A Mathematical Approach, Springer Nature Switzerland.

### **Suggestive Readings**

- Deisenroth, M. P., Faisal, A. A., and Ong, C. S. (2020), Mathematics for Machine Learning, Cambridge University Press.
- Shalev-Shwartz, S., and Ben-David, S. (2014), Understanding Machine Learning - From Theory to Algorithms, Cambridge University Press.
- Phillips, Jeff. (2020), Mathematical Foundations for Data analysis, Springer.
- Goodfellow, I., Bengio, Y., and Courville, A. (2016), Deep Learning, MIT Press.
- Hastie, T.; Tibshirani, R., and Friedman, J. (2001), The Elements of Statistical Learning, Springer New York Inc.

**Practical (30 hours)- Practical work to be performed in computer lab using Python.**

Following lab exercises should be done for at least two classification problems or two regression problems or both, whenever applicable.

Following datasets can be used for classification problems

- [https://scikit-learn.org/stable/datasets/toy\\_dataset.html](https://scikit-learn.org/stable/datasets/toy_dataset.html)  
(Toy datasets, Iris plants dataset, handwritten digits dataset, Wine recognition dataset, Breast cancer diagnostic dataset)
- <https://pypi.org/project/ISLP/>  
(Smarket dataset)

Following datasets can be used for regression problems

- <https://scikit-learn.org/0.15/modules/classes.html#module-sklearn.datasets>  
(Diabetes dataset, Boston house dataset, California housing dataset, Advertising dataset)

Following tasks needs to be performed for the below mentioned ML techniques in scikit learn (<https://scikit-learn.org/stable/>), whenever applicable:

- Split the dataset into two parts: training and test. Create and train model on training set and report model performance on test set.
- Test the model performance using  $k$ -fold cross validation (take  $k = 5$  or  $10$ ) in terms of applicable metrics like – Accuracy, Precision, Recall, MAE, RMSE etc.
- Finding optimal parameters using Grid Search CV, whenever applicable; for example: in case of polynomial regression, employ Grid search CV to find the optimal value of the degree  $d$  for which the MSE is least.

**Practicals List:**

1. Create a Linear regression model. Use one variable at a time, all variable at a time, and statistically significant variables (using co-relation matrix) at a time, and observe the model performance. Preferably work with advertising dataset to predict sales in terms of the above features.
2. Fix a 10th order polynomial and sample 15 noisy data points (that is all 15 points do not lie on this polynomial). This is usually done by adding a white noise  $\epsilon \sim N(0,1)$  to the polynomial  $f(x)$ . Using polynomial regression fit two models: one of order 10 and one of order 2. Compare the in-sample and out-sample errors for both models. Try to observe underfitting-overfitting, if any. In another scenario, take  $f(x)$  to be a polynomial of order 50 and sample 15 noiseless data points (all lie on the graph of  $f(x)$ ) and again fit a polynomial model of order 2 and 10. Compare the in-sample and out-sample errors. (refer to Exercise 4.2 and Problem 4.4 of [1]).
3. On the Smarket data, predict direction based on features Lag1 and Lag2. Split the Smarket dataset into training and testing parts in the ratio 80-20. Fit logistic regression on the training data and evaluate its accuracy on the test data via confusion matrix, ROC, and AUC. Plot decision boundary for the logistic regression in the 2D feature space spanned by Lag1 and Lag2 (you might need to rescale the variables).
4. Create decision tree models for classification and regression. Observe the effect of various parameters like - splitting criterion (Gini index, Cross entropy), max depth (for tree pruning). Examine overfitting-underfitting in the associated tree model. Display a decision tree.
5. Create Random Forest models for classification and regression. Observe the effect of number of estimators in the context of overfitting.

6. Create SVM models for classification and regression. Observe the effect of the parameter - kernel.
7. Create LDA and QDA models and assess them preferably on the digits dataset.
8. Create  $k$ -means cluster model for clustering. Observe the effect of parameter  $k$  (number of clusters). Plot  $k$  versus error to find out best  $k$  (Elbow criterion). Plot clusters in case of 2-dimensional data.
9. Demonstrate Principal Component Analysis (PCA) on a dataset with large number of features.
10. Create an ANN model for both classification and regression. Observe the effect of parameters- hidden layer sizes, activation functions (ReLU, Logistic/Sigmoid, Tanh), optimizers (Adam, Sgd), batch size, learning rate, early stopping, validation fraction, maximum number of iterations. Plot iteration number versus accuracy on training and validation dataset. The mnist dataset may be used to explore real strength of ANN. (<https://www.kaggle.com/datasets/oddrational/mnist-in-csv?resource=download> in csv format).

**B.A. (Prog.) Semester-VIII with Mathematics as Major**  
**Category-II**

**DISCIPLINE SPECIFIC CORE COURSE (DSC-8): TOPICS IN MULTIVARIATE CALCULUS**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Topics in Multivariate Calculus	4	3	0	1	Class XII pass with Mathematics	Calculus

**Learning Objectives:** The primary objective of this course is to introduce the:

- Extension of the studies of single variable differential and integral calculus to functions of two or more independent variables.
- Applications of multivariable calculus tools to physics, economics, and optimization.
- Geometry and visualisation of curves and surfaces in two dimensions (plane) and three dimensions (space).
- Techniques of integration to functions of two and three independent variables.

**Learning Outcomes:** This course will enable the students to:

- Learn the conceptual variations when advancing in calculus from one variable to multivariable discussion.
- Understand the maximization and minimization of multivariable functions subject to the given constraints on variables.
- Learn about inter-relationship amongst the line integral, double and triple integral formulations.
- Familiarize with Green's, Stokes' and Gauss divergence theorems.

**SYLLABUS OF DSC-8**

**UNIT-I: Calculus of Functions of Several Variables (18 hours)**

Basic Concepts, Limits and Continuity, Tangent Planes, Partial Derivatives, Total Differential, Differentiability, Chain Rules, Directional Derivatives and the Gradient, Extrema of Functions of Two Variables, Method of Lagrange multipliers with one constraint.

**UNIT-II: Double and Triple Integrals (15 hours)**

Double integration over rectangular and nonrectangular regions, Double integrals in polar co-ordinates, Triple integral over a parallelopiped and solid regions, Volume by triple integrals, Triple integration in cylindrical and spherical coordinates, Change of variables in double and triple integrals.

**UNIT-III: Green's, Stokes' and Gauss Divergence Theorem****(12 hours)**

Line integrals, Applications of line integrals: Mass and Work, Fundamental theorem for line integrals, Conservative vector fields, Green's theorem, Area as a line integral, Surface integrals, Stokes' theorem, Gauss divergence theorem.

**Essential Reading**

1. Strauss, Monty J., Bradley, Gerald L., & Smith, Karl J. (2007). Calculus (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. (Pearson Education). Delhi. Indian Reprint 2011.

**Suggestive Reading**

- Marsden, J. E., Tromba, A., & Weinstein, A. (2004). Basic Multivariable Calculus. Springer (SIE). First Indian Reprint.

**DSE Courses of B.A. (Prog.) Semester-VIII**  
**Category-II**

**DISCIPLINE SPECIFIC ELECTIVE COURSE – 4(i): APPLIED ALGEBRA**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Applied Algebra	4	3	1	0	Class XII pass with Mathematics	Linear Algebra, Abstract Algebra

**Learning Objectives:** The primary objective of this course is to:

- Introduce the applications of linear algebra in the field of science and arts.
- Develop the analytical and numerical skills to apply the algebraic concepts in real-life situations.
- Understand the identification numbers and different check digit schemes that can be used to reduce the errors during their transmission.

**Learning Outcomes:** This course will enable the students to:

- Understand the system of linear equations, matrices and transformations in the fields of economics, science, engineering and computer science.
- Apply the combinatorics and graph theory in scheduling and reliability theory.
- Learn about identification numbers and using check digits to check for errors after the identification number has been transmitted.

**SYLLABUS OF DSE-4(i)**

**UNIT-I: Applications of Linear Algebra (15 hours)**

Applications of linear systems: Leontief input-output model in economics, Traffic flow, and diet problem; Applications to computer graphics, difference equations and Markov chains; Applications to linear models: Least-squares problems, and least-squares lines.

**UNIT-II: Latin Squares and Graph Models (12 hours)**

Latin squares, Table for a finite group as a Latin square, Latin squares as in design of experiments; Mathematical models for matching jobs, Spelling checker, Network reliability, Street surveillance, Scheduling meetings, Interval graph modeling and Influence model, Pitcher pouring puzzle.

**UNIT-III: Various Check Digit Schemes (18 hours)**

Developing identification numbers, Types of identification numbers, Transmission errors, Check digits, Integer division, Modular arithmetic, US postal money orders, Airline ticket identification numbers, The Universal Product Code check digit scheme, ISBN check digit

scheme, Creating Identification numbers, IBM scheme, Symmetry, Symmetry and Rigid motions, Verhoeff check digit scheme.

### Essential Readings

1. David C. Lay, Steven R. Lay and Judi J. McDonald (2016). Linear Algebra and Its Applications (5th ed.). Pearson.
2. Tucker, Alan (2012). Applied Combinatorics (6th ed.). John Wiley & Sons, Inc.
3. Kirtland, Joseph (2001). Identification Numbers and Check Digit Schemes. Mathematical Association of America.

### Suggestive Readings

- Andirilli, Stephen and Hecker, David (2016). Elementary Linear Algebra (5th ed.). Academic Press, Elsevier.
- Lidl, Rudolf and Pilz, Günter (1998). Applied Abstract Algebra (2nd ed.). Springer. Indian Reprint 2014.
- Strang, Gilbert (2016). Introduction to Linear Algebra (5th ed.). Wellesley-Cambridge.

## DISCIPLINE SPECIFIC ELECTIVE COURSE-4(ii): ELEMENTS OF PARTIAL DIFFERENTIAL EQUATIONS

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Partial Differential Equations	4	3	1	0	Class XII pass with Mathematics	Differential Equations

**Learning Objectives:** The main objective of this course is to introduce:

- Basic concepts of first and second-order linear/nonlinear partial differential equations.
- Methods to solve first-order nonlinear PDEs and determine integral surfaces.
- Linear PDEs with constant coefficients, and finding their solutions using complimentary functions and particular integral.
- Modeling of wave equation, diffusion equation, traffic flow and their solutions.

**Learning Outcomes:** The course will enable the students to learn:

- Charpit's and Jacobi's methods to solve first-order nonlinear partial differential equations in two and three independent variables, respectively.
- Monge's method for integrating PDE of type  $Rr + Ss + Tt = V$ .
- The Cauchy problem and solutions of one-dimensional wave equations with initial boundary-value problems, and vibration of finite string with fixed ends.
- The macroscopic modeling of the traffic flow, where the focus will be on modeling the density of cars and their flow, rather than modeling individual cars and their velocity.

## SYLLABUS OF DSE-4(ii)

### UNIT–I: First-order Partial Differential Equations (18 hours)

Review of basic concepts: Origins of first-order PDEs, Lagrange's method for solving linear equations of first order; Integral surfaces passing through a given curve, and surfaces orthogonal to a given system of surfaces; Nonlinear PDEs of the first order, and compatible systems of first-order PDEs; Charpit's method for solving nonlinear PDEs, special types of first-order PDEs, and solutions satisfying given conditions; Jacobi's method for solving nonlinear PDE with three independent variables.

### UNIT – II: Second-order Partial Differential Equations (15 hours)

Origins of second-order PDEs, and solving linear PDEs with constant coefficients using methods of finding the complementary function and particular integral; Monge's method of integrating nonlinear second-order PDE of type  $Rr + Ss + Tt = V$  with variable coefficients.

### UNIT – III: Applications of Partial Differential Equations (12 hours)

Solution of one-dimensional diffusion equation and wave equation by method of separation of variables, d'Alembert's solution of the Cauchy problem for the one-dimensional wave equation; Solutions of homogeneous one-dimensional wave equations with initial boundary-value problems, and vibration of finite string with fixed ends; Traffic flow model.

### Essential Readings

- 1 Myint-U, Tyn & Debnath, Lokenath. (2007). Linear Partial Differential Equations for Scientists and Engineers (4th ed.). Birkhäuser. Indian Reprint.
- 2 Piaggio, H.T.H. (2004). Differential Equations. CBS Publishers & Distributors, Delhi.
- 3 Sneddon, Ian N. (2006). Elements of Partial Differential Equations, Dover Publications. Indian Reprint.

### Suggestive Readings

- Amaranath T. (2023). An Elementary Course in Partial Differential Equations (3rd ed.). Narosa Publishing House.
- Arrigo, Daniel (2023). An Introduction to Partial Differential Equations (2nd ed.). Springer.
- Kapoor, N. M. (2023). A Text Book of Differential Equations. Pitambar Publishing Company.

## DISCIPLINE SPECIFIC ELECTIVE COURSE-4(iii): MATHEMATICAL STATISTICS

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mathematical Statistics	4	3	1	0	Class XII pass with Mathematics	Probability and Statistics, Multivariate Calculus



**Learning Objectives:** The main objective of this course is to introduce the:

- Joint behavior of several random variables theoretically and through illustrative practical examples.
- Theory underlying modern statistics to give the student a solid grounding in (mathematical) statistics and the principles of statistical inference.
- Application of the theory to the statistical modeling of data from real applications, including model identification, estimation, and interpretation.
- Theory and analysis of multivariate data which covers two-factor analysis of variance, multiple linear regression including models for contingency tables.

**Learning Outcomes:** The course will enable the students to:

- Understand joint distributions of random variables including the multivariate normal distribution.
- Estimate model parameters from the statistical inference based on confidence intervals and hypothesis testing.
- Understand the theory of multiple regression models and contingency tables.
- Apply principles and theory to the statistical modeling and analysis of practical problems in a variety of application areas, and to interpret results and draw conclusions in context.

## **SYLLABUS OF DSE-4(iii)**

### **UNIT–I: Joint Probability Distributions (15 hours)**

Joint probability mass function for two discrete random variables, Marginal probability mass function, Joint probability density function for two continuous random variables, Marginal probability density function, Independent random variables; Expected values, covariance, and correlation; Linear combination of random variables, Moment generating functions of linear combination of random variables; Conditional distributions and conditional expectation, The laws of total expectation and variance; Bivariate normal distribution.

### **UNIT-II: Sampling Distributions and Estimation (12 hours)**

Distribution of important statistics such as the sample totals, sample means, and sample proportions; Joint sampling distribution of sample mean and sample variance,  $t$ -statistic and  $F$ -statistic distributions based on normal random samples; Concepts and criteria for point estimation, The method of moments estimators and maximum likelihood estimation; Interval estimation and basic properties of confidence intervals, One-sample  $t$  confidence interval, Confidence intervals for a population proportion and population variance.

### **UNIT-III: Tests of Hypotheses, ANOVA and Multiple Regression Analysis (18 hours)**

Statistical hypotheses and test procedures, One-sample tests about: population mean, population proportion, and population variance;  $P$ -values for tests; Two-sample  $z$ -confidence interval and  $t$ -confidence interval tests; Single-factor ANOVA, Two-factor ANOVA without replication; Multiple linear regression model and estimating parameters; Chi-squared goodness-of-fit tests, Two-way Contingency tables.

## Essential Reading

1. Devore, Jay L., Berk, Kenneth N. & Carlton Matthew A. (2021). Modern Mathematical Statistics with Applications. Third edition, Springer.

## Suggestive Readings

- Devore, Jay L. (2016). Probability and Statistics for Engineering and the Sciences. Ninth edition, Cengage Learning India Private Limited, Delhi. Fourth impression 2022.
- Hogg, Robert V., McKean, Joseph W., & Craig, Allen T. (2019). Introduction to Mathematical Statistics. Eighth edition, Pearson. Indian Reprint 2020.
- Mood, A.M., Graybill, F.A., & Boes, D.C. (1974). Introduction the Theory of Statistics (3rd ed.). Tata McGraw Hill Pub. Co. Ltd. Reprinted 2017.
- Wackerly, Dennis D., Mendenhall III, William & Scheaffer, Richard L. (2008). Mathematical Statistics with Applications. 7th edition, Cengage Learning.

### DISCIPLINE SPECIFIC ELECTIVE COURSE-4(iv): OPTIMIZATION TECHNIQUES

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Optimization Techniques	4	3	1	0	Class XII pass with Mathematics	Multivariate Calculus

**Learning Objectives:** The primary objective of this course is to introduce:

- Nonlinear optimization problems
- Transshipment and dynamic programming problems
- Integer Programming, fractional programming problems
- Convex and generalized convex functions and their properties

**Learning Outcomes:** This course will enable the students to:

- Nonlinear programming problems and their applications
- Method to solve fractional programming problems with linear constraints
- Methods to solve dynamic programming problems using recursive computations

## SYLLABUS OF DSE-4(iv)

### UNIT-I: Transshipment and Dynamic Programming Problems (15 hours)

Transshipment problem, Shortest-route problem; Dynamic programming, Recursive forward and backward computation, Knapsack/fly-away/cargo-loading problems solution through dynamic programming.

**UNIT-II: Integer Programming Problems****(15 hours)**

Integer programming problem, Gomory's cutting plane method for integer problems, Mixed integer problems, Branch and bound method.

**UNIT-III: Nonlinear Programming Problems****(15 hours)**

Convex functions, Convex programming problems; Generalized convex functions; Linear fractional programming problem, Charnes and Cooper transformation, Simplex algorithm to solve linear fractional programming problem.

**Essential Readings**

1. Chandra, Suresh, Jayadeva and Mehra, Aparna (2009). Numerical Optimization with Applications. Narosa Publishing House Pvt. Ltd. Delhi. Second Reprint 2016.
2. Taha, Hamdy A. (2017). Operations Research: An Introduction (10th ed.). Pearson.

**Suggestive Reading**

- Swarup, K., Gupta, P.K., and Mohan, M. (1984). Operations Research. Sultan Chand.

**DISCIPLINE SPECIFIC ELECTIVE COURSE-4(v): RINGS AND FIELDS****CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
<b>Rings and Fields</b>	<b>4</b>	<b>3</b>	<b>1</b>	<b>0</b>	<b>Class XII pass with Mathematics</b>	<b>Abstract Algebra</b>

**Learning Objectives:** The primary objective of this course is to:

- Understand the basic algebraic structures rings, Euclidean rings, polynomial rings and fields.
- Understand the form of ideals, maximal ideals in the quotient rings and their order preserving correspondence with the parent ring.
- Learn the concept of splitting fields of a polynomial over a field and its existence and uniqueness.
- Gain the knowledge of some geometric constructions using field extensions.

**Learning Outcomes:** This course will enable the students to:

- Have familiar with the algebraic structure rings, its maximal ideals, and quotient rings.
- Understand the polynomial rings in one variable over a field with the help of the concept of Euclidean rings.
- Learn the field extensions and the existence, uniqueness of splitting fields of any polynomial over a field.
- Gain the knowledge of structure of finite fields, constructability of numbers using straightedge and compass.

## **SYLLABUS OF DSE-4(v)**

### **UNIT-I: Ideals in the quotient rings and Euclidean rings (15 hours)**

Ring homomorphism, First Fundamental theorem of ring homomorphism, Ideals in the quotient rings, Maximal ideals, Maximal ideals of rings of all real valued continuous functions on closed unit interval, Field of quotients of an integral domain, Euclidean rings, Units in Euclidean rings, Principal ideal rings, Unique factorization theorem, Prime elements and the ideal generated by them.

### **UNIT-II: Polynomial Rings and Field Extensions (15 hours)**

Ring of Gaussian integers, Polynomial rings in one variable, Division algorithm, Irreducible polynomials and the ideal generated by them, Polynomial rings over the rational field, Gauss' lemma, Eisenstein criterion, Polynomial rings in  $n$  variables.

Extension of Fields: The Fundamental Theorem of Field Theory, Splitting Fields, Zeros of an irreducible polynomial.

### **UNIT-III: Algebraic Extensions (15 hours)**

Characterization of field extensions, Finite extensions, Properties of algebraic extensions;

Classification of Finite Fields, Structure of Finite Fields, Subfields of a Finite Field;

Geometric Constructions: Constructible Numbers, Angle-Trisectors and Circle-Squares.

### **Essential Readings**

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint (2021).
2. Herstein. I. N. (1975). Topics in Algebra (2nd ed.). Wiley India. Reprint 2022.

### **Suggestive Readings**

- Dummit, David S., and Foote, Richard M. (2011). Abstract Algebra (3rd ed.), Wiley.
- Garling, D. J. H. (2021). Galois Theory and Its Algebraic Background (2nd ed.). Cambridge University Press.

**B.Sc. (Physical Sciences/Mathematical Sciences) Semester-VIII**  
**Category-III**

**DISCIPLINE SPECIFIC CORE COURSE – (DSC-8): TOPICS IN MULTIVARIATE CALCULUS**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Topics in Multivariate Calculus	4	3	1	0	Class XII pass with Mathematics	Calculus

**Learning Objectives:** The primary objective of this course is to introduce the:

- Extension of the studies of single variable differential and integral calculus to functions of two or more independent variables.
- Applications of multivariable calculus tools to physics, economics, and optimization.
- Geometry and visualisation of curves and surfaces in two dimensions (plane) and three dimensions (space).
- Techniques of integration to functions of two and three independent variables.

**Learning Outcomes:** This course will enable the students to:

- Learn the conceptual variations when advancing in calculus from one variable to multivariable discussion.
- Understand the maximization and minimization of multivariable functions subject to the given constraints on variables.
- Learn about inter-relationship amongst the line integral, double and triple integral formulations.
- Familiarize with Green's, Stokes' and Gauss divergence theorems.

**SYLLABUS OF DSC-8**

**UNIT-I: Calculus of Functions of Several Variables (18 hours)**

Basic Concepts, Limits and Continuity, Tangent Planes, Partial Derivatives, Total Differential, Differentiability, Chain Rules, Directional Derivatives and the Gradient, Extrema of Functions of Two Variables, Method of Lagrange multipliers with one constraint.

**UNIT-II: Double and Triple Integrals (15 hours)**

Double integration over rectangular and nonrectangular regions, Double integrals in polar co-ordinates, Triple integral over a parallelopiped and solid regions, Volume by triple integrals, Triple integration in cylindrical and spherical coordinates, Change of variables in double and triple integrals.

### **UNIT-III: Green's, Stokes' and Gauss Divergence Theorem**

**(12 hours)**

Line integrals, Applications of line integrals: Mass and Work, Fundamental theorem for line integrals, Conservative vector fields, Green's theorem, Area as a line integral, Surface integrals, Stokes' theorem, Gauss divergence theorem.

#### **Essential Reading**

1. Strauss, Monty J., Bradley, Gerald L., & Smith, Karl J. (2007). Calculus (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. (Pearson Education). Delhi. Indian Reprint 2011.

#### **Suggestive Reading**

- Marsden, J. E., Tromba, A., & Weinstein, A. (2004). Basic Multivariable Calculus. Springer (SIE). First Indian Reprint.

**DSE Courses of B.Sc. (Physical Sciences/Mathematical Sciences) Semester-VIII**  
**Category-III**

**DISCIPLINE SPECIFIC ELECTIVE COURSE – 6(i): APPLIED ALGEBRA**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Applied Algebra	4	3	1	0	Class XII pass with Mathematics	Linear Algebra, Abstract Algebra

**Learning Objectives:** The primary objective of this course is to:

- Introduce the applications of linear algebra in the field of science and arts.
- Develop the analytical and numerical skills to apply the algebraic concepts in real-life situations.
- Understand the identification numbers and different check digit schemes that can be used to reduce the errors during their transmission.

**Learning Outcomes:** This course will enable the students to:

- Understand the system of linear equations, matrices and transformations in the fields of economics, science, engineering and computer science.
- Apply the combinatorics and graph theory in scheduling and reliability theory.
- Learn about identification numbers and using check digits to check for errors after the identification number has been transmitted.

**SYLLABUS OF DSE-6(i)**

**UNIT-I: Applications of Linear Algebra (15 hours)**

Applications of linear systems: Leontief input-output model in economics, Traffic flow, and diet problem; Applications to computer graphics, difference equations and Markov chains; Applications to linear models: Least-squares problems, and least-squares lines.

**UNIT-II: Latin Squares and Graph Models (12 hours)**

Latin squares, Table for a finite group as a Latin square, Latin squares as in design of experiments; Mathematical models for matching jobs, Spelling checker, Network reliability, Street surveillance, Scheduling meetings, Interval graph modeling and Influence model, Pitcher pouring puzzle.

**UNIT-III: Various Check Digit Schemes (18 hours)**

Developing identification numbers, Types of identification numbers, Transmission errors, Check digits, Integer division, Modular arithmetic, US postal money orders, Airline ticket identification numbers, The Universal Product Code check digit scheme, ISBN check digit scheme, Creating Identification numbers, IBM scheme, Symmetry, Symmetry and Rigid motions, Verhoeff check digit scheme.

### Essential Readings

1. David C. Lay, Steven R. Lay and Judi J. McDonald (2016). Linear Algebra and Its Applications (5th ed.). Pearson.
2. Tucker, Alan (2012). Applied Combinatorics (6th ed.). John Wiley & Sons, Inc.
3. Kirtland, Joseph (2001). Identification Numbers and Check Digit Schemes. Mathematical Association of America.

### Suggestive Readings

- Andirilli, Stephen and Hecker, David (2016). Elementary Linear Algebra (5th ed.). Academic Press, Elsevier.
- Lidl, Rudolf and Pilz, Günter (1998). Applied Abstract Algebra (2nd ed.). Springer. Indian Reprint 2014.
- Strang, Gilbert (2016). Introduction to Linear Algebra (5th ed.). Wellesley-Cambridge.

## DISCIPLINE SPECIFIC ELECTIVE COURSE-6(ii): ELEMENTS OF PARTIAL DIFFERENTIAL EQUATIONS

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Partial Differential Equations	4	3	1	0	Class XII pass with Mathematics	Differential Equations

**Learning Objectives:** The main objective of this course is to introduce:

- Basic concepts of first and second-order linear/nonlinear partial differential equations.
- Methods to solve first-order nonlinear PDEs and determine integral surfaces.
- Linear PDEs with constant coefficients, and finding their solutions using complimentary functions and particular integral.
- Modeling of wave equation, diffusion equation, traffic flow and their solutions.

**Learning Outcomes:** The course will enable the students to learn:

- Charpit's and Jacobi's methods to solve first-order nonlinear partial differential equations in two and three independent variables, respectively.
- Monge's method for integrating PDE of type  $Rr + Ss + Tt = V$ .
- The Cauchy problem and solutions of one-dimensional wave equations with initial boundary-value problems, and vibration of finite string with fixed ends.
- The macroscopic modeling of the traffic flow, where the focus will be on modeling the density of cars and their flow, rather than modeling individual cars and their velocity.



## SYLLABUS OF DSE-6(ii)

### UNIT–I: First-order Partial Differential Equations (18 hours)

Review of basic concepts: Origins of first-order PDEs, Lagrange's method for solving linear equations of first order; Integral surfaces passing through a given curve, and surfaces orthogonal to a given system of surfaces; Nonlinear PDEs of the first order, and compatible systems of first-order PDEs; Charpit's method for solving nonlinear PDEs, special types of first-order PDEs, and solutions satisfying given conditions; Jacobi's method for solving nonlinear PDE with three independent variables.

### UNIT – II: Second-order Partial Differential Equations (15 hours)

Origins of second-order PDEs, and solving linear PDEs with constant coefficients using methods of finding the complementary function and particular integral; Monge's method of integrating nonlinear second-order PDE of type  $Rr + Ss + Tt = V$  with variable coefficients.

### UNIT – III: Applications of Partial Differential Equations (12 hours)

Solution of one-dimensional diffusion equation and wave equation by method of separation of variables, d'Alembert's solution of the Cauchy problem for the one-dimensional wave equation; Solutions of homogeneous one-dimensional wave equations with initial boundary-value problems, and vibration of finite string with fixed ends; Traffic flow model.

### Essential Readings

- 1 Myint-U, Tyn & Debnath, Lokenath. (2007). Linear Partial Differential Equations for Scientists and Engineers (4th ed.). Birkhäuser. Indian Reprint.
- 2 Piaggio, H.T.H. (2004). Differential Equations. CBS Publishers & Distributors, Delhi.
- 3 Sneddon, Ian N. (2006). Elements of Partial Differential Equations, Dover Publications. Indian Reprint.

### Suggestive Readings

- Amaranath T. (2023). An Elementary Course in Partial Differential Equations (3rd ed.). Narosa Publishing House.
- Arrigo, Daniel (2023). An Introduction to Partial Differential Equations (2nd ed.). Springer.
- Kapoor, N. M. (2023). A Text Book of Differential Equations. Pitambar Publishing Company.

## DISCIPLINE SPECIFIC ELECTIVE COURSE-6(iii): MATHEMATICAL STATISTICS

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mathematical Statistics	4	3	1	0	Class XII pass with Mathematics	Probability and Statistics, Multivariate Calculus

**Learning Objectives:** The main objective of this course is to introduce the:

- Joint behavior of several random variables theoretically and through illustrative practical examples.
- Theory underlying modern statistics to give the student a solid grounding in (mathematical) statistics and the principles of statistical inference.
- Application of the theory to the statistical modeling of data from real applications, including model identification, estimation, and interpretation.
- Theory and analysis of multivariate data which covers two-factor analysis of variance, multiple linear regression including models for contingency tables.

**Learning Outcomes:** The course will enable the students to:

- Understand joint distributions of random variables including the multivariate normal distribution.
- Estimate model parameters from the statistical inference based on confidence intervals and hypothesis testing.
- Understand the theory of multiple regression models and contingency tables.
- Apply principles and theory to the statistical modeling and analysis of practical problems in a variety of application areas, and to interpret results and draw conclusions in context.

## **SYLLABUS OF DSE-6(iii)**

### **UNIT–I: Joint Probability Distributions (15 hours)**

Joint probability mass function for two discrete random variables, Marginal probability mass function, Joint probability density function for two continuous random variables, Marginal probability density function, Independent random variables; Expected values, covariance, and correlation; Linear combination of random variables, Moment generating functions of linear combination of random variables; Conditional distributions and conditional expectation, The laws of total expectation and variance; Bivariate normal distribution.

### **UNIT-II: Sampling Distributions and Estimation (12 hours)**

Distribution of important statistics such as the sample totals, sample means, and sample proportions; Joint sampling distribution of sample mean and sample variance,  $t$ -statistic and  $F$ -statistic distributions based on normal random samples; Concepts and criteria for point estimation, The method of moments estimators and maximum likelihood estimation; Interval estimation and basic properties of confidence intervals, One-sample  $t$  confidence interval, Confidence intervals for a population proportion and population variance.

### **UNIT-III: Tests of Hypotheses, ANOVA and Multiple Regression Analysis (18 hours)**

Statistical hypotheses and test procedures, One-sample tests about: population mean, population proportion, and population variance;  $P$ -values for tests; Two-sample  $z$ -confidence interval and  $t$ -confidence interval tests; Single-factor ANOVA, Two-factor ANOVA without replication; Multiple linear regression model and estimating parameters; Chi-squared goodness-of-fit tests, Two-way Contingency tables.

## Essential Reading

1. Devore, Jay L., Berk, Kenneth N. & Carlton Matthew A. (2021). Modern Mathematical Statistics with Applications. Third edition, Springer.

## Suggestive Readings

- Devore, Jay L. (2016). Probability and Statistics for Engineering and the Sciences. Ninth edition, Cengage Learning India Private Limited, Delhi. Fourth impression 2022.
- Hogg, Robert V., McKean, Joseph W., & Craig, Allen T. (2019). Introduction to Mathematical Statistics. Eighth edition, Pearson. Indian Reprint 2020.
- Mood, A.M., Graybill, F.A., & Boes, D.C. (1974). Introduction the Theory of Statistics (3rd ed.). Tata McGraw Hill Pub. Co. Ltd. Reprinted 2017.
- Wackerly, Dennis D., Mendenhall III, William & Scheaffer, Richard L. (2008). Mathematical Statistics with Applications. 7th edition, Cengage Learning.

### DISCIPLINE SPECIFIC ELECTIVE COURSE-6(iv): OPTIMIZATION TECHNIQUES

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Optimization Techniques	4	3	1	0	Class XII pass with Mathematics	Multivariate Calculus

**Learning Objectives:** The primary objective of this course is to introduce:

- Nonlinear optimization problems, Transshipment, and dynamic programming problems.
- Integer Programming, and fractional programming problems.
- Convex and generalized convex functions with their properties.

**Learning Outcomes:** This course will enable the students to:

- Nonlinear programming problems and their applications.
- Method to solve fractional programming problems with linear constraints.
- Methods to solve dynamic programming problems using recursive computations.

## SYLLABUS OF DSE-6(iv)

### UNIT-I: Transshipment and Dynamic Programming Problems (15 hours)

Transshipment problem, Shortest-route problem; Dynamic programming, Recursive forward and backward computation, Knapsack/fly-away/cargo-loading problems solution through dynamic programming.

### UNIT-II: Integer Programming Problems (15 hours)

Integer programming problem, Gomory's cutting plane method for integer problems, Mixed integer problems, Branch and bound method.

**UNIT-III: Nonlinear Programming Problems****(15 hours)**

Convex functions, Convex programming problems; Generalized convex functions; Linear fractional programming problem, Charnes and Cooper transformation, Simplex algorithm to solve linear fractional programming problem.

**Essential Readings**

1. Chandra, Suresh, Jayadeva and Mehra, Aparna (2009). Numerical Optimization with Applications. Narosa Publishing House Pvt. Ltd. Second Reprint 2016.
2. Taha, Hamdy A. (2017). Operations Research: An Introduction (10th ed.). Pearson.

**Suggestive Reading**

- Swarup, K., Gupta, P.K., and Mohan, M. (1984). Operations Research. Sultan Chand.

**DISCIPLINE SPECIFIC ELECTIVE COURSE-6(v): RINGS AND FIELDS****CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
<b>Rings and Fields</b>	<b>4</b>	<b>3</b>	<b>1</b>	<b>0</b>	<b>Class XII pass with Mathematics</b>	<b>Abstract Algebra</b>

**Learning Objectives:** The primary objective of this course is to:

- Understand the basic algebraic structures rings, Euclidean rings, polynomial rings and fields.
- Understand the form of ideals, maximal ideals in the quotient rings and their order preserving correspondence with the parent ring.
- Learn the concept of splitting fields of a polynomial over a field and its existence and uniqueness.
- Gain the knowledge of some geometric constructions using field extensions.

**Learning Outcomes:** This course will enable the students to:

- Have familiar with the algebraic structure rings, its maximal ideals, and quotient rings.
- Understand the polynomial rings in one variable over a field with the help of the concept of Euclidean rings.
- Learn the field extensions and the existence, uniqueness of splitting fields of any polynomial over a field.
- Gain the knowledge of structure of finite fields, constructability of numbers using straightedge and compass.

## **SYLLABUS OF DSE-6(v)**

### **UNIT-I: Ideals in the quotient rings and Euclidean rings (15 hours)**

Ring homomorphism, First Fundamental theorem of ring homomorphism, Ideals in the quotient rings, Maximal ideals, Maximal ideals of rings of all real valued continuous functions on closed unit interval, Field of quotients of an integral domain, Euclidean rings, Units in Euclidean rings, Principal ideal rings, Unique factorization theorem, Prime elements and the ideal generated by them.

### **UNIT-II: Polynomial Rings and Field Extensions (15 hours)**

Ring of Gaussian integers, Polynomial rings in one variable, Division algorithm, Irreducible polynomials and the ideal generated by them, Polynomial rings over the rational field, Gauss' lemma, Eisenstein criterion, Polynomial rings in  $n$  variables.

Extension of Fields: The Fundamental Theorem of Field Theory, Splitting Fields, Zeros of an irreducible polynomial.

### **UNIT-III: Algebraic Extensions (15 hours)**

Characterization of field extensions, Finite extensions, Properties of algebraic extensions;

Classification of Finite Fields, Structure of Finite Fields, Subfields of a Finite Field;

Geometric Constructions: Constructible Numbers, Angle-Trisectors and Circle-Squares.

### **Essential Readings**

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint (2021).
2. Herstein. I. N. (1975). Topics in Algebra (2nd ed.). Wiley India. Reprint 2022.

### **Suggestive Readings**

- Dummit, David S., and Foote, Richard M. (2011). Abstract Algebra (3rd ed.), Wiley.
- Garling, D. J. H. (2021). Galois Theory and Its Algebraic Background (2nd ed.). Cambridge University Press.

**COMMON POOL OF GENERIC ELECTIVES (GE) Semester-VIII COURSES OFFERED  
BY DEPARTMENT OF MATHEMATICS**

**Category-IV**

**GENERIC ELECTIVES (GE-8(i)): RINGS AND FIELDS**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
<b>Rings and Fields</b>	<b>4</b>	<b>3</b>	<b>1</b>	<b>0</b>	<b>Class XII pass with Mathematics</b>	<b>Abstract Algebra</b>

**Learning Objectives:** The primary objective of this course is to:

- Understand the basic algebraic structures rings, Euclidean rings, polynomial rings and fields.
- Understand the form of ideals, maximal ideals in the quotient rings and their order preserving correspondence with the parent ring.
- Learn the concept of splitting fields of a polynomial over a field and its existence and uniqueness.
- Gain the knowledge of some geometric constructions using field extensions.

**Learning Outcomes:** This course will enable the students to:

- Have familiar with the algebraic structure rings, its maximal ideals, and quotient rings.
- Understand the polynomial rings in one variable over a field with the help of the concept of Euclidean rings.
- Learn the field extensions and the existence, uniqueness of splitting fields of any polynomial over a field.
- Gain the knowledge of structure of finite fields, constructability of numbers using straightedge and compass.

**SYLLABUS OF GE-8(i)**

**UNIT-I: Ideals in the quotient rings and Euclidean rings (15 hours)**

Ring homomorphism, First Fundamental theorem of ring homomorphism, Ideals in the quotient rings, Maximal ideals, Maximal ideals of rings of all real valued continuous functions on closed unit interval, Field of quotients of an integral domain, Euclidean rings, Units in Euclidean rings, Principal ideal rings, Unique factorization theorem, Prime elements and the ideal generated by them.

**UNIT-II: Polynomial Rings and Field Extensions (15 hours)**

Ring of Gaussian integers, Polynomial rings in one variable, Division algorithm, Irreducible polynomials and the ideal generated by them, Polynomial rings over the rational field, Gauss' lemma, Eisenstein criterion, Polynomial rings in  $n$  variables.

Extension of Fields: The Fundamental Theorem of Field Theory, Splitting Fields, Zeros of an irreducible polynomial.

**UNIT-III: Algebraic Extensions****(15 hours)**

Characterization of field extensions, Finite extensions, Properties of algebraic extensions;  
 Classification of Finite Fields, Structure of Finite Fields, Subfields of a Finite Field;  
 Geometric Constructions: Constructible Numbers, Angle-Trisectors and Circle-Squares.

**Essential Readings**

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint (2021).
2. Herstein. I. N. (1975). Topics in Algebra (2nd ed.). Wiley India. Reprint 2022.

**Suggestive Readings**

- Dummit, David S., and Foote, Richard M. (2011). Abstract Algebra (3rd ed.), Wiley.
- Garling, D. J. H. (2021). Galois Theory and Its Algebraic Background (2nd ed.). Cambridge University Press.

**GENERIC ELECTIVES (GE-8(ii)): ELEMENTS OF PARTIAL DIFFERENTIAL EQUATIONS****CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Partial Differential Equations	4	3	1	0	Class XII pass with Mathematics	Differential Equations

**Learning Objectives:** The main objective of this course is to introduce:

- Basic concepts of first and second-order linear/nonlinear partial differential equations.
- Methods to solve first-order nonlinear PDEs and determine integral surfaces.
- Linear PDEs with constant coefficients, and finding their solutions using complimentary functions and particular integral.
- Modeling of wave equation, diffusion equation, traffic flow and their solutions.

**Learning Outcomes:** The course will enable the students to learn:

- Charpit's and Jacobi's methods to solve first-order nonlinear partial differential equations in two and three independent variables, respectively.
- Monge's method for integrating PDE of type  $Rr + Ss + Tt = V$ .
- The Cauchy problem and solutions of one-dimensional wave equations with initial boundary-value problems, and vibration of finite string with fixed ends.
- The macroscopic modeling of the traffic flow, where the focus will be on modeling the density of cars and their flow, rather than modeling individual cars and their velocity.

## SYLLABUS OF GE-8(ii)

### UNIT–I: First-order Partial Differential Equations (18 hours)

Review of basic concepts: Origins of first-order PDEs, Lagrange's method for solving linear equations of first order; Integral surfaces passing through a given curve, and surfaces orthogonal to a given system of surfaces; Nonlinear PDEs of the first order, and compatible systems of first-order PDEs; Charpit's method for solving nonlinear PDEs, special types of first-order PDEs, and solutions satisfying given conditions; Jacobi's method for solving nonlinear PDE with three independent variables.

### UNIT – II: Second-order Partial Differential Equations (15 hours)

Origins of second-order PDEs, and solving linear PDEs with constant coefficients using methods of finding the complementary function and particular integral; Monge's method of integrating nonlinear second-order PDE of type  $Rr + Ss + Tt = V$  with variable coefficients.

### UNIT – III: Applications of Partial Differential Equations (12 hours)

Solution of one-dimensional diffusion equation and wave equation by method of separation of variables, d'Alembert's solution of the Cauchy problem for the one-dimensional wave equation; Solutions of homogeneous one-dimensional wave equations with initial boundary-value problems, and vibration of finite string with fixed ends; Traffic flow model.

### Essential Readings

- 1 Myint-U, Tyn & Debnath, Lokenath. (2007). Linear Partial Differential Equations for Scientists and Engineers (4th ed.). Birkhäuser. Indian Reprint.
- 2 Piaggio, H.T.H. (2004). Differential Equations. CBS Publishers & Distributors, Delhi.
- 3 Sneddon, Ian N. (2006). Elements of Partial Differential Equations, Dover Publications. Indian Reprint.

### Suggestive Readings

- Amaranath T. (2023). An Elementary Course in Partial Differential Equations (3rd ed.). Narosa Publishing House.
- Arrigo, Daniel (2023). An Introduction to Partial Differential Equations (2nd ed.). Springer.
- Kapoor, N. M. (2023). A Text Book of Differential Equations. Pitambar Publishing Company.

## GENERIC ELECTIVES (GE-8(iii)): ELEMENTS OF COMPLEX ANALYSIS

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Complex Analysis	4	3	1	0	Class XII pass with Mathematics	Metric Spaces, Multivariate Calculus



**Learning Objectives:** The primary objective of this course is to:

- Acquaint with the basic ideas of complex analysis.
- Learn complex-valued functions with visualization through relevant examples.
- Emphasize on Cauchy's theorems, series expansions and calculation of residues.

**Learning Outcomes:** The accomplishment of the course will enable the students to:

- Grasp the significance of differentiability of complex-valued functions leading to the understanding of Cauchy-Riemann equations.
- Study some elementary functions and evaluate the contour integrals.
- Learn the role of Cauchy-Goursat theorem and the Cauchy integral formula.
- Expand some simple functions as their Taylor and Laurent series, classify the nature of singularities, find residues, and apply Cauchy Residue theorem to evaluate integrals.

## **SYLLABUS OF GE-8(iii)**

### **Unit-I: Analytic Functions**

**(15 hours)**

Basic properties of complex numbers and their exponential form; Limits, continuity, and partial derivatives of functions of two variables. Limits, continuity, and partial derivatives of functions of a complex variable; Cauchy-Riemann Equations, Sufficient conditions for differentiability; Analytic functions and their examples; Exponential, logarithmic, and trigonometric functions.

### **Unit-II: Complex Integrals**

**(15 hours)**

Derivatives of functions, Definite integrals of functions, Contours, Contour integrals and examples, Upper bounds for moduli of contour integrals, Antiderivatives; Statement of Cauchy-Goursat theorem; Cauchy integral formula and its extension, Cauchy's inequality, Liouville's theorem and the fundamental theorem of algebra.

### **Unit-III: Series and Residues**

**(15 hours)**

Convergence of sequences and series of complex numbers; Taylor, and Laurent series with examples; Isolated singular points, Residues, Cauchy's residue theorem; Types of isolated singular points, Residues at poles and its examples.

### **Essential Reading**

1. Brown, James Ward & Churchill, Ruel V. (2014). Complex Variables and Applications (9th ed.). McGraw-Hill Education. Indian Reprint.

### **Suggestive Readings**

- Bak, Joseph & Newman, Donald J. (2010). Complex Analysis (3rd ed.). Undergraduate Texts in Mathematics, Springer.
- Mathews, John H., & Howell, Russell W. (2012). Complex Analysis for Mathematics and Engineering (6th ed.). Jones & Bartlett Learning. Narosa, Delhi. Indian Edition.

## GENERIC ELECTIVES (GE-8(iv)): OPTIMIZATION TECHNIQUES

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Optimization Techniques	4	3	1	0	Class XII pass with Mathematics	Multivariate Calculus

**Learning Objectives:** The primary objective of this course is to introduce:

- Nonlinear optimization problems
- Transshipment and dynamic programming problems
- Integer Programming, fractional programming problems
- Convex and generalized convex functions and their properties

**Learning Outcomes:** This course will enable the students to:

- Nonlinear programming problems and their applications
- Method to solve fractional programming problems with linear constraints
- Methods to solve dynamic programming problems using recursive computations

### SYLLABUS OF GE-8(iv)

#### UNIT-I: Transshipment and Dynamic Programming Problems (15 hours)

Transshipment problem, Shortest-route problem; Dynamic programming, Recursive forward and backward computation, Knapsack/fly-away/cargo-loading problems solution through dynamic programming.

#### UNIT-II: Integer Programming Problems (15 hours)

Integer programming problem, Gomory's cutting plane method for integer problems, Mixed integer problems, Branch and bound method.

#### UNIT-III: Nonlinear Programming Problems (15 hours)

Convex functions, Convex programming problems; Generalized convex functions; Linear fractional programming problem, Charnes and Cooper transformation, Simplex algorithm to solve linear fractional programming problem.

#### Essential Readings

1. Chandra, Suresh, Jayadeva, and Mehra, Aparna (2009). Numerical Optimization with Applications. Narosa Publishing House Pvt. Ltd. Delhi. Second Reprint 2016.
2. Taha, Hamdy A. (2017). Operations Research: An Introduction (10th ed.). Pearson.

#### Suggestive Reading

- Swarup, K., Gupta, P.K., and Mohan, M. (1984). Operations Research. Sultan Chand.