Appendix-13
Resolution No. 4-1-2

Curricular Structure and Syllabi of Courses in Second Year of Two-year M.Sc. Mathematics and One-year M.Sc. Mathematics

(Post Graduate Curriculum Framework (PGCF) -2024 based on NEP-2020)

Department of Mathematics
Faculty of Mathematical Sciences
University of Delhi
Delhi – 110007

CONTENTS

Programme Objectives and Outcomes

Programme Objectives

The two-year M.Sc. Mathematics programme's main objectives are to nurture a student to:

- inculcate and develop mathematical aptitude and the ability to think abstractly.
- develop computational abilities and programming skills.
- develop the ability to read, follow and appreciate mathematical text.
- train them to communicate mathematical ideas in a lucid and effective manner.
- orient them to apply their theoretical knowledge to solve real-life problems.
- prepare them for higher education and research in mathematics.

Programme Outcomes

On successful completion of the two-year M.Sc. Mathematics programme, a student will:

- have a strong foundation in core areas of Mathematics, both pure and applied.
- be able to apply mathematical skills and logical reasoning for problem solving.
- communicate mathematical ideas effectively, in writing as well as orally.
- have sound knowledge of mathematical modelling, programming and computational techniques as required for employment in industry.

Details of Courses in First Year of Two-year M.Sc. Mathematics

Semester	Discipline Specific Core (DSC)	Discipline Specific Elective (DSE)	Skill-Based Course (SBC)	Generic Elective (GE) Courses (offered by the Department of Mathematics)
Semester-I	**DSC-1:** Field Theory **DSC-2**: Introduction to Topology **DSC-3**: Ordinary Differential Equations	**DSE-1:** (i) Matrix Analysis (ii) Numerical Analysis **DSE-2*:** (i) Advanced Group Theory (ii) Nonlinear Optimization *Student will opt for DSE-2 or GE-1	Communicating Mathematics	**GE-1:** (i) Matrix Analysis (ii) Nonlinear Optimization
Semester-II	**DSC-4**: Module Theory **DSC-5**: Functional Analysis **DSC-6**: Complex Analysis	**DSE-3:** (i) Algebraic Number Theory (ii) General Topology **DSE-4#:** (i) Fourier Analysis (ii) Integral Equations #Student will opt for DSE-4 or GE-2	Appreciating Mathematics via Workshops and Seminars	**GE-2**: (i) Fourier Analysis (ii) Integral Equations

Curricular Structure for Second Year of Two-year PG Programme under Structure-1 (Only Course work)

Semester	DSC	DSE	2 Credit Course	Dissertation/ Academic Project/ Entrepreneurship	Total Credits
Semester-III	**DSC-7** **DSC-8** (8 Credits)	**DSE-5** **DSE-6** **DSE-7** OR **DSE-3, DSE-4 & GE-3** (12 Credits)	**Skill-based course/ Workshop/ Specialized laboratory/ Internship/ Apprenticeship/ Hands on learning** (2 Credits)	**NIL**	**22**
Semester-IV	**DSC-9** **DSC-10** (8 Credits)	**DSE-8** **DSE-9** **DSE-10** OR **DSE-5, DSE-6 & GE-4** (12 Credits)	**Skill-based course/ Workshop/ Specialized laboratory/ Internship/ Apprenticeship/ Hands on learning** (2 Credits)	**NIL**	**22**

Details of Courses in Second Year of Two-year M.Sc. Mathematics

Semester	DSC	DSE	SBC	GE
Semester-III	**DSC-7**: Fluid Dynamics **DSC-8**: Measure and Integration	**DSE-5:** (i) Algebraic Topology (ii) Commutative Algebra (iii) Dynamical Systems (iv) Theory of Bounded Operators **DSE-6:** (i) Advanced Complex Analysis (ii) Numerical Methods for Ordinary Differential Equations (iii) Representation of Finite Groups (iv) Topological Dynamics **DSE-7*:** (i) Advanced Functional Analysis (ii) Algebraic Coding Theory (iii) Differential Geometry (iv) Finite Element Methods *Student will opt for DSE-7 or GE-3	Developing Mathematical Ideas	**GE-3:** (i) Dynamical Systems (ii) Numerical Methods for Ordinary Differential Equations

Semester	DSC	DSE	SBC	GE
Semester-IV	**DSC-9**: Partial Differential Equations **DSC-10**: Analysis of Several Variables	**DSE-8:** (i) Advanced Fluid Dynamics (ii) Probability Theory (iii) Simplicial Homology Theory (iv) Theory of Unbounded Operators **DSE-9:** (i) Banach and C*-Algebras (ii) Chaos Theory (iii) Character Theory of Finite Groups (iv) Nonsmooth Optimization **DSE-10#:** (i) Computational Fluid Dynamics (ii) Differential Topology (iii) General Measure Theory (iv) Theory of Non-commutative Rings #Student will opt for DSE-10 or GE-4	Workshops and Seminars on Advanced Topics	**GE-4:** (i) Nonsmooth Optimization (ii) Probability Theory

Curricular Structure for Second Year of Two-year PG Programme under Structure-2 (Course work + Research)

Semester	DSC	DSE	2 Credit Course	Dissertation/ Academic Project/ Entrepreneurship	Total Credits
Semester-III	**DSC-7** **DSC-8** (8 Credits)	**DSE-5** **DSE-6** OR **DSE-3 & GE-3** (8 Credits)	**NIL**	(6 Credits)	**22**
Semester-IV	**DSC-9** **DSC-10** (8 Credits)	**DSE-7** **DSE-8** OR **DSE-4 & GE-4** (8 Credits)	**NIL**	(6 Credits)	**22**

Details of Courses in Second Year of Two-year M.Sc. Mathematics

Semester	DSC	DSE	Dissertation	GE
Semester-III	**DSC-7**: Fluid Dynamics **DSC-8**: Measure and Integration	**DSE-5 and DSE-6* (Select two papers, each from a different group)** **Group-1:** (i) Algebraic Topology (ii) Commutative Algebra (iii) Dynamical Systems (iv) Theory of Bounded Operators **Group-2:** (i) Advanced Complex Analysis (ii) Numerical Methods for Ordinary Differential Equations (iii) Representation of Finite Groups (iv) Topological Dynamics **Group-3:** (i) Advanced Functional Analysis (ii) Algebraic Coding Theory (iii) Differential Geometry (iv) Finite Element Methods *Student will opt for DSE-6 or GE-3	(6 Credits)	**GE-3:** (i) Dynamical Systems (ii) Numerical Methods for Ordinary Differential Equations

Semester	DSC	DSE	Dissertation	GE
Semester-IV	**DSC-9**: Partial Differential Equations **DSC-10**: Analysis of Several Variables	**DSE-7 and DSE-8# (Select two papers, each from a different group)** **Group-1:** (i) Advanced Fluid Dynamics (ii) Probability Theory (iii) Simplicial Homology Theory (iv) Theory of Unbounded Operators **Group-2:** (i) Banach and C*-Algebras (ii) Chaos Theory (iii) Character Theory of Finite Groups (iv) Nonsmooth Optimization **Group-3:** (i) Computational Fluid Dynamics (ii) Differential Topology (iii) General Measure Theory (iv) Theory of Non-commutative Rings #Student will opt for DSE-8 or GE-4	(6 Credits)	**GE-4:** (i) Nonsmooth Optimization (ii) Probability Theory

Curricular Structure for Second Year of Two-year PG Programme under Structure-3 (Research)

Semester	DSC	DSE (related to identified research field)	Research Methods/ Tools/ Writing	One Intensive Problem-based Research	Total Credits
Semester-III	**1 DSC** (course related to the area identified for research) **(4 Credits)**	**1 DSE** (course related or allied to the area identified for research) **(4 Credits)**	(a) **Advanced Research Methodology** of the core discipline + (b) **Tools for Research** **(2 x 2 =4 Credits)**	**(10 Credits)**	**22**
Semester-IV	**-**	**1 DSE** Or a DSE of an allied subject related to the area identified for research **(4 Credits)**	**Techniques of Research Writing** **(2 Credits)**	**(16 Credits)**	**22**

Details of Courses in Second Year of Two-year M.Sc. Mathematics

Semester	DSC	DSE	Research Methods/ Tools/ Writing	One Intensive Problem-based Research
Semester-III	**DSC**: Matrix Groups	**Any one of the following** (course related or allied to the area identified for research): (i) Advanced Complex Analysis (ii) Advanced Functional Analysis (iii) Algebraic Coding Theory (iv) Algebraic Topology (v) Commutative Algebra (vi) Differential Geometry (vii) Dynamical Systems (viii) Finite Element Methods (ix) Numerical Methods for Ordinary Differential Equations (x) Representation of Finite Groups (xi) Theory of Bounded Operators (xii) Topological Dynamics	(a) Advanced Research Methodology (b) Tools for Research	(10 Credits)

Semester	DSC	DSE	Research Methods/ Tools/ Writing	One Intensive Problem-based Research
Semester-IV	**-**	**Any one of the following (**course related or allied to the area identified for research)**:** (i) Advanced Fluid Dynamics (ii) Banach and C*-Algebras (iii) Chaos Theory (iv) Character Theory of Finite Groups (v) Computational Fluid Dynamics (vi) Differential Topology (vii) General Measure Theory (viii) Nonsmooth Optimization (ix) Probability Theory (x) Simplicial Homology Theory (xi) Theory of Non-commutative Rings (xii) Theory of Unbounded Operators	Techniques of Research Writing	(16 Credits)

Programme Objectives and Outcomes

Programme Objectives

The one-year M.Sc. Mathematics programme's main objectives are to nurture a student to:

- inculcate and develop mathematical aptitude and the ability to think abstractly.
- develop computational abilities and programming skills.
- develop the ability to read, follow and appreciate mathematical text.
- train them to communicate mathematical ideas in a lucid and effective manner.
- orient them to apply their theoretical knowledge to solve real-life problems.
- prepare them for higher education and research in mathematics.

Programme Outcomes

On successful completion of the one-year M.Sc. Mathematics programme, a student will:

- have a strong foundation in core areas of Mathematics, both pure and applied.
- be able to apply mathematical skills and logical reasoning for problem solving.
- communicate mathematical ideas effectively, in writing as well as orally.
- have sound knowledge of mathematical modelling, programming and computational techniques as required for employment in industry.

Curricular Structure for One-year PG Programme under Structure-1 (Only Course work)

Semester	DSC	DSE	2 Credit Course	Total Credits
Semester-I	**DSC-1** **DSC-2** (8 Credits)	**DSE-1** **DSE-2** **DSE-3** OR **DSE-1, DSE-2 & GE-1** (12 Credits)	**Skill-based course/ Workshop/ Specialized laboratory/ Internship/ Apprenticeship/ Hands on Learning** (2 Credits)	**22**
Semester-II	**DSC-3** **DSC-4** (8 Credits)	**DSE-4** **DSE-5** **DSE-6** OR **DSE-3, DSE-4 & GE-2** (12 Credits)	**Skill-based course/ Workshop/ Specialized laboratory/ Internship/ Apprenticeship/ Hands on Learning** (2 Credits)	**22**

Details of Courses in One-year M.Sc. Mathematics

Semester	DSC	DSE	SBC	GE
Semester-I	**DSC-1**: Fluid Dynamics **DSC-2**: Measure and Integration	**DSE-1:** (i) Commutative Algebra (ii) Dynamical Systems (iii) Introduction to Topology@ (iv) Theory of Bounded Operators **DSE-2:** (i) Numerical Methods for Ordinary Differential Equations (ii) Ordinary Differential Equations@ (iii) Representation of Finite Groups (iv) Topological Dynamics **DSE-3*:** (i) Advanced Functional Analysis (ii) Algebraic Coding Theory (iii) Differential Geometry (iv) Finite Element Methods *Student will opt for DSE-3 or GE-1 @Student is advised to opt for these papers, if not studied earlier	Developing Mathematical Ideas	**GE-1:** (i) Dynamical Systems (ii) Numerical Methods for Ordinary Differential Equations

Semester	DSC	DSE	SBC	GE
Semester-II	**DSC-3**: Partial Differential Equations **DSC-4**: Analysis of Several Variables	**DSE-4:** (i) Advanced Fluid Dynamics (ii) Module Theory@ (iii) Probability Theory (iv) Theory of Unbounded Operators **DSE-5:** (i) Banach and C*-Algebras (ii) Chaos Theory (iii) Complex Analysis@ (iv) Nonsmooth Optimization **DSE-6#:** (i) Computational Fluid Dynamics (ii) Differential Topology (iii) General Measure Theory (iv) Theory of Non-commutative Rings #Student will opt for DSE-6 or GE-2 @Student is advised to opt for these papers, if not studied earlier	Workshops and Seminars on Advanced Topics	**GE-2:** (i) Nonsmooth Optimization (ii) Probability Theory

Curricular Structure for One-year PG Programme under Structure-2 (Course work + Research)

Semester	DSC	DSE	2 Credit Course	Dissertation/ Academic Project/ Entrepreneurship	Total Credits
Semester-I	**DSC-1** **DSC-2** (8 Credits)	**DSE-1** **DSE-2** OR **DSE-1 & GE-1** (8 Credits)	**NIL**	(6 Credits)	**22**
Semester-II	**DSC-3** **DSC-4** (8 Credits)	**DSE-3** **DSE-4** OR **DSE-1 & GE-2** (8 Credits)	**NIL**	(6 Credits)	**22**

Details of Courses in One-year M.Sc. Mathematics

Semester	DSC	DSE	Dissertation	GE
Semester-I	**DSC-1**: Fluid Dynamics **DSC-2**: Measure and Integration	**DSE-1/ DSE-2* (Select two papers, each from a different group)** **Group-1:** (i) Commutative Algebra (ii) Dynamical Systems (iii) Introduction to Topology@ (iv) Theory of Bounded Operators **Group-2:** (i) Numerical Methods for Ordinary Differential Equations (ii) Ordinary Differential Equations@ (iii) Representation of Finite Groups (iv) Topological Dynamics **Group-3:** (i) Advanced Functional Analysis (ii) Algebraic Coding Theory (iii) Differential Geometry (iv) Finite Element Methods *Student will opt for DSE-2 or GE-1 @Student is advised to opt for these papers, if not studied earlier	(6 Credits)	**GE-1:** (i) Dynamical Systems (ii) Numerical Methods for Ordinary Differential Equations

Semester	DSC	DSE	Dissertation	GE
Semester-II	**DSC-3**: Partial Differential Equations **DSC-4**: Analysis of Several Variables	**DSE-3/ DSE-4# (Select two papers, each from a different group)** **Group-1:** (i) Advanced Fluid Dynamics (ii) Module Theory@ (iii) Probability Theory (iv) Theory of Unbounded Operators **Group-2:** (i) Banach and C*-Algebras (ii) Chaos Theory (iii) Complex Analysis@ (iv) Nonsmooth Optimization **Group-3:** (i) Computational Fluid Dynamics (ii) Differential Topology (iii) General Measure Theory (iv) Theory of Non-commutative Rings #Student will opt for DSE-4 or GE-2 @Student is advised to opt for these papers, if not studied earlier	(6 Credits)	**GE-2:** (i) Nonsmooth Optimization (ii) Probability Theory

Curricular Structure for One-year PG Programme under Structure-3 (Research)

Semester	DSC	DSE (related to identified research field)	Research Methods/ Tools/ Writing	One Intensive Problem-based Research	Total Credits
Semester-I	**1 DSC** (course related to the area identified for research) **(4 Credits)**	**1 DSE** (course related or allied to the area identified for research) **(4 Credits)**	(a) **Advanced Research Methodology** of the core discipline + (b) **Tools for Research** **(2 x 2 =4 Credits)**	**(10 Credits)**	**22**
Semester-II	**-**	**1 DSE** Or a DSE of an allied subject related to the area identified for research **(4 Credits)**	**Techniques of Research Writing** **(2 Credits)**	**(16 Credits)**	**22**

Details of Courses in One-year M.Sc. Mathematics

Semester	DSC	DSE	Research Methods/ Tools/ Writing	One Intensive Problem-based Research
Semester-I	**DSC**: Matrix Groups	**Any one of the following** (course related or allied to the area identified for research)**:** (i) Advanced Functional Analysis (ii) Algebraic Coding Theory (iii) Commutative Algebra (iv) Differential Geometry (v) Dynamical Systems (vi) Finite Element Methods (vii) Numerical Methods for Ordinary Differential Equations (viii) Representation of Finite Groups (ix) Theory of Bounded Operators (x) Topological Dynamics	(a) Advanced Research Methodology (b) Tools for Research	(10 Credits)

Semester	DSC	DSE	Research Methods/ Tools/ Writing	One Intensive Problem-based Research
Semester-II	-	**Any one of the following (**course related or allied to the area identified for research)**:** (i) Advanced Fluid Dynamics (ii) Banach and C*-Algebras (iii) Chaos Theory (iv) Computational Fluid Dynamics (v) Differential Topology (vi) General Measure Theory (vii) Nonsmooth Optimization (viii) Probability Theory (ix) Theory of Non-commutative Rings (x) Theory of Unbounded Operators	Techniques of Research Writing	(16 Credits)

Syllabi of Courses in Semester-III of M.Sc. Mathematics under Structure-1 (Only Course work)

Discipline Specific Core (DSC) Courses

DISCIPLINE SPECIFIC CORE – 7: FLUID DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSC-7: Fluid Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Calculus and Partial Differential Equations

Learning Objectives

The objective of this course is to:

- prepare a mathematical foundation to study the motion of fluids.
- develop concepts, models, and techniques to solve problems of fluid flow.
- develop the ability to conduct advanced studies and research in the broad field of fluid dynamics.

Learning Outcomes

After studying this course, the student will be able to:

- understand the concept of fluids, their classification, flow lines, models and approaches to study fluid flow.
- formulate mass and momentum conservation principles and obtain their solution for non-viscous flow.
- know potential flow, Bernoulli's equation, Kelvin's minimum energy and circulation theorems.
- understand two- and three-dimensional motion, complex potential, circle theorem, Blasius theorem, Weiss's and Butler's sphere theorems.
- apply the concept of stress and strain in viscous flow to derive Navier–Stokes equation of motion and energy equation.

Syllabus

Unit – 1 **(10 hours)**

Classification of fluids, Continuum model, Eulerian and Lagrangian approach of description, Differentiation following the fluid motion, Flow lines, vorticity and circulation, Conservation of mass: Equation of continuity, Boundary surface.

Unit – 2 **(12 hours)**

Forces in fluid motion, Conservation of momentum: Euler's equation of motion, Theory of irrotational motion: Integration of Euler's equation under different conditions, Bernoulli's equation, Impulsive motion, Kelvin's minimum energy and circulation theorems, Potential theorem.

Unit – 3 **(13 hours)**

Two-dimensional motion: Complex potential, Line sources, sinks, doublets and vortices, Two-dimensional image system, Milne–Thomson circle theorem, Images with respect to a plane and cylinder, Blasius theorem. Three-dimensional flows, Weiss's sphere theorem, Images with respect to sphere, Axi-symmetric flow, Stokes stream function, Butler's sphere theorem, Flow past spheres and cylinders.

Unit – 4 **(10 hours)**

Stress and strain analysis, Newton's law of viscosity, Laminar flow, Navier–Stokes equation of motion, Steady flow between parallel planes and Poiseuille flow, Constitutive equation, Energy equation.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] F. Chorlton, *Text Book of Fluid Dynamics*, CBS Publisher, 2005.
[2] R. W. Fox, P. J. Pritchard and A. T. McDonald, *Introduction to Fluid Mechanics*, Seventh Edition, John Wiley & Sons, 2009.
[3] P. K. Kundu, I. M. Cohen and D. R. Dowling, *Fluid Mechanics*, Sixth Edition, Academic Press, 2016.

Suggested Readings

(i) L. M. Milne-Thomson, *Theoretical Hydrodynamics*, The Macmillan company, USA, 1969.
(ii) D. E. Rutherford, *Fluid Dynamics*, Oliver and Boyd Ltd., 1978.

DISCIPLINE SPECIFIC CORE – 8: MEASURE AND INTEGRATION

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSC-8: Measure and Integration	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Real Analysis and Riemann Integration

Learning Objectives

The primary objective of this course is to:

- extend the notion of length of an interval with the introduction of the concept of Lebesgue outer measure for any subset of real line.
- investigate the properties of Lebesgue measurable sets and functions.
- familiarize students with the Lebesgue integration of functions and its comparison with Riemann integration.
- generalize the concepts of measure and integration to an abstract space.

Learning Outcomes

This course will enable the students to:

- verify whether a given subset of $\mathbb{R}$ or a real valued function is measurable.
- understand the requirement and the concept of the Lebesgue integral (a generalization of the Riemann integration) along with its properties.
- understand the statements and proofs of the fundamental integral convergence theorems and demonstrate their applications.
- carry out a comprehensive study of functions of bounded variation and their utility in understanding differentiation and integration.
- apply Hölder and Minkowski inequalities in L^p-spaces and understand completeness of L^p-spaces.

Syllabus

Unit – 1 **(14 hours)**

Lebesgue outer measure, Measurable sets, Lebesgue measure, Borel sets, Regularity, Measurable functions, Borel and Lebesgue measurability, Non-measurable sets.

Unit – 2 **(13 hours)**

Integration of nonnegative functions, General integral, Integration of series, Riemann and Lebesgue integrals.

Unit – 3 **(8 hours)**

Functions of bounded variation, Lebesgue's differentiation theorem, Differentiation and integration, Absolute continuity of functions.

Unit – 4 **(10 hours)**

Measures and outer measures, Measure spaces, Integration with respect to a measure, L^p-spaces, Hölder's and Minkowski's inequalities, Completeness of L^p-spaces.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] G. de Barra, *Measure Theory and Integration*, Ellis Horwood Ltd., Chichester, John Wiley & Sons, Inc., New York, 1981 (Indian Reprint, 2014).

Suggested Readings

(i) M. Capinski and P. E. Kopp, *Measure, Integral and Probability*, Springer, 2005.
(ii) E. Hewitt and K. Stromberg, *Real and Abstract Analysis: A Modern Treatment of the Theory of Functions of a Real Variable*, Springer, Berlin, 1975.
(iii) H. L. Royden and P. M. Fitzpatrick, *Real Analysis*, Fourth Edition, Pearson, 2015.

Discipline Specific Elective (DSE) Courses

DISCIPLINE SPECIFIC ELECTIVE – 5 (i): ALGEBRAIC TOPOLOGY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-5 (i): Algebraic Topology	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- understand the concepts of homotopic maps, homotopy type, retracts, and deformation retracts, and their role in algebraic topology.
- compute fundamental groups of basic topological spaces.
- acquire knowledge of covering projections, lifting properties, Borsuk–Ulam theorem, and classification techniques of covering spaces.
- learn free groups and free products to understand and apply the Seifert–Van Kampen theorem.

Learning Outcomes

This course will enable the students to:

- distinguish between spaces with the same homotopy type.
- compute fundamental groups of standard spaces such as n-sphere $\mathbb{S}^n$ and punctured planes.
- apply the concept of fundamental groups to prove Brouwer's fixed-point theorem and the Fundamental theorem of Algebra.
- explain the lifting theorems and their implications in topology.
- classify covering spaces for given base spaces.
- apply Seifert–Van Kampen theorem to compute fundamental groups of glued spaces.

Syllabus

Unit – 1 **(11 hours)**

Homotopic maps, Homotopy type, Retract and deformation retract.

Unit – 2 **(12 hours)**

Fundamental group, Calculation of fundamental groups of n-sphere $\mathbb{S}^n$ and punctured plane, Brouwer's fixed-point theorem, Fundamental theorem of Algebra.

Unit – 3 **(12 hours)**

Covering projections, Lifting theorems, Borsuk–Ulam theorem, Classification of covering spaces.

Unit – 4 **(10 hours)**

Free products, Free groups, Seifert–Van Kampen theorem and its applications.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. R. Munkres, *Elements of Algebraic Topology*, Addison-Wesley Publishing Company, 1984.
[2] T. B. Singh, *Introduction to Topology*, Springer Nature Singapore, 2019.

Suggested Readings

(i) G. E. Bredon, *Geometry and Topology*, Springer, 2014.
(ii) A. Hatcher, *Algebraic Topology*, Cambridge University Press, 2002.
(iii) W. S. Massey, *A Basic Course in Algebraic Topology*, World Publishing Corporation, 2009.
(iv) J. J. Rotman, *An Introduction to Algebraic Topology*, Springer, 2011.
(v) E. H. Spanier, *Algebraic Topology*, Springer-Verlag, 1989.

DISCIPLINE SPECIFIC ELECTIVE – 5 (ii): COMMUTATIVE ALGEBRA

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-5 (ii): Commutative Algebra	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra and Field Theory

Learning Objectives

The objective of this course is to:

- develop a solid understanding of the structure of commutative rings, ideals, their radicals, extension, contraction etc.
- study important constructions such as total quotient rings, localizations.
- develop basic foundation in other areas of mathematics such as algebraic geometry, algebraic number theory.

Learning Outcomes

This course will enable the students to:

- know the localization of rings at a prime ideal that is an algebraic analogue of the geometric notion concentrating attention near a point.
- know more closely the polynomial rings, power series rings in one or more variables over a commutative ring and their prime spectrum.
- define, identify, and elaborate integral closure of rings, valuations rings, discrete valuation rings, structure theorem of Artin rings.

Syllabus

Unit – 1 **(12 hours)**

Radical of an ideal, Prime avoidance lemma, Chinese remainder theorem, Extension and contraction of ideals, Jacobson radical of a ring, Nakayama lemma, Tensor product of modules.

Unit – 2 **(13 hours)**

Rings and modules of fractions, Localization, Local properties, Primary decomposition, First and second uniqueness theorem of primary decomposition, Associated prime ideals of decomposable ideals.

Unit – 3 **(10 hours)**

Integral ring extensions, Going up theorem, Going down theorem, Integrally closed domains, Valuation rings, Hilbert's Nullstellensatz theorem.

Unit – 4 **(10 hours)**

Noetherian rings, Primary decomposition in Noetherian rings, Artin rings, Structure theorem for Artin rings, Discrete valuation rings.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. F. Atiyah and I. G. MacDonald, *Introduction to Commutative Algebra*, CRC Press, Taylor & Francis, 2018.

Suggested Readings

(i) D. Eisenbud, *Commutative Algebra with a View Towards Algebraic Geometry*, Springer, 2004.
(ii) R. Y. Sharp, *Steps in Commutative Algebra*, Cambridge University Press, 2000.
(iii) B. Singh, *Basic Commutative Algebra*, World Scientific, 2011.
(iv) O. Zariski and P. Samuel, *Commutative Algebra*, Volume I & II, Springer, 1975.

DISCIPLINE SPECIFIC ELECTIVE – 5 (iii): DYNAMICAL SYSTEMS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-5 (iii): Dynamical Systems	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology and Ordinary Differential Equations

Learning Objectives
The primary objective of this course is to:
- understand discrete and continuous systems with case studies to study nonlinear systems of ordinary differential equations and dynamical systems.
- understand the concepts, models and techniques to realize the real-world problems and stability of the systems along with the chaotic dynamic behaviour of models by understanding bifurcations.

Learning Outcomes
This course will enable the students to learn:
- formulation of mathematical models with the stability analysis near the equilibrium points.
- how the concept of phase portraits helps to analyse mathematical model graphically.
- the qualitative behaviour of the solution set of a given system of differential equations including the invariant sets and limiting behaviour of the dynamical system or flow defined by the system of differential equations.
- how different bifurcations explain the chaotic behaviour of the system.

Syllabus

Unit – 1 **(13 hours)**
Linear systems: Jordan forms, Stability theory; Nonlinear systems: Fundamental existence-uniqueness theorem, Dependence on initial conditions and parameters, Flow of a differential equation, Linearization, Stable manifold theorem, Hartman–Grobman theorem.

Unit – 2 **(10 hours)**
Stability and Lyapunov functions, Saddle points, Nodes, Foci, Centers and nonhyperbolic critical points, Center manifold theorem.

Unit – 3 **(12 hours)**
Limit sets and attractors, Periodic orbits and limit cycles, Poincaré map, Stable manifold theorem for periodic orbits, Poincare-Bendixson theorem.

Unit – 4 **(10 hours)**
Bifurcations at nonhyperbolic equilibrium points, Saddle node, Transcritical and Pitchfork bifurcations, Hopf bifurcation.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. W. Hirsch, S. Smale and R. L. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, Third Edition, Academic Press, 2013.
[2] L. Perko, *Differential Equations and Dynamical Systems*, Third Edition, Springer Verlag, 2001.

Suggested Readings

(i) R. L. Devaney, *A First Course in Chaotic Dynamical Systems: Theory and Experiment*, CRC Press, Taylor & Francis, 2018.
(ii) S. H. Strogatz, *Nonlinear Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 2018.
(iii) S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, TAM Volume 2, Springer-Verlag, NY, 1990.

DISCIPLINE SPECIFIC ELECTIVE – 5 (iv): THEORY OF BOUNDED OPERATORS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-5 (iv): Theory of Bounded Operators	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis

Learning Objectives
The primary objective of this course is to:
- introduce some classes of bounded linear operators which play a central role in both pure and applied mathematics.
- study the properties and spectral theory of these operators.

Learning Outcomes
This course will enable the students to understand:
- the spectrum and sub-divisions of spectrum of standard operators like shifts and multiplication.
- the spectral theorem for some classes of bounded linear operators.
- the concepts of compactness, self-adjointness and positivity of bounded linear operators.
- trace class and Hilbert–Schmidt operators.

Syllabus

Unit – 1 **(11 hours)**
Properties of spectrum and resolvent of bounded operators, Subdivision of the spectrum including point, approximate and compression spectrum.

Unit – 2 **(10 hours)**
Operators on Hilbert spaces, Adjoint operator, Projections and idempotents, Operations with projections, Invariant and reducing subspaces.

Unit – 3 **(14 hours)**
Compact operators on Hilbert spaces, Diagonalisation of compact self-adjoint operators, Spectral theorem and functional calculus for Compact normal operators, Positive operators, Compact operators on Banach spaces, Spectral theory of compact operators.

Unit – 4 **(10 hours)**
Polar decomposition, Singular values, Trace class operators, Trace norm and Hilbert Schmidt operators.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] R. Bhatia, *Notes on Functional Analysis*, Hindustan Book Agency, 2009.
[2] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.

Suggested Readings

(i) E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, 2006.
(ii) B. Simon, *Operator Theory: A Comprehensive Course in Analysis*, Part 4, American Mathematical Society, 2015.
(iii) S. R. Garcia, J. Mashregi and W. T. Ross, *Operator Theory by Example*, Oxford University Press, 2023.

DISCIPLINE SPECIFIC ELECTIVE – 6 (i): ADVANCED COMPLEX ANALYSIS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-6 (i): Advanced Complex Analysis	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Complex Analysis

Learning Objectives
The primary objective of this course is to:
- explore the metric space structure of the spaces of continuous and analytic functions, through Arzela–Ascoli, Hurwitz and Montel theorems.
- investigate various characterizations of simply connected regions, with a special focus on the Riemann mapping theorem.
- use Runge's and Mittag–Leffler's theorems to approximate and interpolate analytic and meromorphic functions.
- analyze the range of analytic functions using Bloch's/ Landau's constants and Picard's theorem.

Learning Outcomes
This course will enable the students to:
- apply various variants of maximum modulus theorem, encompassing Hadamard's three circles theorem and Phragmen–Lindelöf theorem.
- comprehend the notions of normality, compactness, equicontinuity and local boundedness for the spaces of continuous and analytic functions.
- construct and factorize entire functions using infinite products, including special functions like gamma and zeta.
- analyze the harmonic functions on a disk using Poisson kernel, which in turn, solves the Dirichlet problem for a unit disk.

Syllabus

Unit – 1 **(12 hours)**
Convex functions and Hadamard's three circles theorem, Maximum-Modulus theorem (third version), Phragmen–Lindelöf theorem, Spaces of continuous functions, Normal and equicontinuous families, Arzela–Ascoli theorem.

Unit – 2 **(11 hours)**
Spaces of analytic functions, Hurwitz's theorem, Montel's theorem, Riemann mapping theorem, Infinite products, Weierstrass factorization theorem, Factorization of sine function.

Unit – 3 **(10 hours)**
Gamma and Riemann zeta function, Runge's theorem, Characterizations of simple connectedness, Mittag–Leffler's theorem.

Unit – 4 **(12 hours)**
Harmonic functions, Mean value property, Maximum and minimum principles, Harmonic function on a disk, Harnack's theorem, Range of an analytic function, Bloch's theorem, Bloch's and Landau's constants, Picard's theorem.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] J. B. Conway, *Functions of One Complex Variable*, Second Edition, Narosa Publishing House, New Delhi, 2002.

Suggested Readings
(i) L. V. Ahlfors, *Complex Analysis*, Mc Graw Hill Co., Indian Edition, 2017.
(ii) L. Hahn and B. Epstein, *Classical Complex Analysis*, Jones and Bartlett, 1996.
(iii) E. M. Stein and R. Shakarchi, *Complex Analysis*, Princeton University Press, 2003.
(iv) D. Ullrich, *Complex Made Simple*, Volume 97, American Mathematical Society, 2008.

DISCIPLINE SPECIFIC ELECTIVE – 6 (ii): NUMERICAL METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-6 (ii): Numerical Methods for Ordinary Differential Equations	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Ordinary Differential Equations

Learning Objectives

The primary objective of this course is to:

- develop the basic theory underlying the numerical solution of differential equations.
- introduce the concepts of consistency, stability and convergence of finite difference methods.
- execute the numerical schemes for the solution of differential equations.

Learning Outcomes

This course will enable the students to:

- gain a thorough understanding of the fundamental concepts involved in the construction and analysis of finite difference schemes for solving ordinary differential equations (ODEs).
- apply various numerical methods based on finite difference approaches to obtain approximate solutions for both initial value problems (IVPs) and boundary value problems (BVPs).
- develop the ability to select appropriate finite difference methods for specific types of problems and effectively apply them to real world applications.

Syllabus

Unit – 1 **(11 hours)**

Initial value problems: Existence and uniqueness of solution, Finite difference equation, Truncation error, Single step methods for first order IVPs and system of IVPs- Family of explicit and implicit Runge–Kutta methods, Taylor series methods, Derivation, Truncation error, Consistency, Stability and convergence analysis.

Unit – 2 **(12 hours)**

IVPs for the system of ODEs, Consistency, Zero stability and convergence of linear multistep methods, Routh–Hurwitz criterion, Order and error constant, First Dahlquist Barrier, Local truncation error and global truncation error, Error bounds, Local error, Linear stability theory, Higher order differential equations.

Unit – 3 **(12 hours)**

Derivation of explicit and implicit multistep methods for IVPs and system of IVPs, Truncation error, Stability and convergence analysis of family of Nystrom method, Adams–Bashforth method,

Adams–Moulton method, Milne–Simpson method, Predictor corrector method, and Modified predictor corrector method, Hybrid method, Multistep methods for second order IVPs.

Unit – 4 **(10 hours)**

Linear BVPs for second order ordinary differential equations, Shooting method, Finite difference method, Collocation method, Derivative boundary conditions, Nonlinear two-point BVPs, Higher order finite difference methods, Stability, Truncation error and convergence analysis.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. K. Jain, S. R. K. Iyenger and R. K. Jain, *Numerical Methods for Scientific and Engineering Computations*, Seventh Edition, New Age International Publisher, 2019.

[2] J. D. Lambert, *Numerical Methods for Ordinary Differential Systems: The Initial Value Problem*, John Wiley & Sons, 1991.

Suggested Readings

(i) K. E. Atkinson, W. Han and D. E. Stewart, *Numerical Solution of Ordinary Differential Equations*, John Wiley & Sons, 2009.

(ii) J. C. Butcher, *The Numerical Analysis of Ordinary Differential Equations*, Second Edition, Wiley, New York, 2008.

(iii) L. Collatz, *The Numerical Treatment of Differential Equations*, Springer-Verlag, 1966.

DISCIPLINE SPECIFIC ELECTIVE – 6 (iii): REPRESENTATION OF FINITE GROUPS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-6 (iii): Representation of Finite Groups	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra and Group Theory

Learning Objectives

The primary objective of this course is to:

- represent finite groups as groups of matrices (via homomorphisms) and apply the tools of linear algebra to study the group structure.
- introduce the notion of Group algebra, which plays an essential role in classifying representations of groups.
- to discuss some applications of representations of finite groups, such as the Burnside's theorem.

Learning Outcomes

This course will enable the students to:

- define and construct examples of group representations, *FG*-modules, group algebras.
- grasp key concepts and tools of representation theory and establish a link between *FG*-modules and group representations.
- prove and apply Maschke's theorem and Schur's lemma to describe all irreducible representations of finite groups over the field of complex numbers.
- apply the theory of characters and group representations to gain insight into group structure, such as normal subgroups, and the solubility of groups.

Syllabus

Unit – 1 **(11 hours)**

Representation of groups, *FG*-modules and *FG*-submodules, and reducibility, Permutation modules, *FG*-modules and equivalent representations, Reducible and irreducible *FG*-modules, Group algebra of *G*, Regular *FG*-module and regular representations, *FG*-homomorphisms, Direct sum of *FG*-modules.

Unit – 2 **(11 hours)**

Maschke's theorem for *FG*-modules and consequences. Schur's lemma and its converse, Application of Schur's lemma, Irreducible modules and group algebra, Structure of group algebra and space of *CG*-homomorphisms.

Unit – 3 **(10 hours)**

Characters and their properties, Permutation and regular characters, Inner product, Number of irreducible characters, Orthogonality relations and finding normal subgroups.

Unit – 4 **(13 hours)**

Algebraic numbers, Algebraic integers and their properties, Character values*,* The Burnside's *(p,q)-*theorem and solubility of some particular groups.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] G. James and M. Liebeck, *Representations and Characters of Groups*, Second Edition, Cambridge University Press, 2005.

Suggested Readings

(i) C. W. Curtis and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras*, American Mathematical Society, 2006.
(ii) W. Fulton and J. Harris, *Representation Theory - A First Course*, Springer-Verlag, 2004.
(iii) I. M. Issacs, *Character Theory of Finite Groups*, American Mathematical Society reprint, 2006.

DISCIPLINE SPECIFIC ELECTIVE – 6 (iv): TOPOLOGICAL DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-6 (iv): Topological Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- provide a strong background of topological dynamical systems including their applications.
- develop some useful and interesting dynamical properties like expansivity, shadowing and topological stability with supporting examples and results from symbolic and topological dynamics.
- introduce the celebrated Sarkovskii's theorem.

Learning Outcomes

This course will enable the students to:

- construct interesting examples of dynamical systems and topological conjugacy.
- visualize stable sets, omega sets and alpha limit sets.
- understand the applications of Sarkovskii's theorem.
- use subshifts of finite type to characterize irreducible matrices.
- prove key results on expansivity and shadowing regarding existence/non-existence, product, subspace and their different characterizations etc.
- find the class of topologically stable homeomorphisms.

Syllabus

Unit – 1 **(10 hours)**

Definition and examples (including real life examples) of dynamical systems, Orbits, Types of orbits, Topological conjugacy and orbits, Phase portrait-graphical analysis of orbit, Periodic points and stable sets, Omega and alpha limit sets and their properties.

Unit – 2 **(10 hours)**

Sarkovskii's theorem, Shift spaces and subshift, Subshift of finite type, Subshift represented by a matrix, Characterizations of irreducible matrices.

Unit – 3 **(13 hours)**

Definition and examples of expansive homeomorphisms, Properties of expansive homeomorphisms, Non-existence of expansive homeomorphism on the unit interval and unit circle, Generators and weak generators, Generators and expansive homeomorphisms.

Unit – 4 **(12 hours)**

Converging semi-orbits for expansive homeomorphisms, Definition, examples and properties of maps having shadowing property, Topological Anosov homeomorphisms and topological stability.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] N. Aoki and K. Hiraide, *Topological Theory of Dynamical Systems: Recent Advances*, North Holland Publications, 1994.
[2] M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge University Press, 2004.

Suggested Readings

(i) D. C. Hanselman and B. Little field, *Mastering MATLAB*, Pearson, 2012.
(ii) D. Lind and B. Marcus, *An Introduction to Symbolic Dynamics and Coding*, Cambridge University Press, 1996.
(iii) C. Robinson, *Dynamical Systems*, *Stability, Symbolic Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 1998.
(iv) J. de. Vries, *Elements of Topological Dynamics*, Springer, 1993.

DISCIPLINE SPECIFIC ELECTIVE – 7 (i): ADVANCED FUNCTIONAL ANALYSIS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-7 (i): Advanced Functional Analysis	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis and Topology

Learning Objectives

The primary objective of this course is to:

- define and explain the structure of a topological vector space and its fundamental properties.
- differentiate between normed, metrizable, locally convex and Hausdorff topological vector spaces.
- introduce the foundational theorems of functional analysis, including the Hahn–Banach, Banach–Steinhaus, Open mapping, and Closed graph theorems in the context of locally convex spaces.
- explain some applications of Banach–Alaoglu theorem and Krein–Milman theorem.

Learning Outcomes

This course will enable the students to:

- appreciate types of topological vector spaces and their separation properties.
- understand quotient spaces, weak topology and weak*-topology.
- analyze concepts of continuity, boundedness, and convergence for linear operators and functionals on topological vector spaces.
- understand the notion of local convexity and the role of seminorms in defining locally convex topologies.

Syllabus

Unit – 1 **(12 hours)**

Topological vector spaces, Types of Topological vector spaces, Separation properties, Linear mappings, Finite dimensional spaces, Metrization, Boundedness and continuity, Seminorms and local convexity, Normability.

Unit – 2 **(11 hours)**

Quotient spaces, Seminorms and quotient spaces, Examples, Baire category theorem, Banach–Steinhaus theorem, The open mapping theorem and the closed graph theorem on topological vector spaces.

Unit – 3 **(11 hours)**

Hahn–Banach separation theorem on topological vector spaces, Continuous extension theorem, Weak topologies, Weak topology and convexity, Weak topology and metrizability, Weak*-topology of a dual space, Compact convex sets, Banach–Alaoglu theorem and applications, Goldstine theorem.

Unit – 4 **(11 hours)**

Extreme points, Krein–Milman theorem, Convex hull of compact sets, Applications of Krein–Milman theorem: Stone–Weierstrass theorem, Markov–Kakutani fixed point theorem.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.
[2] W. Rudin, *Functional Analysis*, Second Edition, Tata Mc Graw-Hill, 2011.

Suggested Readings

(i) V. I. Bogachev and O. G. Smolyanov, *Topological Vector Spaces and Their Applications*, Springer, 2017.
(ii) S. Kantorovitz, *Introduction to Modern Analysis*, Oxford Graduate Texts in Mathematics, Oxford University Press, 2006.
(iii) J. Voigt, *A Course on Topological Vector Spaces*, Birkhäuser, 2020.

DISCIPLINE SPECIFIC ELECTIVE – 7 (ii): ALGEBRAIC CODING THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-7 (ii): Algebraic Coding Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra, Groups and Rings

Learning Objectives

The primary objective of this course is to:

- provide an introduction to algebraic coding theory, particularly linear codes.
- discuss bounds on the parameters along with cyclic codes.
- describe some well-known codes, such as Reed–Muller and Golay codes.
- explore the algebraic structure of Cyclic and Quadratic residue codes over fields and rings.

Learning Outcomes

This course will enable the students to:

- get an insight into the matrix representation of a code, as well as encoding and decoding.
- understand Hamming, MDS and Reed–Muller codes.
- describe cyclic codes and their generator polynomial.
- learn about special cyclic codes, such as Quadratic residue codes, and their properties over the ring $\mathbb{Z}_4$.

Syllabus

Unit – 1 **(10 hours)**

Error detecting and error correcting codes, Maximum likelihood decoding, Hamming distance, Linear codes, Hamming weight, Generator matrix, Parity check matrix, Equivalence of linear codes, Encoding and decoding of linear codes, Syndrome decoding.

Unit – 2 **(11 hours)**

Bounds on codes, Sphere covering bound, Hamming bound, Perfect codes, Binary Hamming codes, Binary Golay codes, Singleton bound and MDS codes. Propagation rules, Reed–Muller codes.

Unit – 3 **(12 hours)**

Cyclic codes, Cyclic codes as ideals, Generator polynomial of cyclic codes, Generator and parity-check matrices of cyclic codes, Decoding of cyclic codes, Burst error correcting codes.

Unit – 4 **(12 hours)**

Quadratic residue codes: QR codes over fields of characteristic 2 and 3, Cyclic codes and their generating polynomial over $\mathbb{Z}_4$, QR codes over $\mathbb{Z}_4$.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] S. Ling and C. Xing, *Coding Theory: A First Course*, Cambridge University Press, 2004.
[2] W. C. Huffman and V. Pless, *Fundamentals of Error Correcting Codes*, Cambridge University Press, 2010.

Suggested Readings
(i) R. Hill, *A First Course in Coding Theory*, Oxford University Press, 1986.
(ii) F. J. Mac William and N. J. A. Sloane, *Theory of Error Correcting Codes, Part I & II,* Elsevier/North-Holland, Amsterdam, 1977.

DISCIPLINE SPECIFIC ELECTIVE – 7 (iii): DIFFERENTIAL GEOMETRY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-7 (iii): Differential Geometry	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra, Multivariate Calculus and Differential Equations

Learning Objectives

The primary objective of this course is to introduce:

- surfaces and parametrized surfaces.
- orientation on connected surfaces.
- geodesics on surfaces.
- Weingarten maps on oriented surfaces.
- arc length and curvature of oriented plane curves.
- curvatures of oriented surfaces.

Learning Outcomes

This course will enable the students to:

- understand the concepts of level sets and graphs of functions, smooth vector fields, tangent spaces of level sets.
- appreciate surfaces and parametrized surfaces, Gauss map, geodesics and parallel transport on oriented surfaces.
- know what the Weingarten map of an oriented surface is, realize it as shape operator and use it to compute curvature of oriented plane curves.
- find global parametrization and hence arc length of an oriented plane curve.
- compute various types of curvatures of surfaces.

Syllabus

Unit – 1 **(10 hours)**

Level sets in $\mathbb{R}^{n+1}$ and graphs of functions, Smooth vector fields and existence and uniqueness of their integral curves, Tangent spaces of level sets at regular points, Surfaces in $\mathbb{R}^{n+1}$ as inverse images of regular values of smooth functions, Necessary condition for extrema of functions on surfaces-Lagrange multipliers, Existence of a normal vector field on a connected surface, Orientation, Gauss map.

Unit – 2 **(13 hours)**

The notion of a geodesic on a surface, Existence and uniqueness of a geodesic on a surface through a given point with a given velocity vector thereof, Covariant derivative of a smooth vector field, Parallel vector field along a curve, Existence and uniqueness of a parallel vector field along a curve with a given initial vector, Weingarten map of a surface at a point, Local parametrization and curvature of a plane curve.

Unit – 3 **(10 hours)**

Global parametrization and arc length of an oriented plane curve, Differential 1-forms, Line integral of 1-forms over parametrized curves.

Unit – 4 **(12 hours)**

Parametrized surfaces with examples, Curvature of surfaces, Normal curvature of a surface at a point in a given direction, Principal curvatures, First and second fundamental forms, Gauss-Kronecker curvature and mean curvature.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] A. Pressley, *Elementary Differential Geometry*, Springer-Verlag London Limited, 2012.
[2] J. A. Thorpe, *Elementary Topics in Differential Geometry*, Springer (India) Pvt. Limited, 2004.

Suggested Readings

(i) W. Kuhnel, *Differential Geometry: Curves-Surfaces-Manifolds*, Third Edition, American Mathematical Society, 2015.
(ii) B. O’ Neill, *Elementary Differential Geometry*, Second Edition, Academic Press INC., Academic Press, New York, 2006.

DISCIPLINE SPECIFIC ELECTIVE – 7 (iv): FINITE ELEMENT METHODS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-7 (iv): Finite Element Methods	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Differential Equations

Learning Objectives

The primary objective of this course is to:

- introduce basic aspects of finite element methods (FEM) including domain discretization, polynomial interpolation, application of boundary conditions, assembly of global arrays, and solution of the resulting algebraic systems.
- discuss the use of finite element methods in solving engineering problems in the domain of solid mechanics, fluid mechanics, heat transfer and electromagnetism.

Learning Outcomes

This course will enable the students to:

- use integral statement to deduce finite element approximations for the underlying linear partial differential equations.
- write special-purpose finite element programs within a procedural programming environment.
- use finite element methods to solve engineering problems in solids mechanics, fluid mechanics, heat transfer, and electromagnetism.
- assess the accuracy and reliability of finite element solutions and troubleshoot problems arising from errors in a given finite element analysis.

Syllabus

Unit – 1 **(12 hours)**

Basic concepts of weak formulation, Variational formulation of a one dimensional model equation, Basis function and finite element solutions, Collocation method, Ritz method, Least square method, Standard Galerkin method, FEM for model problem, Error estimate for FEM for model equation, Convergence analysis.

Unit – 2 **(11 hours)**

Various shapes of finite element, Higher order basis functions, Finite element methods for elliptic problems: Variational methods, Standard Galerkin method, Error estimate for FEM for elliptic problem, FEM for Poisson equation.

Unit – 3 **(12 hours)**

Finite element methods for parabolic problems: One dimensional model problems, Semi-discretization in space, Error estimates, Discretization in space and time, Galerkin method, Finite element methods for hyperbolic problems: Standard Galerkin method, Standard Galerkin method with strongly and weakly imposed boundary conditions.

Unit – 4 **(10 hours)**

Applications of the FEM to second order BVPs in one dimension, Applications of the FEM to linear elliptic, parabolic and hyperbolic equations.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] G. Evans, J. Blackledge and P. Yardley, *Numerical Methods for Partial Differential Equations*, Springer-Verlag, London, 2000.
[2] C. Johnson, *Numerical Solutions of Partial Differential Equations by Finite Element Methods*, *Cambridge University Press*, Cambridge, 1987.
[3] J. Whiteley, *Finite Element Methods - A Practical Guide*, Springer, 2016.

Suggested Readings

(i) Z. Chen, *Finite Element Methods and Their Applications*, Springer-Verlag, New York, 2005.
(ii) V. Thomee, *Galerkin Finite Element Methods for Parabolic Problems*, Second Edition, Springer-Verlag, Berlin, 2006.

Skill-Based Course (SBC)

DEVELOPING MATHEMATICAL IDEAS

Course Title	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Developing Mathematical Ideas	**2**	**0**	**0**	**2**	**Same as for entry to M.Sc. Mathematics**	**NIL**

Learning Objectives

This course will train students

- to develop skills to create new mathematical ideas independently.
- to present these ideas adeptly.

Learning Outcomes

Students will be able to

- hone their analytical skills and their ability to think critically.
- learn to work collaboratively.
- acquire skills that help them to create and develop new mathematical ideas.

Methodology

We plan to form groups of students, say of 5-6 each, who will be assigned a piece of mathematical work (article/ paper published in reputed journals/ periodicals/book chapters). The designated groups will be required to read and understand this mathematical work under the supervision of a faculty member. They will be further encouraged to pose meaningful questions/problems in the context of the mathematics they have read and possibly offer solutions. Presentations will be conducted for these groups as a part of their assessment process.

Appropriate material for the study will be provided by the department/faculty. Papers/articles for example, may be chosen from resources like *Involve* (Link: https://msp.org/involve/), *SIAM Undergraduate Research Online* (Link: https://www.siam.org/publications/siuro), *The American Mathematical Monthly* (Mathematical Association of America, Taylor and Francis, Link: https://www.tandfonline.com/journals/uamm20), *Mathematics Magazine* (Mathematical Association of America, Taylor and Francis, Link: https://www.tandfonline.com/journals/umma20), *The Mathematics Student* (Indian Mathematical Society, Link: https://www.indianmathsoc.org/MS.html) and *The Mathematical Intelligencer* (Springer Nature, Link: https://link.springer.com/journal/283). Advanced topics beyond the prescribed syllabus from textbooks/ research monographs may also be chosen.

Generic Elective (GE) Courses

GENERIC ELECTIVE – 3 (i): DYNAMICAL SYSTEMS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
GE-3 (i): Dynamical Systems	4	3	1	0	Class XII pass with Mathematics	Basics of Topology and Ordinary Differential Equations

Learning Objectives

The primary objective of this course is to:

- understand discrete and continuous systems with case studies to study nonlinear systems of ordinary differential equations and dynamical systems.
- understand the concepts, models and techniques to realize the real-world problems and stability of the systems along with the chaotic dynamic behaviour of models by understanding bifurcations.

Learning Outcomes

This course will enable the students to learn:

- formulation of mathematical models with the stability analysis near the equilibrium points.
- how the concept of phase portraits helps to analyse mathematical model graphically.
- the qualitative behaviour of the solution set of a given system of differential equations including the invariant sets and limiting behaviour of the dynamical system or flow defined by the system of differential equations.
- how different bifurcations explain the chaotic behaviour of the system.

Syllabus

Unit – 1 **(13 hours)**

Linear systems: Jordan forms, Stability theory; Nonlinear systems: Fundamental existence-uniqueness theorem, Dependence on initial conditions and parameters, Flow of a differential equation, Linearization, Stable manifold theorem, Hartman–Grobman theorem.

Unit – 2 **(10 hours)**

Stability and Lyapunov functions, Saddle points, Nodes, Foci, Centers and nonhyperbolic critical points, Center manifold theorem.

Unit – 3 **(12 hours)**

Limit sets and attractors, Periodic orbits and limit cycles, Poincaré map, Stable manifold theorem for periodic orbits, Poincare–Bendixson theorem.

Unit – 4 **(10 hours)**

Bifurcations at nonhyperbolic equilibrium points, Saddle node, Transcritical and Pitchfork bifurcations, Hopf bifurcation.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. W. Hirsch, S. Smale and R. L. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, Third Edition, Academic Press, 2013.
[2] L. Perko, *Differential Equations and Dynamical Systems*, Third Edition, Springer Verlag, 2001.

Suggested Readings

(i) R. L. Devaney, *A First Course in Chaotic Dynamical Systems: Theory and Experiment*, CRC Press, Taylor & Francis, 2018.
(ii) S. H. Strogatz, *Nonlinear Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 2018.
(iii) S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, TAM Volume 2, Springer-Verlag, NY, 1990.

GENERIC ELECTIVE – 3 (ii): NUMERICAL METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
GE-3 (ii): Numerical Methods for Ordinary Differential Equations	4	3	1	0	Class XII pass with Mathematics	Basics of Ordinary Differential Equations

Learning Objectives

The primary objective of this course is to:

- develop the basic theory underlying the numerical solution of differential equations.
- introduce the concepts of consistency, stability and convergence of finite difference methods.
- execute the numerical schemes for the solution of differential equations.

Learning Outcomes

This course will enable the students to:

- gain a thorough understanding of the fundamental concepts involved in the construction and analysis of finite difference schemes for solving ordinary differential equations (ODEs).
- apply various numerical methods based on finite difference approaches to obtain approximate solutions for both initial value problems (IVPs) and boundary value problems (BVPs).
- develop the ability to select appropriate finite difference methods for specific types of problems and effectively apply them to real world applications.

Syllabus

Unit – 1 **(11 hours)**

Initial value problems: Existence and uniqueness of solution, Finite difference equation, Truncation error, Single step methods for first order IVPs and system of IVPs- Family of explicit and implicit Runge–Kutta methods, Taylor series methods, Derivation, Truncation error, Consistency, Stability and convergence analysis.

Unit – 2 **(12 hours)**

IVPs for the system of ODEs, Consistency, Zero stability and convergence of linear multistep methods, Routh–Hurwitz criterion, Order and error constant, First Dahlquist Barrier, Local truncation error and global truncation error, Error bounds, Local error, Linear stability theory, Higher order differential equations.

Unit – 3 **(12 hours)**

Derivation of explicit and implicit multistep methods for IVPs and system of IVPs, Truncation error, Stability and convergence analysis of family of Nystrom method, Adams–Bashforth method,

Adams–Moulton method, Milne–Simpson method, Predictor corrector method, and Modified predictor corrector method, Hybrid method, Multistep methods for second order IVPs.

Unit – 4 **(10 hours)**
Linear BVPs for second order ordinary differential equations, Shooting method, Finite difference method, Collocation method, Derivative boundary conditions, Nonlinear two-point BVPs, Higher order finite difference methods, Stability, Truncation error and convergence analysis.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] M. K. Jain, S. R. K. Iyenger and R. K. Jain, *Numerical Methods for Scientific and Engineering Computations*, Seventh Edition, New Age International Publisher, 2019.
[2] J. D. Lambert, *Numerical Methods for Ordinary Differential Systems: The Initial Value Problem*, John Wiley & Sons, 1991.

Suggested Readings
(i) K. E. Atkinson, W. Han and D. E. Stewart, *Numerical Solution of Ordinary Differential Equations*, John Wiley & Sons, 2009.
(ii) J. C. Butcher, *The Numerical Analysis of Ordinary Differential Equations*, Second Edition, Wiley, New York, 2008.
(iii) L. Collatz, *The Numerical Treatment of Differential Equations*, Springer-Verlag, 1966.

Syllabi of Courses in Semester-IV of M.Sc. Mathematics under Structure-1 (Only Course work)

Discipline Specific Core (DSC) Courses

DISCIPLINE SPECIFIC CORE – 9: PARTIAL DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSC-9: Partial Differential Equations	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Multivariate Calculus and Differential Equations

Learning Objectives

The main objective of this course is to introduce:

- well-posedness, fundamental solutions, existence and uniqueness of solutions for Laplace equation, Poisson equation and heat equation.
- solution for wave equation by spherical means.
- characteristics, complete integrals, envelopes and conservation laws for first-order nonlinear partial differential equations.
- classical solution techniques such as Green's function, similarity solutions and transform methods.

Learning Outcomes

This course will enable the students to:

- understand Laplace equation, Poisson equation, and Heat equation, their fundamental solutions, uniqueness principles, mean value properties, and Green's function.
- apply the method of spherical means to solve homogeneous and nonhomogeneous wave equations.
- use characteristics to solve nonlinear partial differential equations, construct complete integrals and envelopes, and understand conservation laws.
- implement various techniques such as similarity solutions and transform methods to derive solutions of different types of partial differential equations.

Syllabus

Unit – 1 **(12 hours)**

Well-posed problems, Classical solution, Laplace equation, Poisson equation, Fundamental solution, Strong maximum principle and uniqueness of solution, Mean value formulas, Representation formula, Green's function, Poisson's formula.

Unit – 2 **(10 hours)**

Heat equation, Fundamental solution for homogeneous and nonhomogeneous initial-value problems, Mean value formula, Strong maximum principle and uniqueness of solution, Local estimates for the solution.

Unit – 3 **(13 hours)**

Wave equation: Solution of homogeneous and nonhomogeneous problems by spherical means,

Nonlinear first order partial differential equations: Complete integrals and envelopes, Characteristics, Introduction to conservation laws.

Unit – 4 **(10 hours)**
Other solution methods: Similarity solutions, Fourier transform and Laplace transform, Cole–Hopf transformation, Potential function, Hodograph and Legendre transform.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] L. C. Evans, *Partial Differential Equations*, American Mathematical Society, Providence, RI, 1998.
[2] F. John, *Partial Differential Equations*, Fourth Edition, Springer-Verlag, New York, 1982.

Suggested Readings
(i) P. R. Garabedian, *Partial Differential Equations*, John Wiley & Sons, Inc., New York- London-Sydney, 1964.
(ii) A. K. Nandakumaran and P. S. Datti, *Partial Differential Equations: Classical Theory with a Modern Touch*, Cambridge University Press, 2020.

DISCIPLINE SPECIFIC CORE – 10: ANALYSIS OF SEVERAL VARIABLES

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSC-10: Analysis of Several Variables	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Calculus, Real Analysis including Riemann Integration

Learning Objectives

The primary objective of this course is to:

- introduce differentiation of vector valued functions on $\mathbb{R}^n$ and their properties.
- familiarize students with integration of functions over rectangles and bounded sets in $\mathbb{R}^n$.
- extend integration of functions to unbounded sets in $\mathbb{R}^n$.
- study change of variables and its applications.

Learning Outcomes

This course will enable the students to:

- check differentiability of vector valued functions on $\mathbb{R}^n$, understand the relation between directional derivative and differentiability, apply chain rule, mean value theorems, inverse and implicit function theorems.
- understand higher order derivatives and be able to apply Taylor's formulas with integral remainder, Lagrange's remainder and Peano's remainder.
- master the concepts of integration over rectangles and bounded sets in $\mathbb{R}^n$.
- generalize the integration theory to unbounded sets in $\mathbb{R}^n$.
- grasp the effect of change of variables in integration.

Syllabus

Unit– 1 **(12 hours)**

The differentiability of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$, Partial derivatives and differentiability, Directional derivatives and differentiability, Chain rule, Mean value theorems, Inverse function theorem and Implicit function theorem.

Unit– 2 **(11 hours)**

Derivatives of higher order, Taylor's formulas with integral remainder, Lagrange's remainder and Peano's remainder, Integral over a rectangle, Existence of the integral.

Unit– 3 **(10 hours)**

Evaluation of the integral, Fubini's theorem, Integral over a bounded set.

Unit– 4 **(12 hours)**

Rectifiable sets, Improper integrals, Change of variable theorem, Applications of change of variables.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] M. Giaquinta and G. Modica, *Mathematical Analysis: An Introduction to Functions of Several Variables*, Birkhäuser, 2009.
[2] J. R. Munkres, *Analysis on Manifolds*, CRC Press, Taylor & Francis, 2018.

Suggested Readings
(i) W. Rudin, *Principles of Mathematical Analysis*, Third Edition, Mc Graw Hill, 1986.
(ii) M. Spivak, *Calculus on Manifolds: A Modern Approach to Classical Theorems of Advanced Calculus*, Taylor & Francis, 2018.

Discipline Specific Elective (DSE) Courses

DISCIPLINE SPECIFIC ELECTIVE – 8 (i): ADVANCED FLUID DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-8 (i): Advanced Fluid Dynamics	**4**	**3**	**1**	**0**	**Same as for entry to M.Sc. Mathematics**	**Basics of Calculus and Partial Differential Equations**

Learning Objectives

The primary objective of this course is to:

- prepare a foundation for advanced studies in compressible flow, boundary layer theory and magnetohydrodynamics.
- develop concepts, models, and techniques that enable problem-solving in compressible flow, boundary layer theory and magnetohydrodynamics.
- equip students with concepts and techniques to conduct research in the above mentioned domains.

Learning Outcomes

This course will enable the students to:

- learn conservation laws, first and second laws of thermodynamics, internal energy and entropy, different forms of energy equations and dimensional analysis.
- know about compressibility in real fluids, wave motion, sound waves, hyperbolic and dispersive waves, shock waves, their formation, properties and elementary analysis.
- know the concepts of boundary layer, boundary layer equations and their solutions, measurements of boundary layer thickness.
- understand the interaction between hydrodynamic processes and electromagnetic phenomena.
- formulate the basic equations of motion in inviscid and viscous conducting fluid flow and explain Alfven's theorem and magnetohydrodynamic (MHD) waves and MHD shocks.

Syllabus

Unit – 1 **(11 hours)**

Flow characteristics, Conservation laws, Equation of state of a substance, First and second law of thermodynamics, Internal energy and entropy, Energy equation, Nondimensionalizing the basic equations of incompressible viscous fluid flow, Nondimensional numbers.

Unit – 2 **(12 hours)**

Compressibility effects in real fluids, Equations of motion, Sound wave, Hyperbolic and dispersive waves, Isentropic gas flow, Flow through a nozzle, Method of characteristics, Shock jump conditions, Non-linear plane waves, Shock waves and their elementary analysis, Similarity solutions.

Unit – 3 **(11 hours)**
Boundary layer concept, Estimation of boundary layer thickness and friction forces, Prandtl's boundary layer equations, Boundary layer along a flat plate, Boundary layer thickness, General properties of the boundary layer equations, Similar solutions, Momentum and energy integral equations for the boundary layer.

Unit – 4 **(11 hours)**
Maxwell's electromagnetic field equations, Magnetohydrodynamic (MHD) approximations, Magnetic field equation, Magnetic Reynolds number, Magnetic body force, Equations of Motions of conducting fluid, Alfven's theorem, MHD waves, MHD shock waves.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] F. Chorlton, *Text Book of Fluid Dynamics*, CBS Publisher, 2005.
[2] H. Schlichting and K. Gersten, *Boundary Layer Theory*, Ninth Edition, Springer, 2017.
[3] G. B. Witham, *Linear and Nonlinear Waves*, John Wiley & Sons, 1999.

Suggested Readings
(i) K. R. Cramer and S. I. Pai, *Magnetofluid Dynamics for Engineers and Applied Physics*, McGraw Hill Book Co., New York, 1973.
(ii) Y. Shao-Wen, *Foundations of Fluid Mechanics*, PHI, New Delhi, 1960.

DISCIPLINE SPECIFIC ELECTIVE – 8 (ii): PROBABILITY THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-8 (ii): Probability Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Measure Theory

Learning Objectives
The primary objective of this course is to introduce:
- probability space as a measure space and random variables as measurable functions.
- expectation and moments of random variables.
- notion of convergence in probability.
- conditioning on sub-σ-algebra.

Learning Outcomes
This course will enable the students to learn:
- about probability or uncertainty in abstract setting.
- moments and expectation of random variables which help to understand applications of probability in industry.
- how to apply the idea of convergence in probability.
- weak law and strong law of large numbers and their applications.

Syllabus

Unit – 1 **(11 hours)**
Probability: σ-algebra, Constructing probability triples, The extension theorem, Random variables, Independence of events, Continuity of probabilities, Limit events, The Borel–Cantelli lemma.

Unit – 2 **(10 hours)**
Expected values: Simple, general non-negative and arbitrary random variables, Moment generating functions, Markov's inequality, Chebyshev's inequality.

Unit – 3 **(12 hours)**
Convergence of random variables: Convergence almost surely, Convergence in probability, Weak law of large numbers, Strong law of large numbers.

Unit – 4 **(12 hours)**
Distributions of random variables: Examples of distributions, Characteristic functions, The central limit theorem, Conditional probability, Conditioning on random variable, Conditioning on a sub-σ-algebra, Conditional variance.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. S. Rosenthal, *A First Look at Rigorous Probability Theory*, Second Edition, World Scientific, Singapore, 2006.

Suggested Readings

(i) W. Feller, *An Introduction to Probability Theory and its Applications*, Volume 1, Third Edition, Wiley, 2008.
(ii) J. E. Michael and J. S. Rosenthal, *Probability and Statistics: The Science of Uncertainty*, Second Edition, W. H. Freeman & Co Ltd., 2009.
(iii) S. Ross, *A First Course in Probability*, Tenth Edition, Pearson Education, 2022.
(iv) D. W. Stroock, *Probability Theory, An Analytic View*, Cambridge University Press, 2024.

DISCIPLINE SPECIFIC ELECTIVE – 8 (iii): SIMPLICIAL HOMOLOGY THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-8 (iii): Simplicial Homology Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- introduce the foundations of simplicial complexes and homology theory.
- develop an understanding of chain mappings and induced homomorphisms.
- apply these concepts to establish fundamental results in topology such as Euler–Poincaré theorem, Brouwer's and Lefschetz fixed-point theorems.

Learning Outcomes

This course will enable the students to:

- identify hyperplanes, simplexes and finite simplicial complexes as subsets of a Euclidean space.
- learn the idea of compact triangulable spaces as geometric carriers of finite simplicial complexes (polyhedra).
- learn the use of homological algebra to associate simplicial homology groups and illustrate it by computing simplicial homology groups of some well-known compact polyhedral.
- prove important applications of simplicial homology theory like invariance of dimension, Euler's formula, Lefschetz and Brouwer's fixed point theorems.

Syllabus

Unit – 1 **(11 hours)**

Geometric simplexes, Geometric complexes and polyhedra, Simplicial maps, Simplicial approximation of continuous maps between two polyhedral.

Unit – 2 **(12 hours)**

Orientation of geometric complexes, Chain complexes, Simplicial homology groups, Structure of homology groups, Relative homology groups, Computation of homology groups, Homology groups of n-sphere.

Unit – 3 **(12 hours)**

Chain mappings, Chain derivation, Chain homotopy, Contiguous maps, Homomorphism induced by continuous maps between two polyhedra, Functorial property of induced homomorphisms, Topological and homotopy invariance of homology groups.

Unit – 4 **(10 hours)**

Euler–Poincaré theorem and Euler's formula, Invariance of dimension, Brouwer's fixed point theorem, Degree of self-mappings of $\mathbb{S}^n$, Brouwer's degree theorem, Existence of eigen values, Lefschetz fixed point theorem.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] F. H. Croom, *Basic Concepts of Algebraic Topology*, Springer, 1978.

Suggested Readings

(i) M. K. Agoston, *Algebraic Topology: A First Course*, Marcel Dekker, 1976.
(ii) M. A. Armstrong, *Basic Topology*, Springer, 1983.
(iii) S. Deo, *Algebraic Topology - A Primer*, Second Edition, Hindustan Book Agency, 2018.

DISCIPLINE SPECIFIC ELECTIVE – 8 (iv): THEORY OF UNBOUNDED OPERATORS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-8 (iv): Theory of Unbounded Operators	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis and Bounded Operators

Learning Objectives

The primary objective of this course is to:

- introduce the notion of unbounded operators.
- develop the theory of operator semigroups and understand their role in applications, particularly for solving differential equations.

Learning Outcomes

This course will enable the students to:

- identify closed and closable linear operators on Banach spaces.
- compute adjoints of unbounded linear operators.
- understand spectral properties of some unbounded operators.
- comprehend the role unbounded operators and semigroups play in applications, particularly in studying solutions of differential equations.

Syllabus

Unit – 1 **(10 hours)**

Unbounded linear operators, Hilbert adjoints, Hellinger–Toeplitz theorem, Hermitian, symmetric and self-adjoint linear operators, Closed linear operators, Closable operators and their closures on Banach spaces.

Unit – 2 **(12 hours)**

Cayley transform, Deficiency indices, Spectral properties of self-adjoint operators, Multiplication and differentiation operators and their spectra.

Unit – 3 **(11 hours)**

Analytic properties of exponential functions, Matrix Semigroups, Uniformly continuous semigroups, Semigroups on Hilbert spaces, Strongly continuous semigroups.

Unit – 4 **(12 hours)**

Generators of semigroups and their resolvents, Hille–Yosida theorem (for contraction semigroup), Dissipative operators and their properties, Lumer–Phillips theorem, Generators of Group, Stone's theorem.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] K. J. Engle and R. Nagel, *One-parameter Semigroups for Linear Evolution Equations*, Springer-Verlag, New York, 2000.

[2] E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, 2006.

Suggested Readings

(i) S. Goldberg, *Unbounded Linear Operators: Theory and Applications*, Dover Publications, 2006.

(ii) E. Hille and R. S. Phillips, *Functional Analysis and Semi-groups*. American Mathematical Society, Providence, RI, 1957.

(iii) A. Pazy, *Semigroups of Linear Operators and Applications to Partial Differential Equations*, Springer, 1983.

(iv) M. Schechter, *Principles of Functional Analysis*, Second Edition, American Mathematical Society, 2001.

(v) J. Weidmann, *Linear Operators in Hilbert Spaces, Graduate Texts in Mathematics*, Springer, New York, 1980.

DISCIPLINE SPECIFIC ELECTIVE – 9 (i): BANACH AND C*-ALGEBRAS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-9 (i): Banach and C*-Algebras	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis and Topology

Learning Objectives

The primary objective of this course is to introduce:

- Banach algebras and C*-algebras.
- various ways to construct new operator algebras using given ones.
- spectrum of elements in Banach algebras and to study its properties.
- Gelfand representations of commutative Banach algebras and of C*-algebras.

Learning Outcomes

This course will enable the students to:

- familiarize with the representations of operator algebras.
- realize commutative Banach algebras and abelian C*-algebras as space of continuous functions on locally compact groups.
- understand the powerful tool of functional calculus.
- identify any C*-algebra as closed *-subalgebra of space of bounded linear operators on a Hilbert space.

Syllabus

Unit – 1 **(11 hours)**

Elementary properties and examples of Banach algebras, Ideals and quotients, Invertible elements, Spectrum and spectral radius, Spectral radius formula, Spectral mapping theorem (for polynomials), Gelfand–Mazur theorem.

Unit – 2 **(11 hours)**

Multiplicative linear functionals, Commutative Banach algebra, w*-topology, Gelfand transform of an element, Structure space, Gelfand representation.

Unit – 3 **(12 hours)**

Elementary properties and examples of C*-algebras, Unitization, Gelfand–Naimark representation of commutative C*-algebras, Continuous functional calculus, Spectral mapping theorem for normal elements, Positive elements of C*-algebras.

Unit – 4 **(11 hours)**

Ideals in C*-algebras, Approximate units, Quotients, Positive linear functionals, Gelfand–Naimark–Segal representation of C*-algebras.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.
[2] G. J. Murphy, *C*-algebras and Operator Theory*, Academic Press Inc., 1990.

Suggested Readings

(i) J. B. Conway, *A Course in Operator Theory, Graduate Texts in Mathematics*, Springer, 2007.
(ii) J. Dixmier, *C*-algebras*, North-Holland Publishing Company, 1977.
(iii) R. G. Douglas, *Banach Algebra Techniques in Operator Theory, Graduate Texts in Mathematics*, Springer, 1998.
(iv) E. Kaniuth, *A Course on Commutative Banach Algebras*, Graduate Texts in Mathematics, Springer, 2009.
(v) S. Kantorovitz, *Introduction to Modern Analysis*, Oxford Graduate Texts in Mathematics, Oxford University Press, 2006.
(vi) M. Takesaki, *Theory of Operator Algebras I*, Springer, 2002.

DISCIPLINE SPECIFIC ELECTIVE – 9 (ii): CHAOS THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-9 (ii): Chaos Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- introduce some useful and interesting notions like Topological Transitivity and Sensitive dependence on initial conditions.
- study different types of chaos including Devaney's chaos and finding their interrelationships.
- know classical result that period three implies chaos on intervals.
- relate chaos and decomposition theorems.
- study Topological entropy through open covers and also Bowen's definition of entropy, equivalence of these two definitions on compact metric spaces.
- study various interesting results related to topological entropy.

Learning Outcomes

This course will enable the students to:

- construct interesting examples of Topological transitive maps, Topological mixing maps etc.
- know classical examples of Devaney's chaotic maps like tent map, shift maps, logistic maps.
- study and compare different types of chaos.
- find relation between transitivity and chaos on intervals.
- relate chaos theory and classical decomposition theorems.
- study very useful notion of Topological entropy including its properties.
- calculate entropy of any homeomorphism of closed unit interval and of unit circle.

Syllabus

Unit – 1 **(12 hours)**

Topological Transitivity, Locally eventually onto maps, Topological mixing, Sensitive dependence on initial conditions, Devaney's definition of chaos, Transitivity and limit sets for continuous interval maps.

Unit – 2 **(11 hours)**

Characterizing topological mixing in terms of topological transitivity for continuous interval maps, Topological Weakly Mixing, Totally Transitive maps, Relation between transitivity and chaos on intervals, Logistic maps and shift maps as chaotic maps.

Unit – 3 **(12 hours)**

Various other definitions of chaos and their interrelationships. Period three implies chaos, Chaos and decomposition theorems including Bowen's decomposition theorem, Topological Entropy:

Definition using open covers, Examples and properties, Bowen's definition of topological entropy, Equivalence of two definitions, Topological version of Kolmogorov–Sinai theorem.

Unit – 4 **(10 hours)**

Topological Entropy of maps on a compact metric space, Topological Entropy of product maps, of iterations of a map, Topological entropy of an expansive homeomorphism on a compact metric space, of the two-sided shift, of any homeomorphism of the unit interval and of the unit circle.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] N. Aoki and K. Hiraide, *Topological Theory of Dynamical Systems: Recent Advances*, North Holland Publications, 1994.
[2] R. L. Devaney, *A First Course in Chaotic Dynamical Systems*, CRC Press, 2018.
[3] S. Ruette, *Chaos for Continuous Interval Maps: A Survey of Relationship Between Various Kinds of Chaos*, 2018.
[4] Peter Walters, *An Introduction to Ergodic Theory*, Springer, 2000.

Suggested Readings

(i) L. Alsedà, J. Llibre and M. Misiurewicz, *Combinatorial Dynamics and Entropy in Dimension One*, Advanced Series in Nonlinear Dynamics, World Scientific, 2000.
(ii) L. S. Block and W. A. Coppel, *Dynamics in One Dimension*, Springer, 2014.
(iii) M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge University Press, 2015.

DISCIPLINE SPECIFIC ELECTIVE – 9 (iii): CHARACTER THEORY OF FINITE GROUPS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-9 (iii): Character Theory of Finite Groups	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra and Group Theory

Learning Objectives

The primary objective of this course is to:

- introduce characters of finite groups, class functions, and find the number of irreducible characters.
- find character tables of S_5, using characters of tensor products of CG-modules and powers of characters.
- define restricted characters and prove Clifford's theorem and its application to find the character table of A_5.
- explore some arithmetic properties of character values and introduce real characters.

Learning Outcomes

This course will enable the students to:

- define and construct examples of characters, prove some fundamental properties of characters, and calculate character tables of some small groups and symmetric groups, etc.
- construct new characters from given characters and understand the notion of induced and restricted characters.
- prove the Frobenius reciprocity theorem and the Frobenius–Schur count of involutions.

Syllabus

Unit – 1 **(10 hours)**

Group characters and their properties, Inner product of characters, Class functions and number of irreducible characters, Character tables and some orthogonality relations, Normal subgroups and lifted characters, Linear characters, Character tables of D_6, S_4, A_4.

Unit – 2 **(11 hours)**

Character of tensor products of CG-modules, Powers of characters, Decomposition of power of a character, Character table of symmetric group S_5, Character table of direct product of groups, Restricted characters, Constituents of a restricted character, Clifford's theorem, Restriction of symmetric groups to alternate groups in general normal subgroups of index two, Character table of A_5.

Unit – 3 **(12 hours)**

Induced CG-modules and their characters: Homomorphisms, Transitivity of induction, Frobenius reciprocity theorem, Values of induced characters. Algebraic integers, Some properties of degrees of irreducible characters and arithmetic properties of character values.

Unit – 4 **(12 hours)**

Real representations, Real conjugacy classes and real characters, Characters which can be realized over the reals, RG-modules and CG-modules, G-invariant symmetric bilinear form, Indicator function, The Frobenius–Schur count of involutions.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] G. James and M. Liebeck, *Representations and Characters of Groups*, Second Edition, Cambridge University Press, 2005.

Suggested Readings

(i) I. M. Issacs, *Character Theory of Finite Groups*, American Mathematical Society reprint, 2006.
(ii) W. Ledermann, *Introduction to Group Characters*, Second Edition, Cambridge University Press, 1987.

DISCIPLINE SPECIFIC ELECTIVE – 9 (iv): NONSMOOTH OPTIMIZATION

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-9 (iv): Nonsmooth Optimization	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Nonlinear Optimization

Learning Objectives

The primary objective of this course is to:

- understand the tools to deal with nonsmooth convex functions.
- study conjugate duality in terms of conjugate functions for constrained nonlinear optimization problems.
- introduce numerical techniques to solve constrained nonlinear optimization problems.

Learning Outcomes

This course will enable the students to learn:

- the notions of subgradients and subdifferentials for nonsmooth convex functions.
- the use of conjugate functions to develop the theory of conjugate duality.
- about numerical methods like gradient descent method, gradient projection method, Newton's method and conjugate gradient method.
- penalty approach technique to solve constrained nonlinear optimization problems.

Syllabus

Unit – 1 **(11 hours)**
Extended real valued functions, Proper convex functions, Closure of convex functions, Differential derivatives, Subgradients and subdifferentials.

Unit – 2 **(12 hours)**
Conjugate functions, Biconjugate functions, Perturbation functions, Closure of convex functions, Directional derivatives, Subgradients and subdifferentials.

Unit – 3 **(12 hours)**
Gradient descent method, Gradient projection method, Newton's method, Conjugate gradient method.

Unit – 4 **(10 hours)**
Penalty function methods, Exterior penalty function, Interior penalty functions.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. Avriel, *Nonlinear Programming: Analysis and Methods*, Dover Publications, 2003.

[2] O. Güler, *Foundations of Optimization*, Springer, 2010.

Suggested Readings

(i) A. Bagirov, N. Karitsa and M. M. Makela, *Introduction to Nonsmooth Optimization: Theory, Practice and Software*, Springer, 2014.
(ii) M. S. Bazaraa, H. D. Sherali and C. M. Shetty, *Nonlinear Programming: Theory and Algorithms,* Third Edition, John Wiley & Sons, 2013.
(iii) D. G. Luenberger and Y. Ye, *Linear and Nonlinear Programming*, Springer, 2008.

DISCIPLINE SPECIFIC ELECTIVE – 10 (i): COMPUTATIONAL FLUID DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-10 (i): Computational Fluid Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Partial Differential Equation (Undergraduate level)

Learning Objectives

The primary objective of this course is to teach:

- various numerical schemes on finite difference and finite volume methods for solving PDEs.
- discretization errors and grid dependence.
- some real-world applications of PDEs and fluid dynamics.
- discretization of governing equations of diffusion, convection-diffusion, fluid flow and thereby computing the numerical solutions using the flow variables using algorithms.

Learning Outcomes

This course will enable the students to learn:

- techniques for solving the PDEs along with some initial and boundary conditions by using the finite difference and finite volume methods.
- the basic conservation principles of mass, momentum, energy, discretization of governing equations.
- discretization techniques.
- some popular algorithms like SIMPLE and SIMPLER used to obtain the solutions of steady and unsteady flow problems by finite volume methods.

Syllabus

Unit – 1 **(12 hours)**

Basics of discretization using finite differences, Various single and multi-step explicit and implicit finite difference schemes for 1-D and 2-D parabolic and hyperbolic initial boundary value problems, Alternating Direction Implicit schemes (ADI) for 2-D parabolic and hyperbolic equations, Order of accuracy, Consistency, Stability and convergence of a finite difference scheme, Courant Friedrich Lewy condition.

Unit – 2 **(12 hours)**

Finite difference schemes for second and fourth order 2-D elliptic boundary value problem and applications, Finite volume method for diffusion and convection-diffusion equations, Discretization of one and two-dimensional steady state diffusion and convection-diffusion equations, Central difference, Upwind, Exponential, Hybrid, Power-law and QUICK schemes and their properties.

Unit – 3 **(11 hours)**

Flow field calculation, Pressure-velocity coupling, Vorticity-stream function approach, Primitive variables, Staggered grid, Pressure and velocity corrections, Pressure correction equation, SIMPLE and SIMPLER algorithms.

Unit – 4 **(10 hours)**

Finite volume methods for unsteady flows, Discretization of one-dimensional transient heat conduction, Explicit, fully implicit and Crank–Nicolson schemes, Implementation of boundary conditions.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. C. Strikweda, *Finite Difference Schemes and Partial Differential Equations*, Second Edition, SIAM, 2004.
[2] H. K. Versteeg and W. Malalasekera, *An Introduction to Computational Fluid Dynamics: The Finite Volume Method*, Second Edition, Pearson, 2008.

Suggested Readings

(i) J. D. Anderson, *Computational Fluid Dynamics*, McGraw-Hill, 1995.
(ii) S. V. Patankar, *Numerical Heat Transfer and Fluid Flow*, CRC Press, Taylor and Francis, Indian Edition, 2017.
(iii) R. H. Pletcher, J. C. Tannehill and D. A. Anderson, *Computational Fluid Mechanics and Heat Transfer*, CRC Press, Taylor and Francis, 2013.
(iv) J. W. Thomas, *Numerical Partial Differential Equations: Finite Difference Methods*, Springer, 2013.

DISCIPLINE SPECIFIC ELECTIVE – 10 (ii): DIFFERENTIAL TOPOLOGY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-10 (ii): Differential Topology	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- introduce the concepts of topological manifolds, smooth structures, smooth manifolds, and manifolds with boundary.
- develop an understanding of smooth functions, smooth maps, diffeomorphisms, and tangent spaces.
- explain the Inverse function theorem, immersions and submersions.
- develop the fundamental concepts of 2-manifolds and distinguish between orientable and non-orientable surfaces.
- explore the properties of compact and connected surfaces.

Learning Outcomes

This course will enable the students to:

- identify and construct examples of topological manifolds, smooth structures and manifolds with and without boundary.
- demonstrate understanding of diffeomorphisms and tangent spaces.
- apply the Inverse function theorem, immersions and submersions.
- define key concepts such as 2-manifolds, orientability, compactness, connectedness and boundary of a surface.
- differentiate between orientable and non-orientable surfaces using examples such as the sphere, torus, Möbius strip and Klein bottle.

Syllabus

Unit – 1 **(12 hours)**

Topological manifolds, Topological properties of manifolds, Smooth structures, Examples of smooth manifolds, Manifolds with boundary.

Unit – 2 **(11 hours)**

Smooth functions and smooth maps, Lie groups, Diffeomorphisms.

Unit – 3 **(10 hours)**

Derivatives and tangents, Inverse function theorem, Immersions and submersions.

Unit – 4 **(12 hours)**

Complexes, Connected sum of two surfaces, Non-orientable surfaces (2- Manifolds), Compact and connected surfaces, Classification of compact and connected surfaces with and without boundary.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] V. Guillemin and Alan Pollack, *Differential Topology*, Prentice-Hall, 1974.
[2] L. C. Kinsey, *Topology of Surfaces*, Springer Verlag, 1997.
[3] J. M. Lee, *Introduction to Smooth Manifolds*, Second Edition, Springer, 2013.

Suggested Readings

(i) L. Conlon, *Differentiable Manifolds*, Second Edition, Birkhäuser Advanced Texts, 2001.
(ii) M. Spivak, *A Comprehensive Introduction to Differential Geometry*, Volume 1, Third Edition, Publish or Perish, Huston, Texas, 1999.
(iii) L. W. Tu, *Introduction to Manifolds*, Second Edition, Springer, 2011.
(iv) F. W. Warner, *Foundations of Differentiable Manifolds and Lie Group*, Springer-Verlag, 1983.

DISCIPLINE SPECIFIC ELECTIVE – 10 (iii): GENERAL MEASURE THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-10 (iii): General Measure Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Measure Theory and Topology

Learning Objectives

The primary objective of this course is to introduce:

- real valued and complex valued measures.
- decomposition of measure spaces and of measures.
- extension of a premeasure to a measure, Lebesgue measure on Euclidean spaces.
- representation of measures and functionals in terms of integrals.
- product measures.

Learning Outcomes

This course will enable the students to:

- appreciate signed measures and complex measures, mutual singularity of measures, Hahn and Jordan decompositions, Lebesgue decomposition, Radon–Nikodym theorem.
- verify conditions under which a set function defined on a collection of subsets of a set has an extension to a measure on a sigma-algebra.
- apply Riesz representation theorem for bounded linear functionals on L^p-spaces.
- understand product measure and the results of Fubini and Tonelli, and express the Lebesgue measure on Euclidean spaces as a product measure.
- apply Riesz–Markov representation theorem for the bounded linear functionals on the space of continuous functions.

Syllabus

Unit – 1 **(13 hours)**

Signed measures, Hahn and Jordan decompositions, Mutually singular measures, Radon–Nikodym theorem, Lebesgue decomposition, Complex measure.

Unit – 2 **(10 hours)**

The Carathéodory extension theorem, Lebesgue measure on $\mathbb{R}^n$, Regularity and translation invariance of Lebesgue measure on $\mathbb{R}^n$.

Unit – 3 **(10 hours)**

Riesz representation theorem for the dual of L^p-spaces, Product measures, Fubini's theorem, Tonelli's theorem.

Unit – 4 **(12 hours)**

Locally compact Hausdorff spaces and construction of Radon measure, Riesz–Markov representation theorem for positive linear functionals on $C_c(X)$, Riesz representation theorem for the dual of $C(X)$.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] H. L. Royden and P. M. Fitzpatrick, *Real Analysis*, Fourth Edition, Pearson, 2015.
[2] M. E. Taylor, *Measure Theory and Integration*, American Mathematical Society, 2006.

Suggested Readings

(i) G. B. Folland, *Real Analysis: Modern Techniques and Their Applications*, Second Edition, Wiley, New York, 1999.
(ii) P. R. Halmos, *Measure Theory*, Springer Science + Business Media, LLC, 2014.
(iii) E. M. Stein and R. Shakarchi, *Real Analysis: Measure Theory, Integration and Hilbert Spaces*, New Age International Publishers, New Delhi, 2010.

DISCIPLINE SPECIFIC ELECTIVE – 10 (iv): THEORY OF NON-COMMUTATIVE RINGS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-10 (iv): Theory of Non-commutative Rings	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Groups and Rings

Learning Objectives

The primary objective of this course is to:

- give students an understanding of Wedderburn–Artin theory of semisimple rings.
- develop Jacobson's general theory of radicals, prime and semiprime rings, and primitive and semiprimitive rings.
- introduce the structure of primitive rings as a generalisation of the Wedderburn–Artin theorem on Artinian simple rings.

Learning Outcomes

This course will enable the students to:

- know about an extensive variety of rings, including free rings, Weyl algebra, Hilbert twist and triangular ring.
- understand the module theoretic definition of semisimple rings and how it leads to the Wedderburn–Artin structure theorem on their complete classification.
- know Jacobson's general theory of radicals, semiprime rings, prime, primitive and semiprimitive rings and their structures.
- understand the significance of the fundamental result 'Density Theorem' and its consequences on the structure of primitive rings.

Syllabus

Unit – 1 **(11 hours)**

Simple rings, Reduced rings, Dedekind-finite rings, Algebra, Quaternions, Free k-rings, Rings with generators and relations, Weyl algebra, Formal power series ring, Hilbert's twist ring, Differential polynomial rings, Derivation and inner derivation on a ring, Triangular rings, Characterization of one-sided and two-sided ideals in such rings.

Unit – 2 **(11 hours)**

Noetherian and Artinian rings, Examples of one-sided Noetherian and Artinian triangular rings, Twisted polynomial ring and Quotient of free $\mathbb{Z}$-ring, Semisimple rings, Structure of semisimple rings: Wedderburn–Artin's theorem.

Unit – 3 **(10 hours)**

Structure theorem of simple left Artinian rings, Jacobson radical, J-semisimple rings, Nil and nilpotent ideals, Connection between semisimple and J-semisimple rings, Hopkins–Levitzki theorem, Nakayama's lemma.

Unit – 4 **(13 hours)**
Prime radical, Characterisation of prime and semiprime ideals, Prime and semiprime rings, Structure theorem of primitive rings, Density theorem.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] T.-Y. Lam, *A First Course in Noncommutative Rings*, Springer, 2001.

Suggested Readings
(i) I. N. Herstein, *Noncommutative Rings*, The Mathematical Association of America, 2005.
(ii) T. W. Hungerford, *Algebra*, Springer-Verlag, New York, 1981.
(iii) L. H. Rowen, *Ring Theory*, Student Edition, Academic Press, 1991.

Skill-Based Course (SBC)

WORKSHOPS AND SEMINARS ON ADVANCED TOPICS

Course Title	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Workshops and Seminars on Advanced Topics	2	0	0	2	Same as for entry to M.Sc. Mathematics	NIL

Learning Objectives

In this course, we aim to train students to

- understand and assimilate mathematical ideas and techniques presented and discussed in workshops and seminars on varied advanced topics in mathematics.
- communicate mathematical ideas succinctly.

Learning Outcomes

This course will enable students to

- efficiently identify the core ideas in any mathematical discourse, particularly outside the classroom setting.
- write a concise summary of these ideas.

Methodology

The students will attend a requisite number of workshops and seminars organized by the Department through the semester. The workshops and seminars will be on advanced topics, building on the classroom courses offered by the Department. The students will be expected to prepare and submit summaries of a few of these (as mandated by the Department) and also give presentations for assessment.

Generic Elective (GE) Courses

GENERIC ELECTIVE – 4 (i): NONSMOOTH OPTIMIZATION

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
GE-4 (i): Nonsmooth Optimization	4	3	1	0	Class XII pass with Mathematics	Basics of Nonlinear Optimization

Learning Objectives
The primary objective of this course is to:
- understand the tools to deal with nonsmooth convex functions.
- study conjugate duality in terms of conjugate functions for constrained nonlinear optimization problems.
- introduce numerical techniques to solve constrained nonlinear optimization problems.

Learning Outcomes
This course will enable the students to learn:
- the notions of subgradients and subdifferentials for nonsmooth convex functions.
- the use of conjugate functions to develop the theory of conjugate duality.
- about numerical methods like gradient descent method, gradient projection method, Newton's method and conjugate gradient method.
- penalty approach technique to solve constrained nonlinear optimization problems.

Syllabus

Unit – 1 **(11 hours)**
Extended real valued functions, Proper convex functions, Closure of convex functions, Differential derivatives, Subgradients and subdifferentials.

Unit – 2 **(12 hours)**
Conjugate functions, Biconjugate functions, Perturbation functions, Closure of convex functions, Directional derivatives, Subgradients and subdifferentials.

Unit – 3 **(12 hours)**
Gradient descent method, Gradient projection method, Newton's method, Conjugate gradient method.

Unit – 4 **(10 hours)**
Penalty function methods, Exterior penalty function, Interior penalty functions.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. Avriel, *Nonlinear Programming: Analysis and Methods*, Dover Publications, 2003.
[2] O. Güler, *Foundations of Optimization*, Springer, 2010.

Suggested Readings

(i) A. Bagirov, N. Karitsa and M. M. Makela, *Introduction to Nonsmooth Optimization: Theory, Practice and Software*, Springer, 2014.
(ii) M. S. Bazaraa, H. D. Sherali and C. M. Shetty, *Nonlinear Programming: Theory and Algorithms,* Third Edition, John Wiley & Sons, 2013.
(iii) D. G. Luenberger and Y. Ye, *Linear and Nonlinear Programming*, Springer, 2008.

GENERIC ELECTIVE – 4 (ii): PROBABILITY THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
GE-4 (ii): Probability Theory	4	3	1	0	Class XII pass with Mathematics	Basics of Integration

Learning Objectives
The primary objective of this course is to introduce:
- probability space as a measure space and random variables as measurable functions.
- expectation and moments of random variables.
- notion of convergence in probability.
- conditioning on sub-σ-algebra.

Learning Outcomes
This course will enable the students to learn:
- about probability or uncertainty in abstract setting.
- moments and expectation of random variables which help to understand applications of probability in industry.
- how to apply the idea of convergence in probability.
- weak law and strong law of large numbers and their applications.

Syllabus

Unit – 1 **(11 hours)**
Probability: σ-algebra, Constructing probability triples, The extension theorem, Random variables, Independence of events, Continuity of probabilities, Limit events, The Borel–Cantelli lemma.

Unit – 2 **(10 hours)**
Expected values: Simple, general non-negative and arbitrary random variables, Moment generating functions, Markov’s inequality, Chebyshev’s inequality.

Unit – 3 **(12 hours)**
Convergence of random variables: Convergence almost surely, Convergence in probability, Weak law of large numbers, Strong law of large numbers.

Unit – 4 **(12 hours)**
Distributions of random variables: Examples of distributions, Characteristic functions, The central limit theorem, Conditional probability, Conditioning on random variable, Conditioning on a sub-σ-algebra, Conditional variance.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. S. Rosenthal, *A First Look at Rigorous Probability Theory*, Second Edition, World Scientific, Singapore, 2006.

Suggested Readings

(i) W. Feller, *An Introduction to Probability Theory and its Applications*, Volume 1, Third Edition, Wiley, 2008.
(ii) J. E. Michael and J. S. Rosenthal, *Probability and Statistics: The Science of Uncertainty*, Second Edition, W. H. Freeman & Co Ltd., 2009.
(iii) S. Ross, *A First Course in Probability*, Tenth Edition, Pearson Education, 2022.
(iv) D. W. Stroock, *Probability Theory, An Analytic View*, Cambridge University Press, 2024.

Syllabi of Courses
in
Semester-III
of
M.Sc. Mathematics under
Structure-2
(Course work + Research)

Discipline Specific Core (DSC) Courses

DISCIPLINE SPECIFIC CORE – 7: FLUID DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSC-7: Fluid Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Calculus and Partial Differential Equations

Learning Objectives

The objective of this course is to:

- prepare a mathematical foundation to study the motion of fluids.
- develop concepts, models, and techniques to solve problems of fluid flow.
- develop the ability to conduct advanced studies and research in the broad field of fluid dynamics.

Learning Outcomes

After studying this course, the student will be able to:

- understand the concept of fluids, their classification, flow lines, models and approaches to study fluid flow.
- formulate mass and momentum conservation principles and obtain their solution for non-viscous flow.
- know potential flow, Bernoulli's equation, Kelvin's minimum energy and circulation theorems.
- understand two- and three-dimensional motion, complex potential, circle theorem, Blasius theorem, Weiss's and Butler's sphere theorems.
- apply the concept of stress and strain in viscous flow to derive Navier–Stokes equation of motion and energy equation.

Syllabus

Unit – 1 **(10 hours)**

Classification of fluids, Continuum model, Eulerian and Lagrangian approach of description, Differentiation following the fluid motion, Flow lines, vorticity and circulation, Conservation of mass: Equation of continuity, Boundary surface.

Unit – 2 **(12 hours)**

Forces in fluid motion, Conservation of momentum: Euler's equation of motion, Theory of irrotational motion: Integration of Euler's equation under different conditions, Bernoulli's equation, Impulsive motion, Kelvin's minimum energy and circulation theorems, Potential theorem.

Unit – 3 **(13 hours)**

Two-dimensional motion: Complex potential, Line sources, sinks, doublets and vortices, Two-dimensional image system, Milne–Thomson circle theorem, Images with respect to a plane and cylinder, Blasius theorem. Three-dimensional flows, Weiss's sphere theorem, Images with respect to sphere, Axi-symmetric flow, Stokes stream function, Butler's sphere theorem, Flow past spheres and cylinders.

Unit – 4 **(10 hours)**

Stress and strain analysis, Newton's law of viscosity, Laminar flow, Navier–Stokes equation of motion, Steady flow between parallel planes and Poiseuille flow, Constitutive equation, Energy equation.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] F. Chorlton, *Text Book of Fluid Dynamics*, CBS Publisher, 2005.
[2] R. W. Fox, P. J. Pritchard and A. T. McDonald, *Introduction to Fluid Mechanics*, Seventh Edition, John Wiley & Sons, 2009.
[3] P. K. Kundu, I. M. Cohen and D. R. Dowling, *Fluid Mechanics*, Sixth Edition, Academic Press, 2016.

Suggested Readings

(i) L. M. Milne-Thomson, *Theoretical Hydrodynamics*, The Macmillan company, USA, 1969.
(ii) D. E. Rutherford, *Fluid Dynamics*, Oliver and Boyd Ltd., 1978.

DISCIPLINE SPECIFIC CORE – 8: MEASURE AND INTEGRATION

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSC-8: Measure and Integration	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Real Analysis and Riemann Integration

Learning Objectives

The primary objective of this course is to:

- extend the notion of length of an interval with the introduction of the concept of Lebesgue outer measure for any subset of real line.
- investigate the properties of Lebesgue measurable sets and functions.
- familiarize students with the Lebesgue integration of functions and its comparison with Riemann integration.
- generalize the concepts of measure and integration to an abstract space.

Learning Outcomes

This course will enable the students to:

- verify whether a given subset of $\mathbb{R}$ or a real valued function is measurable.
- understand the requirement and the concept of the Lebesgue integral (a generalization of the Riemann integration) along with its properties.
- understand the statements and proofs of the fundamental integral convergence theorems and demonstrate their applications.
- carry out a comprehensive study of functions of bounded variation and their utility in understanding differentiation and integration.
- apply Hölder and Minkowski inequalities in L^p-spaces and understand completeness of L^p-spaces.

Syllabus

Unit – 1 **(14 hours)**

Lebesgue outer measure, Measurable sets, Lebesgue measure, Borel sets, Regularity, Measurable functions, Borel and Lebesgue measurability, Non-measurable sets.

Unit – 2 **(13 hours)**

Integration of nonnegative functions, General integral, Integration of series, Riemann and Lebesgue integrals.

Unit – 3 **(8 hours)**

Functions of bounded variation, Lebesgue's differentiation theorem, Differentiation and integration, Absolute continuity of functions.

Unit – 4 **(10 hours)**

Measures and outer measures, Measure spaces, Integration with respect to a measure, L^p-spaces, Hölder's and Minkowski's inequalities, Completeness of L^p-spaces.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] G. de Barra, *Measure Theory and Integration*, Ellis Horwood Ltd., Chichester, John Wiley & Sons, Inc., New York, 1981 (Indian Reprint, 2014).

Suggested Readings

(i) M. Capinski and P. E. Kopp, *Measure, Integral and Probability*, Springer, 2005.
(ii) E. Hewitt and K. Stromberg, *Real and Abstract Analysis: A Modern Treatment of the Theory of Functions of a Real Variable*, Springer, Berlin, 1975.
(iii) H. L. Royden and P.M. Fitzpatrick, *Real Analysis*, Fourth Edition, Pearson, 2015.

Discipline Specific Elective (DSE) Courses

DSE-5 and DSE-6

Group-1

DISCIPLINE SPECIFIC ELECTIVE: ALGEBRAIC TOPOLOGY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Algebraic Topology	**4**	**3**	**1**	**0**	**Same as for entry to M.Sc. Mathematics**	**Basics of Topology**

Learning Objectives
The primary objective of this course is to:
- understand the concepts of homotopic maps, homotopy type, retracts, and deformation retracts, and their role in algebraic topology.
- compute fundamental groups of basic topological spaces.
- acquire knowledge of covering projections, lifting properties, Borsuk–Ulam theorem, and classification techniques of covering spaces.
- learn free groups and free products to understand and apply the Seifert–Van Kampen theorem.

Learning Outcomes
This course will enable the students to:
- distinguish between spaces with the same homotopy type.
- compute fundamental groups of standard spaces such as n-sphere $\mathbb{S}^n$ and punctured planes.
- apply the concept of fundamental groups to prove Brouwer's fixed-point theorem and the Fundamental theorem of Algebra.
- explain the lifting theorems and their implications in topology.
- classify covering spaces for given base spaces.
- apply Seifert–Van Kampen theorem to compute fundamental groups of glued spaces.

Syllabus

Unit – 1 **(11 hours)**
Homotopic maps, Homotopy type, Retract and deformation retract.

Unit – 2 **(12 hours)**
Fundamental group, Calculation of fundamental groups of n-sphere $\mathbb{S}^n$ and punctured plane, Brouwer's fixed-point theorem, Fundamental theorem of Algebra.

Unit – 3 **(12 hours)**
Covering projections, Lifting theorems, Borsuk–Ulam theorem, Classification of covering spaces.

Unit – 4 **(10 hours)**
Free products, Free groups, Seifert–Van Kampen theorem and its applications.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] J. R. Munkres, *Elements of Algebraic Topology*, Addison-Wesley Publishing Company, 1984.
[2] T. B. Singh, *Introduction to Topology*, Springer Nature Singapore, 2019.

Suggested Readings
(i) G. E. Bredon, *Geometry and Topology*, Springer, 2014.
(ii) A. Hatcher, *Algebraic Topology*, Cambridge University Press, 2002.
(iii) W. S. Massey, *A Basic Course in Algebraic Topology*, World Publishing Corporation, 2009.
(iv) J. J. Rotman, *An Introduction to Algebraic Topology*, Springer, 2011.
(v) E. H. Spanier, *Algebraic Topology*, Springer-Verlag, 1989.

DISCIPLINE SPECIFIC ELECTIVE: COMMUTATIVE ALGEBRA

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Commutative Algebra	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra and Field Theory

Learning Objectives

The objective of this course is to:

- develop a solid understanding of the structure of commutative rings, ideals, their radicals, extension, contraction etc.
- study important constructions such as total quotient rings, localizations.
- develop basic foundation in other areas of mathematics such as algebraic geometry, algebraic number theory.

Learning Outcomes

This course will enable the students to:

- know the localization of rings at a prime ideal that is an algebraic analogue of the geometric notion concentrating attention near a point.
- know more closely the polynomial rings, power series rings in one or more variables over a commutative ring and their prime spectrum.
- define, identify, and elaborate integral closure of rings, valuations rings, discrete valuation rings, structure theorem of Artin rings.

Syllabus

Unit – 1 **(12 hours)**

Radical of an ideal, Prime avoidance lemma, Chinese remainder theorem, Extension and contraction of ideals, Jacobson radical of a ring, Nakayama lemma, Tensor product of modules.

Unit – 2 **(13 hours)**

Rings and modules of fractions, Localization, Local properties, Primary decomposition, First and second uniqueness theorem of primary decomposition, Associated prime ideals of decomposable ideals.

Unit – 3 **(10 hours)**

Integral ring extensions, Going up theorem, Going down theorem, Integrally closed domains, Valuation rings, Hilbert's Nullstellensatz theorem.

Unit – 4 **(10 hours)**

Noetherian rings, Primary decomposition in Noetherian rings, Artin rings, Structure theorem for Artin rings, Discrete valuation rings.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. F. Atiyah and I. G. MacDonald, *Introduction to Commutative Algebra*, CRC Press, Taylor & Francis, 2018.

Suggested Readings

(i) D. Eisenbud, *Commutative Algebra with a View Towards Algebraic Geometry*, Springer, 2004.
(ii) R. Y. Sharp, *Steps in Commutative Algebra*, Cambridge University Press, 2000.
(iii) B. Singh, *Basic Commutative Algebra*, World Scientific, 2011.
(iv) O. Zariski and P. Samuel, *Commutative Algebra*, Volume I & II, Springer, 1975.

DISCIPLINE SPECIFIC ELECTIVE: DYNAMICAL SYSTEMS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Dynamical Systems	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology and Ordinary Differential Equations

Learning Objectives

The primary objective of this course is to:

- understand discrete and continuous systems with case studies to study nonlinear systems of ordinary differential equations and dynamical systems.
- understand the concepts, models and techniques to realize the real-world problems and stability of the systems along with the chaotic dynamic behaviour of models by understanding bifurcations.

Learning Outcomes

This course will enable the students to learn:

- formulation of mathematical models with the stability analysis near the equilibrium points.
- how the concept of phase portraits helps to analyse mathematical model graphically.
- the qualitative behaviour of the solution set of a given system of differential equations including the invariant sets and limiting behaviour of the dynamical system or flow defined by the system of differential equations.
- how different bifurcations explain the chaotic behaviour of the system.

Syllabus

Unit – 1 **(13 hours)**

Linear systems: Jordan forms, Stability theory; Nonlinear systems: Fundamental existence-uniqueness theorem, Dependence on initial conditions and parameters, Flow of a differential equation, Linearization, Stable manifold theorem, Hartman–Grobman theorem.

Unit – 2 **(10 hours)**

Stability and Lyapunov functions, Saddle points, Nodes, Foci, Centers and nonhyperbolic critical points, Center manifold theorem.

Unit – 3 **(12 hours)**

Limit sets and attractors, Periodic orbits and limit cycles, Poincaré map, Stable manifold theorem for periodic orbits, Poincare-Bendixson theorem.

Unit – 4 **(10 hours)**

Bifurcations at nonhyperbolic equilibrium points, Saddle node, Transcritical and Pitchfork bifurcations, Hopf bifurcation.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. W. Hirsch, S. Smale and R. L. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, Third Edition, Academic Press, 2013.
[2] L. Perko, *Differential Equations and Dynamical Systems*, Third Edition, Springer Verlag, 2001.

Suggested Readings

(i) R. L. Devaney, *A First Course in Chaotic Dynamical Systems: Theory and Experiment*, CRC Press, Taylor & Francis, 2018.
(ii) S. H. Strogatz, *Nonlinear Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 2018.
(iii) S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, TAM Volume 2, Springer-Verlag, NY, 1990.

DISCIPLINE SPECIFIC ELECTIVE: THEORY OF BOUNDED OPERATORS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Theory of Bounded Operators	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis

Learning Objectives

The primary objective of this course is to:

- introduce some classes of bounded linear operators which play a central role in both pure and applied mathematics.
- study the properties and spectral theory of these operators.

Learning Outcomes

This course will enable the students to understand:

- the spectrum and sub-divisions of spectrum of standard operators like shifts and multiplication.
- the spectral theorem for some classes of bounded linear operators.
- the concepts of compactness, self-adjointness and positivity of bounded linear operators.
- trace class and Hilbert–Schmidt operators.

Syllabus

Unit – 1 **(11 hours)**

Properties of spectrum and resolvent of bounded operators, Subdivision of the spectrum including point, approximate and compression spectrum.

Unit – 2 **(10 hours)**

Operators on Hilbert spaces, Adjoint operator, Projections and idempotents, Operations with projections, Invariant and reducing subspaces.

Unit – 3 **(14 hours)**

Compact operators on Hilbert spaces, Diagonalisation of compact self-adjoint operators, Spectral theorem and functional calculus for Compact normal operators, Positive operators, Compact operators on Banach spaces, Spectral theory of compact operators.

Unit – 4 **(10 hours)**

Polar decomposition, Singular values, Trace class operators, Trace norm and Hilbert Schmidt operators.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] R. Bhatia, *Notes on Functional Analysis*, Hindustan Book Agency, 2009.
[2] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.

Suggested Readings

(i) E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, 2006.
(ii) B. Simon, *Operator Theory: A Comprehensive Course in Analysis*, Part 4, American Mathematical Society, 2015.
(iii) S. R. Garcia, J. Mashregi and W. T. Ross, *Operator Theory by Example*, Oxford University Press, 2023.

Group-2

DISCIPLINE SPECIFIC ELECTIVE: ADVANCED COMPLEX ANALYSIS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Advanced Complex Analysis	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Complex Analysis

Learning Objectives

The primary objective of this course is to:

- explore the metric space structure of the spaces of continuous and analytic functions, through Arzela–Ascoli, Hurwitz and Montel theorems.
- investigate various characterizations of simply connected regions, with a special focus on the Riemann mapping theorem.
- use Runge's and Mittag–Leffler's theorems to approximate and interpolate analytic and meromorphic functions.
- analyze the range of analytic functions using Bloch's/ Landau's constants and Picard's theorem.

Learning Outcomes

This course will enable the students to:

- apply various variants of maximum modulus theorem, encompassing Hadamard's three circles theorem and Phragmen–Lindelöf theorem.
- comprehend the notions of normality, compactness, equicontinuity and local boundedness for the spaces of continuous and analytic functions.
- construct and factorize entire functions using infinite products, including special functions like gamma and zeta.
- analyze the harmonic functions on a disk using Poisson kernel, which in turn, solves the Dirichlet problem for a unit disk.

Syllabus

Unit – 1 **(12 hours)**

Convex functions and Hadamard's three circles theorem, Maximum-Modulus theorem (third version), Phragmen–Lindelöf theorem, Spaces of continuous functions, Normal and equicontinuous families, Arzela–Ascoli theorem.

Unit – 2 **(11 hours)**

Spaces of analytic functions, Hurwitz's theorem, Montel's theorem, Riemann mapping theorem, Infinite products, Weierstrass factorization theorem, Factorization of sine function.

Unit – 3 **(10 hours)**
Gamma and Riemann zeta function, Runge's theorem, Characterizations of simple connectedness, Mittag–Leffler's theorem.

Unit – 4 **(12 hours)**
Harmonic functions, Mean value property, Maximum and minimum principles, Harmonic function on a disk, Harnack's theorem, Range of an analytic function, Bloch's theorem, Bloch's and Landau's constants, Picard's theorem.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] J. B. Conway, *Functions of One Complex Variable*, Second Edition, Narosa Publishing House, New Delhi, 2002.

Suggested Readings
(i) L. V. Ahlfors, *Complex Analysis*, Mc Graw Hill Co., Indian Edition, 2017.
(ii) L. Hahn and B. Epstein, *Classical Complex Analysis*, Jones and Bartlett, 1996.
(iii) E. M. Stein and R. Shakarchi, *Complex Analysis*, Princeton University Press, 2003.
(iv) D. Ullrich, *Complex Made Simple*, Volume 97, American Mathematical Society, 2008.

DISCIPLINE SPECIFIC ELECTIVE: NUMERICAL METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Numerical Methods for Ordinary Differential Equations	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Ordinary Differential Equations

Learning Objectives

The primary objective of this course is to:

- develop the basic theory underlying the numerical solution of differential equations.
- introduce the concepts of consistency, stability and convergence of finite difference methods.
- execute the numerical schemes for the solution of differential equations.

Learning Outcomes

This course will enable the students to:

- gain a thorough understanding of the fundamental concepts involved in the construction and analysis of finite difference schemes for solving ordinary differential equations (ODEs).
- apply various numerical methods based on finite difference approaches to obtain approximate solutions for both initial value problems (IVPs) and boundary value problems (BVPs).
- develop the ability to select appropriate finite difference methods for specific types of problems and effectively apply them to real world applications.

Syllabus

Unit – 1 **(11 hours)**

Initial value problems: Existence and uniqueness of solution, Finite difference equation, Truncation error, Single step methods for first order IVPs and system of IVPs- Family of explicit and implicit Runge–Kutta methods, Taylor series methods, Derivation, Truncation error, Consistency, Stability and convergence analysis.

Unit – 2 **(12 hours)**

IVPs for the system of ODEs, Consistency, Zero stability and convergence of linear multistep methods, Routh–Hurwitz criterion, Order and error constant, First Dahlquist Barrier, Local truncation error and global truncation error, Error bounds, Local error, Linear stability theory, Higher order differential equations.

Unit – 3 **(12 hours)**

Derivation of explicit and implicit multistep methods for IVPs and system of IVPs, Truncation error, Stability and convergence analysis of family of Nystrom method, Adams–Bashforth method,

Adams–Moulton method, Milne–Simpson method, Predictor corrector method, and Modified predictor corrector method, Hybrid method, Multistep methods for second order IVPs.

Unit – 4 **(10 hours)**

Linear BVPs for second order ordinary differential equations, Shooting method, Finite difference method, Collocation method, Derivative boundary conditions, Nonlinear two-point BVPs, Higher order finite difference methods, Stability, Truncation error and convergence analysis.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. K. Jain, S. R. K. Iyenger and R. K. Jain, *Numerical Methods for Scientific and Engineering Computations*, Seventh Edition, New Age International Publisher, 2019.

[2] J. D. Lambert, *Numerical Methods for Ordinary Differential Systems: The Initial Value Problem*, John Wiley & Sons, 1991.

Suggested Readings

(i) K. E. Atkinson, W. Han and D. E. Stewart, *Numerical Solution of Ordinary Differential Equations*, John Wiley & Sons, 2009.

(ii) J. C. Butcher, *The Numerical Analysis of Ordinary Differential Equations*, Second Edition, Wiley, New York, 2008.

(iii) L. Collatz, *The Numerical Treatment of Differential Equations*, Springer-Verlag, 1966.

DISCIPLINE SPECIFIC ELECTIVE: REPRESENTATION OF FINITE GROUPS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Representation of Finite Groups	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra and Group Theory

Learning Objectives

The primary objective of this course is to:

- represent finite groups as groups of matrices (via homomorphisms) and apply the tools of linear algebra to study the group structure.
- introduce the notion of Group algebra, which plays an essential role in classifying representations of groups.
- to discuss some applications of representations of finite groups, such as the Burnside's theorem.

Learning Outcomes

This course will enable the students to:

- define and construct examples of group representations, *FG*-modules, group algebras.
- grasp key concepts and tools of representation theory and establish a link between *FG*-modules and group representations.
- prove and apply Maschke's theorem and Schur's lemma to describe all irreducible representations of finite groups over the field of complex numbers.
- apply the theory of characters and group representations to gain insight into group structure, such as normal subgroups, and the solubility of groups.

Syllabus

Unit – 1 **(11 hours)**

Representation of groups, *FG*-modules and *FG*-submodules, and reducibility, Permutation modules, *FG*-modules and equivalent representations, Reducible and irreducible *FG*-modules, Group algebra of *G*, Regular *FG*-module and regular representations, *FG*-homomorphisms, Direct sum of *FG*-modules.

Unit – 2 **(11 hours)**

Maschke's theorem for *FG*-modules and consequences. Schur's lemma and its converse, Application of Schur's lemma, Irreducible modules and group algebra, Structure of group algebra and space of *CG*-homomorphisms.

Unit – 3 **(10 hours)**

Characters and their properties, Permutation and regular characters, Inner product, Number of irreducible characters, Orthogonality relations and finding normal subgroups.

Unit – 4 **(13 hours)**

Algebraic numbers, Algebraic integers and their properties, Character values, The Burnside's *(p,q)*-theorem and solubility of some particular groups.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] G. James and M. Liebeck, *Representations and Characters of Groups*, Second Edition, Cambridge University Press, 2005.

Suggested Readings

(i) C. W. Curtis and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras*, American Mathematical Society, 2006.
(ii) W. Fulton and J. Harris, *Representation Theory - A First Course*, Springer-Verlag, 2004.
(iii) I. M. Issacs, *Character Theory of Finite Groups*, American Mathematical Society reprint, 2006.

DISCIPLINE SPECIFIC ELECTIVE: TOPOLOGICAL DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Topological Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- provide a strong background of topological dynamical systems including their applications.
- develop some useful and interesting dynamical properties like expansivity, shadowing and topological stability with supporting examples and results from symbolic and topological dynamics.
- introduce the celebrated Sarkovskii's theorem.

Learning Outcomes

This course will enable the students to:

- construct interesting examples of dynamical systems and topological conjugacy.
- visualize stable sets, omega sets and alpha limit sets.
- understand the applications of Sarkovskii's theorem.
- use subshifts of finite type to characterize irreducible matrices.
- prove key results on expansivity and shadowing regarding existence/non-existence, product, subspace and their different characterizations etc.
- find the class of topologically stable homeomorphisms.

Syllabus

Unit – 1 **(10 hours)**

Definition and examples (including real life examples) of dynamical systems, Orbits, Types of orbits, Topological conjugacy and orbits, Phase portrait-graphical analysis of orbit, Periodic points and stable sets, Omega and alpha limit sets and their properties.

Unit – 2 **(10 hours)**

Sarkovskii's theorem, Shift spaces and subshift, Subshift of finite type, Subshift represented by a matrix, Characterizations of irreducible matrices.

Unit – 3 **(13 hours)**

Definition and examples of expansive homeomorphisms, Properties of expansive homeomorphisms, Non-existence of expansive homeomorphism on the unit interval and unit circle, Generators and weak generators, Generators and expansive homeomorphisms.

Unit – 4 **(12 hours)**

Converging semi-orbits for expansive homeomorphisms, Definition, examples and properties of maps having shadowing property, Topological Anosov homeomorphisms and topological stability.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] N. Aoki and K. Hiraide, *Topological Theory of Dynamical Systems: Recent Advances*, North Holland Publications, 1994.
[2] M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge University Press, 2004.

Suggested Readings

(i) D. C. Hanselman and B. Little field, *Mastering MATLAB*, Pearson, 2012.
(ii) D. Lind and B. Marcus, *An Introduction to Symbolic Dynamics and Coding*, Cambridge University Press, 1996.
(iii) C. Robinson, *Dynamical Systems*, *Stability, Symbolic Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 1998.
(iv) J. de. Vries, *Elements of Topological Dynamics*, Springer, 1993.

Group-3

DISCIPLINE SPECIFIC ELECTIVE: ADVANCED FUNCTIONAL ANALYSIS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Advanced Functional Analysis	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis and Topology

Learning Objectives

The primary objective of this course is to:

- define and explain the structure of a topological vector space and its fundamental properties.
- differentiate between normed, metrizable, locally convex and Hausdorff topological vector spaces.
- introduce the foundational theorems of functional analysis, including the Hahn–Banach, Banach–Steinhaus, Open mapping, and Closed graph theorems in the context of locally convex spaces.
- explain some applications of Banach–Alaoglu theorem and Krein–Milman theorem.

Learning Outcomes

This course will enable the students to:

- appreciate types of topological vector spaces and their separation properties.
- understand quotient spaces, weak topology and weak*-topology.
- analyze concepts of continuity, boundedness, and convergence for linear operators and functionals on topological vector spaces.
- understand the notion of local convexity and the role of seminorms in defining locally convex topologies.

Syllabus

Unit – 1 **(12 hours)**

Topological vector spaces, Types of Topological vector spaces, Separation properties, Linear mappings, Finite dimensional spaces, Metrization, Boundedness and continuity, Seminorms and local convexity, Normability.

Unit – 2 **(11 hours)**

Quotient spaces, Seminorms and quotient spaces, Examples, Baire category theorem, Banach–Steinhaus theorem, The open mapping theorem and the closed graph theorem on topological vector spaces.

Unit – 3 **(11 hours)**

Hahn–Banach separation theorem on topological vector spaces, Continuous extension theorem, Weak topologies, Weak topology and convexity, Weak topology and metrizability, Weak*-

topology of a dual space, Compact convex sets, Banach–Alaoglu theorem and applications, Goldstine theorem.

Unit – 4 **(11 hours)**

Extreme points, Krein–Milman theorem, Convex hull of compact sets, Applications of Krein–Milman theorem: Stone–Weierstrass theorem, Markov–Kakutani fixed point theorem.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.
[2] W. Rudin, *Functional Analysis*, Second Edition, Tata Mc Graw-Hill, 2011.

Suggested Readings

(i) V. I. Bogachev and O. G. Smolyanov, *Topological Vector Spaces and Their Applications*, Springer, 2017.
(ii) S. Kantorovitz, *Introduction to Modern Analysis*, Oxford Graduate Texts in Mathematics, Oxford University Press, 2006.
(iii) J. Voigt, *A Course on Topological Vector Spaces*, Birkhäuser, 2020.

DISCIPLINE SPECIFIC ELECTIVE: ALGEBRAIC CODING THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Algebraic Coding Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra, Groups and Rings

Learning Objectives
The primary objective of this course is to:
- provide an introduction to algebraic coding theory, particularly linear codes.
- discuss bounds on the parameters along with cyclic codes.
- describe some well-known codes, such as Reed–Muller and Golay codes.
- explore the algebraic structure of Cyclic and Quadratic residue codes over fields and rings.

Learning Outcomes
This course will enable the students to:
- get an insight into the matrix representation of a code, as well as encoding and decoding.
- understand Hamming, MDS and Reed–Muller codes.
- describe cyclic codes and their generator polynomial.
- learn about special cyclic codes, such as Quadratic residue codes, and their properties over the ring $\mathbb{Z}_4$.

Syllabus

Unit – 1 **(10 hours)**
Error detecting and error correcting codes, Maximum likelihood decoding, Hamming distance, Linear codes, Hamming weight, Generator matrix, Parity check matrix, Equivalence of linear codes, Encoding and decoding of linear codes, Syndrome decoding.

Unit – 2 **(11 hours)**
Bounds on codes, Sphere covering bound, Hamming bound, Perfect codes, Binary Hamming codes, Binary Golay codes, Singleton bound and MDS codes. Propagation rules, Reed–Muller codes.

Unit – 3 **(12 hours)**
Cyclic codes, Cyclic codes as ideals, Generator polynomial of cyclic codes, Generator and parity-check matrices of cyclic codes**,** Decoding of cyclic codes, Burst error correcting codes.

Unit – 4 **(12 hours)**
Quadratic residue codes: QR codes over fields of characteristic 2 and 3, Cyclic codes and their generating polynomial over $\mathbb{Z}_4$, QR codes over $\mathbb{Z}_4$.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] S. Ling and C. Xing, *Coding Theory: A First Course*, Cambridge University Press, 2004.
[2] W. C. Huffman and V. Pless, *Fundamentals of Error Correcting Codes*, Cambridge University Press, 2010.

Suggested Readings

(i) R. Hill, *A First Course in Coding Theory*, Oxford University Press, 1986.
(ii) F. J. Mac William and N. J. A. Sloane, *Theory of Error Correcting Codes, Part I & II,* Elsevier/North-Holland, Amsterdam, 1977.

DISCIPLINE SPECIFIC ELECTIVE: DIFFERENTIAL GEOMETRY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Differential Geometry	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra, Multivariate Calculus and Differential Equations

Learning Objectives

The primary objective of this course is to introduce:

- surfaces and parametrized surfaces.
- orientation on connected surfaces.
- geodesics on surfaces.
- Weingarten maps on oriented surfaces.
- arc length and curvature of oriented plane curves.
- curvatures of oriented surfaces.

Learning Outcomes

This course will enable the students to:

- understand the concepts of level sets and graphs of functions, smooth vector fields, tangent spaces of level sets.
- appreciate surfaces and parametrized surfaces, Gauss map, geodesics and parallel transport on oriented surfaces.
- know what the Weingarten map of an oriented surface is, realize it as shape operator and use it to compute curvature of oriented plane curves.
- find global parametrization and hence arc length of an oriented plane curve.
- compute various types of curvatures of surfaces.

Syllabus

Unit – 1 **(10 hours)**

Level sets in $\mathbb{R}^{n+1}$ and graphs of functions, Smooth vector fields and existence and uniqueness of their integral curves, Tangent spaces of level sets at regular points, Surfaces in $\mathbb{R}^{n+1}$ as inverse images of regular values of smooth functions, Necessary condition for extrema of functions on surfaces-Lagrange multipliers, Existence of a normal vector field on a connected surface, Orientation, Gauss map.

Unit – 2 **(13 hours)**

The notion of a geodesic on a surface, Existence and uniqueness of a geodesic on a surface through a given point with a given velocity vector thereof, Covariant derivative of a smooth vector field, Parallel vector field along a curve, Existence and uniqueness of a parallel vector field along a curve with a given initial vector, Weingarten map of a surface at a point, Local parametrization and curvature of a plane curve.

Unit – 3 **(10 hours)**
Global parametrization and arc length of an oriented plane curve, Differential 1-forms, Line integral of 1-forms over parametrized curves.

Unit – 4 **(12 hours)**
Parametrized surfaces with examples, Curvature of surfaces, Normal curvature of a surface at a point in a given direction, Principal curvatures, First and second fundamental forms, Gauss-Kronecker curvature and mean curvature.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] A. Pressley, *Elementary Differential Geometry*, Springer-Verlag London Limited, 2012.
[2] J. A. Thorpe, *Elementary Topics in Differential Geometry*, Springer (India) Pvt. Limited, 2004.

Suggested Readings
(i) W. Kuhnel, *Differential Geometry: Curves-Surfaces-Manifolds*, Third Edition, American Mathematical Society, 2015.
(ii) B. O' Neill, *Elementary Differential Geometry*, Second Edition, Academic Press INC., Academic Press, New York, 2006.

DISCIPLINE SPECIFIC ELECTIVE: FINITE ELEMENT METHODS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Finite Element Methods	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Differential Equations

Learning Objectives

The primary objective of this course is to:

- introduce basic aspects of finite element methods (FEM) including domain discretization, polynomial interpolation, application of boundary conditions, assembly of global arrays, and solution of the resulting algebraic systems.
- discuss the use of finite element methods in solving engineering problems in the domain of solid mechanics, fluid mechanics, heat transfer and electromagnetism.

Learning Outcomes

This course will enable the students to:

- use integral statement to deduce finite element approximations for the underlying linear partial differential equations.
- write special-purpose finite element programs within a procedural programming environment.
- use finite element methods to solve engineering problems in solids mechanics, fluid mechanics, heat transfer, and electromagnetism.
- assess the accuracy and reliability of finite element solutions and troubleshoot problems arising from errors in a given finite element analysis.

Syllabus

Unit – 1 **(12 hours)**

Basic concepts of weak formulation, Variational formulation of a one dimensional model equation, Basis function and finite element solutions, Collocation method, Ritz method, Least square method, Standard Galerkin method, FEM for model problem, Error estimate for FEM for model equation, Convergence analysis.

Unit – 2 **(11 hours)**

Various shapes of finite element, Higher order basis functions, Finite element methods for elliptic problems: Variational methods, Standard Galerkin method, Error estimate for FEM for elliptic problem, FEM for Poisson equation.

Unit – 3 **(12 hours)**

Finite element methods for parabolic problems: One dimensional model problems, Semi-discretization in space, Error estimates, Discretization in space and time, Galerkin method, Finite element methods for hyperbolic problems: Standard Galerkin method, Standard Galerkin method with strongly and weakly imposed boundary conditions.

Unit – 4 **(10 hours)**
Applications of the FEM to second order BVPs in one dimension, Applications of the FEM to linear elliptic, parabolic and hyperbolic equations.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] G. Evans, J. Blackledge and P. Yardley, *Numerical Methods for Partial Differential Equations*, Springer-Verlag, London, 2000.
[2] C. Johnson, *Numerical Solutions of Partial Differential Equations by Finite Element Methods*, *Cambridge University Press*, Cambridge, 1987.
[3] J. Whiteley, *Finite Element Methods - A Practical Guide*, Springer, 2016.

Suggested Readings
(i) Z. Chen, *Finite Element Methods and Their Applications*, Springer-Verlag, New York, 2005.
(ii) V. Thomee, *Galerkin Finite Element Methods for Parabolic Problems*, Second Edition, Springer-Verlag, Berlin, 2006.

Generic Elective (GE) Courses

GENERIC ELECTIVE – 3 (i): DYNAMICAL SYSTEMS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
GE-3 (i): Dynamical Systems	4	3	1	0	Class XII pass with Mathematics	Basics of Topology and Ordinary Differential Equations

Learning Objectives

The primary objective of this course is to:

- understand discrete and continuous systems with case studies to study nonlinear systems of ordinary differential equations and dynamical systems.
- understand the concepts, models and techniques to realize the real-world problems and stability of the systems along with the chaotic dynamic behaviour of models by understanding bifurcations.

Learning Outcomes

This course will enable the students to learn:

- formulation of mathematical models with the stability analysis near the equilibrium points.
- how the concept of phase portraits helps to analyse mathematical model graphically.
- the qualitative behaviour of the solution set of a given system of differential equations including the invariant sets and limiting behaviour of the dynamical system or flow defined by the system of differential equations.
- how different bifurcations explain the chaotic behaviour of the system.

Syllabus

Unit – 1 **(13 hours)**

Linear systems: Jordan forms, Stability theory; Nonlinear systems: Fundamental existence-uniqueness theorem, Dependence on initial conditions and parameters, Flow of a differential equation, Linearization, Stable manifold theorem, Hartman–Grobman theorem.

Unit – 2 **(10 hours)**

Stability and Lyapunov functions, Saddle points, Nodes, Foci, Centers and nonhyperbolic critical points, Center manifold theorem.

Unit – 3 **(12 hours)**

Limit sets and attractors, Periodic orbits and limit cycles, Poincaré map, Stable manifold theorem for periodic orbits, Poincare-Bendixson theorem.

Unit – 4 **(10 hours)**

Bifurcations at nonhyperbolic equilibrium points, Saddle node, Transcritical and Pitchfork bifurcations, Hopf bifurcation.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. W. Hirsch, S. Smale and R. L. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, Third Edition, Academic Press, 2013.
[2] L. Perko, *Differential Equations and Dynamical Systems*, Third Edition, Springer Verlag, 2001.

Suggested Readings

(i) R. L. Devaney, *A First Course in Chaotic Dynamical Systems: Theory and Experiment*, CRC Press, Taylor & Francis, 2018.
(ii) S. H. Strogatz, *Nonlinear Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 2018.
(iii) S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, TAM Volume 2, Springer-Verlag, NY, 1990.

GENERIC ELECTIVE – 3 (ii): NUMERICAL METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
GE-3 (ii): Numerical Methods for Ordinary Differential Equations	4	3	1	0	Class XII pass with Mathematics	Basics of Ordinary Differential Equations

Learning Objectives

The primary objective of this course is to:

- develop the basic theory underlying the numerical solution of differential equations.
- introduce the concepts of consistency, stability and convergence of finite difference methods.
- execute the numerical schemes for the solution of differential equations.

Learning Outcomes

This course will enable the students to:

- gain a thorough understanding of the fundamental concepts involved in the construction and analysis of finite difference schemes for solving ordinary differential equations (ODEs).
- apply various numerical methods based on finite difference approaches to obtain approximate solutions for both initial value problems (IVPs) and boundary value problems (BVPs).
- develop the ability to select appropriate finite difference methods for specific types of problems and effectively apply them to real world applications.

Syllabus

Unit – 1 **(11 hours)**

Initial value problems: Existence and uniqueness of solution, Finite difference equation, Truncation error, Single step methods for first order IVPs and system of IVPs- Family of explicit and implicit Runge–Kutta methods, Taylor series methods, Derivation, Truncation error, Consistency, Stability and convergence analysis.

Unit – 2 **(12 hours)**

IVPs for the system of ODEs, Consistency, Zero stability and convergence of linear multistep methods, Routh–Hurwitz criterion, Order and error constant, First Dahlquist Barrier, Local truncation error and global truncation error, Error bounds, Local error, Linear stability theory, Higher order differential equations.

Unit – 3 **(12 hours)**

Derivation of explicit and implicit multistep methods for IVPs and system of IVPs, Truncation error, Stability and convergence analysis of family of Nystrom method, Adams–Bashforth method,

Adams–Moulton method, Milne–Simpson method, Predictor corrector method, and Modified predictor corrector method, Hybrid method, Multistep methods for second order IVPs.

Unit – 4 **(10 hours)**
Linear BVPs for second order ordinary differential equations, Shooting method, Finite difference method, Collocation method, Derivative boundary conditions, Nonlinear two-point BVPs, Higher order finite difference methods, Stability, Truncation error and convergence analysis.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] M. K. Jain, S. R. K. Iyenger and R. K. Jain, *Numerical Methods for Scientific and Engineering Computations*, Seventh Edition, New Age International Publisher, 2019.
[2] J. D. Lambert, *Numerical Methods for Ordinary Differential Systems: The Initial Value Problem*, John Wiley & Sons, 1991.

Suggested Readings
(i) K. E. Atkinson, W. Han and D. E. Stewart, *Numerical Solution of Ordinary Differential Equations*, John Wiley & Sons, 2009.
(ii) J. C. Butcher, *The Numerical Analysis of Ordinary Differential Equations*, Second Edition, Wiley, New York, 2008.
(iii) L. Collatz, *The Numerical Treatment of Differential Equations*, Springer-Verlag, 1966.

Syllabi of Courses in Semester-IV of M.Sc. Mathematics under Structure-2 (Course work + Research)

Discipline Specific Core (DSC) Courses

DISCIPLINE SPECIFIC CORE – 9: PARTIAL DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSC-9: Partial Differential Equations	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Multivariate Calculus and Differential Equations

Learning Objectives

The main objective of this course is to introduce:

- well-posedness, fundamental solutions, existence and uniqueness of solutions for Laplace equation, Poisson equation and heat equation.
- solution for wave equation by spherical means.
- characteristics, complete integrals, envelopes and conservation laws for first-order nonlinear partial differential equations.
- classical solution techniques such as Green's function, similarity solutions and transform methods.

Learning Outcomes

This course will enable the students to:

- understand Laplace equation, Poisson equation, and Heat equation, their fundamental solutions, uniqueness principles, mean value properties, and Green's function.
- apply the method of spherical means to solve homogeneous and nonhomogeneous wave equations.
- use characteristics to solve nonlinear partial differential equations, construct complete integrals and envelopes, and understand conservation laws.
- implement various techniques such as similarity solutions and transform methods to derive solutions of different types of partial differential equations.

Syllabus

Unit – 1 **(12 hours)**

Well-posed problems, Classical solution, Laplace equation, Poisson equation, Fundamental solution, Strong maximum principle and uniqueness of solution, Mean value formulas, Representation formula, Green's function, Poisson's formula.

Unit – 2 **(10 hours)**

Heat equation, Fundamental solution for homogeneous and nonhomogeneous initial-value problems, Mean value formula, Strong maximum principle and uniqueness of solution, Local estimates for the solution.

Unit – 3 **(13 hours)**

Wave equation: Solution of homogeneous and nonhomogeneous problems by spherical means,

Nonlinear first order partial differential equations: Complete integrals and envelopes, Characteristics, Introduction to conservation laws.

Unit – 4 **(10 hours)**
Other solution methods: Similarity solutions, Fourier transform and Laplace transform, Cole–Hopf transformation, Potential function, Hodograph and Legendre transform.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] L. C. Evans, *Partial Differential Equations*, American Mathematical Society, Providence, RI, 1998.
[2] F. John, *Partial Differential Equations*, Fourth Edition, Springer-Verlag, New York, 1982.

Suggested Readings
(i) P. R. Garabedian, *Partial Differential Equations*, John Wiley & Sons, Inc., New York- London-Sydney, 1964.
(ii) A. K. Nandakumaran and P. S. Datti, *Partial Differential Equations: Classical Theory with a Modern Touch*, Cambridge University Press, 2020.

DISCIPLINE SPECIFIC CORE – 10: ANALYSIS OF SEVERAL VARIABLES

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSC-10: Analysis of Several Variables	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Calculus, Real Analysis including Riemann Integration

Learning Objectives

The primary objective of this course is to:

- introduce differentiation of vector valued functions on $\mathbb{R}^n$ and their properties.
- familiarize students with integration of functions over rectangles and bounded sets in $\mathbb{R}^n$.
- extend integration of functions to unbounded sets in $\mathbb{R}^n$.
- study change of variables and its applications.

Learning Outcomes

This course will enable the students to:

- check differentiability of vector valued functions on $\mathbb{R}^n$, understand the relation between directional derivative and differentiability, apply chain rule, mean value theorems, inverse and implicit function theorems.
- understand higher order derivatives and be able to apply Taylor's formulas with integral remainder, Lagrange's remainder and Peano's remainder.
- master the concepts of integration over rectangles and bounded sets in $\mathbb{R}^n$.
- generalize the integration theory to unbounded sets in $\mathbb{R}^n$.
- grasp the effect of change of variables in integration.

Syllabus

Unit– 1 **(12 hours)**

The differentiability of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$, Partial derivatives and differentiability, Directional derivatives and differentiability, Chain rule, Mean value theorems, Inverse function theorem and Implicit function theorem.

Unit– 2 **(11 hours)**

Derivatives of higher order, Taylor's formulas with integral remainder, Lagrange's remainder and Peano's remainder, Integral over a rectangle, Existence of the integral.

Unit– 3 **(10 hours)**

Evaluation of the integral, Fubini's theorem, Integral over a bounded set.

Unit– 4 **(12 hours)**

Rectifiable sets, Improper integrals**,** Change of variable theorem, Applications of change of variables.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. Giaquinta and G. Modica, *Mathematical Analysis: An Introduction to Functions of Several Variables*, Birkhäuser, 2009.
[2] J. R. Munkres, *Analysis on Manifolds*, CRC Press, Taylor & Francis, 2018.

Suggested Readings

(i) W. Rudin, *Principles of Mathematical Analysis*, Third Edition, Mc Graw Hill, 1986.
(ii) M. Spivak, *Calculus on Manifolds: A Modern Approach to Classical Theorems of Advanced Calculus*, Taylor & Francis, 2018.

Discipline Specific Elective (DSE) Courses

DSE-7 and DSE-8

Group-1

DISCIPLINE SPECIFIC ELECTIVE: ADVANCED FLUID DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Advanced Fluid Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Calculus and Partial Differential Equations

Learning Objectives

The primary objective of this course is to:

- prepare a foundation for advanced studies in compressible flow, boundary layer theory and magnetohydrodynamics.
- develop concepts, models, and techniques that enable problem-solving in compressible flow, boundary layer theory and magnetohydrodynamics.
- equip students with concepts and techniques to conduct research in the above mentioned domains.

Learning Outcomes

This course will enable the students to:

- learn conservation laws, first and second laws of thermodynamics, internal energy and entropy, different forms of energy equations and dimensional analysis.
- know about compressibility in real fluids, wave motion, sound waves, hyperbolic and dispersive waves, shock waves, their formation, properties and elementary analysis.
- know the concepts of boundary layer, boundary layer equations and their solutions, measurements of boundary layer thickness.
- understand the interaction between hydrodynamic processes and electromagnetic phenomena.
- formulate the basic equations of motion in inviscid and viscous conducting fluid flow and explain Alfven's theorem and magnetohydrodynamic (MHD) waves and MHD shocks.

Syllabus

Unit – 1 **(11 hours)**

Flow characteristics, Conservation laws, Equation of state of a substance, First and second law of thermodynamics, Internal energy and entropy, Energy equation, Nondimensionalizing the basic equations of incompressible viscous fluid flow, Nondimensional numbers.

Unit – 2 **(12 hours)**

Compressibility effects in real fluids, Equations of motion, Sound wave, Hyperbolic and dispersive waves, Isentropic gas flow, Flow through a nozzle, Method of characteristics, Shock jump conditions, Non-linear plane waves, Shock waves and their elementary analysis, Similarity solutions.

Unit – 3 **(11 hours)**

Boundary layer concept, Estimation of boundary layer thickness and friction forces, Prandtl's boundary layer equations, Boundary layer along a flat plate, Boundary layer thickness, General properties of the boundary layer equations, Similar solutions, Momentum and energy integral equations for the boundary layer.

Unit – 4 **(11 hours)**

Maxwell's electromagnetic field equations, Magnetohydrodynamic (MHD) approximations, Magnetic field equation, Magnetic Reynolds number, Magnetic body force, Equations of Motions of conducting fluid, Alfven's theorem, MHD waves, MHD shock waves.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] F. Chorlton, *Text Book of Fluid Dynamics*, CBS Publisher, 2005.
[2] H. Schlichting and K. Gersten, *Boundary Layer Theory*, Ninth Edition, Springer, 2017.
[3] G. B. Witham, *Linear and Nonlinear Waves*, John Wiley & Sons, 1999.

Suggested Readings

(i) K. R. Cramer and S. I. Pai, *Magnetofluid Dynamics for Engineers and Applied Physics*, McGraw Hill Book Co., New York, 1973.
(ii) Y. Shao-Wen, *Foundations of Fluid Mechanics*, PHI, New Delhi, 1960.

DISCIPLINE SPECIFIC ELECTIVE: PROBABILITY THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Probability Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Measure Theory

Learning Objectives

The primary objective of this course is to introduce:

- probability space as a measure space and random variables as measurable functions.
- expectation and moments of random variables.
- notion of convergence in probability.
- conditioning on sub-σ-algebra.

Learning Outcomes

This course will enable the students to learn:

- about probability or uncertainty in abstract setting.
- moments and expectation of random variables which help to understand applications of probability in industry.
- how to apply the idea of convergence in probability.
- weak law and strong law of large numbers and their applications.

Syllabus

Unit – 1 **(11 hours)**

Probability: σ-algebra, Constructing probability triples, The extension theorem, Random variables, Independence of events, Continuity of probabilities, Limit events, The Borel–Cantelli Lemma.

Unit – 2 **(10 hours)**

Expected values: Simple, general non-negative and arbitrary random variables, Moment generating functions, Markov's inequality, Chebyshev's inequality.

Unit – 3 **(12 hours)**

Convergence of random variables: Convergence almost surely, Convergence in probability, Weak law of large numbers, Strong law of large numbers.

Unit – 4 **(12 hours)**

Distributions of random variables: Examples of distributions, Characteristic functions, The central limit theorem, Conditional probability, Conditioning on random variable, Conditioning on a sub-σ-algebra, Conditional variance.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] J. S. Rosenthal, *A First Look at Rigorous Probability Theory*, Second Edition, World Scientific, Singapore, 2006.

Suggested Readings
(i) W. Feller, *An Introduction to Probability Theory and its Applications*, Volume 1, Third Edition, Wiley, 2008.
(ii) J. E. Michael and J. S. Rosenthal, *Probability and Statistics: The Science of Uncertainty*, Second Edition, W. H. Freeman & Co Ltd., 2009.
(iii) S. Ross, *A First Course in Probability*, Tenth Edition, Pearson Education, 2022.
(iv) D. W. Stroock, *Probability Theory, An Analytic View*, Cambridge University Press, 2024.

DISCIPLINE SPECIFIC ELECTIVE: SIMPLICIAL HOMOLOGY THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Simplicial Homology Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- introduce the foundations of simplicial complexes and homology theory.
- develop an understanding of chain mappings and induced homomorphisms.
- apply these concepts to establish fundamental results in topology such as Euler–Poincaré theorem, Brouwer's and Lefschetz fixed-point theorems.

Learning Outcomes

This course will enable the students to:

- identify hyperplanes, simplexes and finite simplicial complexes as subsets of a Euclidean space.
- learn the idea of compact triangulable spaces as geometric carriers of finite simplicial complexes (polyhedra).
- learn the use of homological algebra to associate simplicial homology groups and illustrate it by computing simplicial homology groups of some well-known compact polyhedral.
- prove important applications of simplicial homology theory like invariance of dimension, Euler's formula, Lefschetz and Brouwer's fixed point theorems.

Syllabus

Unit – 1 **(11 hours)**

Geometric simplexes, Geometric complexes and polyhedra, Simplicial maps, Simplicial approximation of continuous maps between two polyhedral.

Unit – 2 **(12 hours)**

Orientation of geometric complexes, Chain complexes, Simplicial homology groups, Structure of homology groups, Relative homology groups, Computation of homology groups, Homology groups of n-sphere.

Unit – 3 **(12 hours)**

Chain mappings, Chain derivation, Chain homotopy, Contiguous maps, Homomorphism induced by continuous maps between two polyhedra, Functorial property of induced homomorphisms, Topological and homotopy invariance of homology groups.

Unit – 4 **(10 hours)**

Euler–Poincaré theorem and Euler's formula, Invariance of dimension, Brouwer's fixed point theorem, Degree of self-mappings of $\mathbb{S}^n$, Brouwer's degree theorem, Existence of eigen values, Lefschetz fixed point theorem.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] F. H. Croom, *Basic Concepts of Algebraic Topology*, Springer, 1978.

Suggested Readings

(i) M. K. Agoston, *Algebraic Topology: A First Course*, Marcel Dekker, 1976.
(ii) M. A. Armstrong, *Basic Topology*, Springer, 1983.
(iii) S. Deo, *Algebraic Topology - A Primer*, Second Edition, Hindustan Book Agency, 2018.

DISCIPLINE SPECIFIC ELECTIVE: THEORY OF UNBOUNDED OPERATORS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Theory of Unbounded Operators	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis and Bounded Operators

Learning Objectives
The primary objective of this course is to:
- introduce the notion of unbounded operators.
- develop the theory of operator semigroups and understand their role in applications, particularly for solving differential equations.

Learning Outcomes
This course will enable the students to:
- identify closed and closable linear operators on Banach spaces.
- compute adjoints of unbounded linear operators.
- understand spectral properties of some unbounded operators.
- comprehend the role unbounded operators and semigroups play in applications, particularly in studying solutions of differential equations.

Syllabus

Unit – 1 **(10 hours)**
Unbounded linear operators, Hilbert adjoints, Hellinger–Toeplitz theorem, Hermitian, symmetric and self-adjoint linear operators, Closed linear operators, Closable operators and their closures on Banach spaces.

Unit – 2 **(12 hours)**
Cayley transform, Deficiency indices, Spectral properties of self-adjoint operators, Multiplication and differentiation operators and their spectra.

Unit – 3 **(11 hours)**
Analytic properties of exponential functions, Matrix Semigroups, Uniformly continuous semigroups, Semigroups on Hilbert spaces, Strongly continuous semigroups.

Unit – 4 **(12 hours)**
Generators of semigroups and their resolvents, Hille–Yosida theorem (for contraction semigroup), Dissipative operators and their properties, Lumer–Phillips theorem, Generators of Group, Stone's theorem.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] K. J. Engle and R. Nagel, *One-parameter Semigroups for Linear Evolution Equations*, Springer-Verlag, New York, 2000.
[2] E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, 2006.

Suggested Readings

(i) S. Goldberg, *Unbounded Linear Operators: Theory and Applications*, Dover Publications, 2006.
(ii) E. Hille and R. S. Phillips, *Functional Analysis and Semi-groups*. American Mathematical Society, Providence, RI, 1957.
(iii) A. Pazy, *Semigroups of Linear Operators and Applications to Partial Differential Equations*, Springer, 1983.
(iv) M. Schechter, *Principles of Functional Analysis*, Second Edition, American Mathematical Society, 2001.
(v) J. Weidmann, *Linear Operators in Hilbert Spaces, Graduate Texts in Mathematics*, Springer, New York, 1980.

Group-2

DISCIPLINE SPECIFIC ELECTIVE: BANACH AND C*-ALGEBRAS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Banach and C*-Algebras	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis and Topology

Learning Objectives

The primary objective of this course is to introduce:

- Banach algebras and C*-algebras.
- various ways to construct new operator algebras using given ones.
- spectrum of elements in Banach algebras and to study its properties.
- Gelfand representations of commutative Banach algebras and of C*-algebras.

Learning Outcomes

This course will enable the students to:

- familiarize with the representations of operator algebras.
- realize commutative Banach algebras and abelian C*-algebras as space of continuous functions on locally compact groups.
- understand the powerful tool of functional calculus.
- identify any C*-algebra as closed *-subalgebra of space of bounded linear operators on a Hilbert space.

Syllabus

Unit – 1 **(11 hours)**

Elementary properties and examples of Banach algebras, Ideals and quotients, Invertible elements, Spectrum and spectral radius, Spectral radius formula, Spectral mapping theorem (for polynomials), Gelfand–Mazur theorem.

Unit – 2 **(11 hours)**

Multiplicative linear functionals, Commutative Banach algebra, w*-topology, Gelfand transform of an element, Structure space, Gelfand representation.

Unit – 3 **(12 hours)**

Elementary properties and examples of C*-algebras, Unitization, Gelfand–Naimark representation of commutative C*-algebras, Continuous functional calculus, Spectral mapping theorem for normal elements, Positive elements of C*-algebras.

Unit – 4 **(11 hours)**
Ideals in C*-algebras, Approximate units, Quotients, Positive linear functionals, Gelfand–Naimark–Segal representation of C*-algebras.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.
[2] G. J. Murphy, *C*-algebras and Operator Theory*, Academic Press Inc., 1990.

Suggested Readings
(i) J. B. Conway, *A Course in Operator Theory, Graduate Texts in Mathematics*, Springer, 2007.
(ii) J. Dixmier, *C*-algebras*, North-Holland Publishing Company, 1977.
(iii) R. G. Douglas, *Banach Algebra Techniques in Operator Theory, Graduate Texts in Mathematics*, Springer, 1998.
(iv) E. Kaniuth, *A Course on Commutative Banach Algebras*, Graduate Texts in Mathematics, Springer, 2009.
(v) S. Kantorovitz, *Introduction to Modern Analysis*, Oxford Graduate Texts in Mathematics, Oxford University Press, 2006.
(vi) M. Takesaki, *Theory of Operator Algebras I*, Springer, 2002.

DISCIPLINE SPECIFIC ELECTIVE: CHAOS THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Chaos Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- introduce some useful and interesting notions like Topological Transitivity and Sensitive dependence on initial conditions.
- study different types of chaos including Devaney's chaos and finding their interrelationships.
- know classical result that period three implies chaos on intervals.
- relate chaos and decomposition theorems.
- study Topological entropy through open covers and also Bowen's definition of entropy, equivalence of these two definitions on compact metric spaces.
- study various interesting results related to topological entropy.

Learning Outcomes

This course will enable the students to:

- construct interesting examples of Topological transitive maps, Topological mixing maps etc.
- know classical examples of Devaney's chaotic maps like tent map, shift maps, logistic maps.
- study and compare different types of chaos.
- find relation between transitivity and chaos on intervals.
- relate chaos theory and classical decomposition theorems.
- study very useful notion of Topological entropy including its properties.
- calculate entropy of any homeomorphism of closed unit interval and of unit circle.

Syllabus

Unit – 1 **(12 hours)**

Topological Transitivity, Locally eventually onto maps, Topological mixing, Sensitive dependence on initial conditions, Devaney's definition of chaos, Transitivity and limit sets for continuous interval maps.

Unit – 2 **(11 hours)**

Characterizing topological mixing in terms of topological transitivity for continuous interval maps, Topological Weakly Mixing, Totally Transitive maps, Relation between transitivity and chaos on intervals, Logistic maps and shift maps as chaotic maps.

Unit – 3 **(12 hours)**

Various other definitions of chaos and their interrelationships. Period three implies chaos, Chaos and decomposition theorems including Bowen's decomposition theorem, Topological Entropy:

Definition using open covers, Examples and properties, Bowen's definition of topological entropy, Equivalence of two definitions, Topological version of Kolmogorov–Sinai theorem.

Unit – 4 **(10 hours)**
Topological Entropy of maps on a compact metric space, Topological Entropy of product maps, of iterations of a map, Topological entropy of an expansive homeomorphism on a compact metric space, of the two-sided shift, of any homeomorphism of the unit interval and of the unit circle.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] N. Aoki and K. Hiraide, *Topological Theory of Dynamical Systems: Recent Advances*, North Holland Publications, 1994.
[2] R. L. Devaney, *A First Course in Chaotic Dynamical Systems*, CRC Press, 2018.
[3] S. Ruette, *Chaos for Continuous Interval Maps: A Survey of Relationship Between Various Kinds of Chaos*, 2018.
[4] Peter Walters, *An Introduction to Ergodic Theory*, Springer, 2000.

Suggested Readings
(i) L. Alsedà, J. Llibre and M. Misiurewicz, *Combinatorial Dynamics and Entropy in Dimension One*, Advanced Series in Nonlinear Dynamics, World Scientific, 2000.
(ii) L. S. Block and W. A. Coppel, *Dynamics in One Dimension*, Springer, 2014.
(iii) M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge University Press, 2015.

DISCIPLINE SPECIFIC ELECTIVE: CHARACTER THEORY OF FINITE GROUPS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Character Theory of Finite Groups	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra and Group Theory

Learning Objectives

The primary objective of this course is to:

- introduce characters of finite groups, class functions, and find the number of irreducible characters.
- find character tables of S_5, using characters of tensor products of CG-modules and powers of characters.
- define restricted characters and prove Clifford's theorem and its application to find the character table of A_5.
- explore some arithmetic properties of character values and introduce real characters.

Learning Outcomes

This course will enable the students to:

- define and construct examples of characters, prove some fundamental properties of characters, and calculate character tables of some small groups and symmetric groups, etc.
- construct new characters from given characters and understand the notion of induced and restricted characters.
- prove the Frobenius reciprocity theorem and the Frobenius–Schur count of involutions.

Syllabus

Unit – 1 **(10 hours)**

Group characters and their properties, Inner product of characters, Class functions and number of irreducible characters, Character tables and some orthogonality relations, Normal subgroups and lifted characters, Linear characters, Character tables of $D_{6,}$ $S_{4,}$ A_4.

Unit – 2 **(11 hours)**

Character of tensor products of CG-modules, Powers of characters, Decomposition of power of a character, Character table of symmetric group S_5, Character table of direct product of groups, Restricted characters, Constituents of a restricted character, Clifford's theorem, Restriction of symmetric groups to alternate groups in general normal subgroups of index two, Character table of A_5.

Unit – 3 **(12 hours)**

Induced CG-modules and their characters: Homomorphisms, Transitivity of induction, Frobenius reciprocity theorem, Values of induced characters. Algebraic integers, Some properties of degrees of irreducible characters and arithmetic properties of character values.

Unit – 4 **(12 hours)**

Real representations, Real conjugacy classes and real characters, Characters which can be realized over the reals, RG-modules and CG-modules, G-invariant symmetric bilinear form, Indicator function, The Frobenius–Schur count of involutions.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] G. James and M. Liebeck, *Representations and Characters of Groups*, Second Edition, Cambridge University Press, 2005.

Suggested Readings

(i) I. M. Issacs, *Character Theory of Finite Groups*, American Mathematical Society reprint, 2006.
(ii) W. Ledermann, *Introduction to Group Characters*, Second Edition, Cambridge University Press, 1987.

DISCIPLINE SPECIFIC ELECTIVE: NONSMOOTH OPTIMIZATION

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Nonsmooth Optimization	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Nonlinear Optimization

Learning Objectives
The primary objective of this course is to:
- understand the tools to deal with nonsmooth convex functions.
- study conjugate duality in terms of conjugate functions for constrained nonlinear optimization problems.
- introduce numerical techniques to solve constrained nonlinear optimization problems.

Learning Outcomes
This course will enable the students to learn:
- the notions of subgradients and subdifferentials for nonsmooth convex functions.
- the use of conjugate functions to develop the theory of conjugate duality.
- about numerical methods like gradient descent method, gradient projection method, Newton's method and conjugate gradient method.
- penalty approach technique to solve constrained nonlinear optimization problems.

Syllabus

Unit – 1 **(11 hours)**
Extended real valued functions, Proper convex functions, Closure of convex functions, Differential derivatives, Subgradients and subdifferentials.

Unit – 2 **(12 hours)**
Conjugate functions, Biconjugate functions, Perturbation functions, Closure of convex functions, Directional derivatives, Subgradients and subdifferentials.

Unit – 3 **(12 hours)**
Gradient descent method, Gradient projection method, Newton's method, Conjugate gradient method.

Unit – 4 **(10 hours)**
Penalty function methods, Exterior penalty function, Interior penalty functions.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] M. Avriel, *Nonlinear Programming: Analysis and Methods*, Dover Publications, 2003.

[2] O. Güler, *Foundations of Optimization*, Springer, 2010.

Suggested Readings

(i) A. Bagirov, N. Karitsa and M. M. Makela, *Introduction to Nonsmooth Optimization: Theory, Practice and Software*, Springer, 2014.
(ii) M. S. Bazaraa, H. D. Sherali and C. M. Shetty, *Nonlinear Programming: Theory and Algorithms,* Third Edition, John Wiley & Sons, 2013.
(iii) D. G. Luenberger and Y. Ye, *Linear and Nonlinear Programming*, Springer, 2008.

Group-3

DISCIPLINE SPECIFIC ELECTIVE: COMPUTATIONAL FLUID DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Computational Fluid Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Partial Differential Equation (Undergraduate level)

Learning Objectives

The primary objective of this course is to teach:

- various numerical schemes on finite difference and finite volume methods for solving PDEs.
- discretization errors and grid dependence.
- some real-world applications of PDEs and fluid dynamics.
- discretization of governing equations of diffusion, convection-diffusion, fluid flow and thereby computing the numerical solutions using the flow variables using algorithms.

Learning Outcomes

This course will enable the students to learn:

- techniques for solving the PDEs along with some initial and boundary conditions by using the finite difference and finite volume methods.
- the basic conservation principles of mass, momentum, energy, discretization of governing equations.
- discretization techniques.
- some popular algorithms like SIMPLE and SIMPLER used to obtain the solutions of steady and unsteady flow problems by finite volume methods.

Syllabus

Unit – 1 **(12 hours)**

Basics of discretization using finite differences, Various single and multi-step explicit and implicit finite difference schemes for 1-D and 2-D parabolic and hyperbolic initial boundary value problems, Alternating Direction Implicit schemes (ADI) for 2-D parabolic and hyperbolic equations, Order of accuracy, Consistency, Stability and convergence of a finite difference scheme, Courant Friedrich Lewy condition.

Unit – 2 **(12 hours)**

Finite difference schemes for second and fourth order 2-D elliptic boundary value problem and applications, Finite volume method for diffusion and convection-diffusion equations, Discretization of one and two-dimensional steady state diffusion and convection-diffusion equations, Central difference, Upwind, Exponential, Hybrid, Power-law and QUICK schemes and their properties.

Unit – 3 **(11 hours)**
Flow field calculation, Pressure-velocity coupling, Vorticity-stream function approach, Primitive variables, Staggered grid, Pressure and velocity corrections, Pressure correction equation, SIMPLE and SIMPLER algorithms.

Unit – 4 **(10 hours)**
Finite volume methods for unsteady flows, Discretization of one-dimensional transient heat conduction, Explicit, fully implicit and Crank–Nicolson schemes, Implementation of boundary conditions.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] J. C. Strikweda, *Finite Difference Schemes and Partial Differential Equations*, Second Edition, SIAM, 2004.
[2] H. K. Versteeg and W. Malalasekera, *An Introduction to Computational Fluid Dynamics: The Finite Volume Method*, Second Edition, Pearson, 2008.

Suggested Readings
(i) J. D. Anderson, *Computational Fluid Dynamics*, McGraw-Hill, 1995.
(ii) S. V. Patankar, *Numerical Heat Transfer and Fluid Flow*, CRC Press, Taylor and Francis, Indian Edition, 2017.
(iii) R. H. Pletcher, J. C. Tannehill and D. A. Anderson, *Computational Fluid Mechanics and Heat Transfer*, CRC Press, Taylor and Francis, 2013.
(iv) J. W. Thomas, *Numerical Partial Differential Equations: Finite Difference Methods*, Springer, 2013.

DISCIPLINE SPECIFIC ELECTIVE: DIFFERENTIAL TOPOLOGY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Differential Topology	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- introduce the concepts of topological manifolds, smooth structures, smooth manifolds, and manifolds with boundary.
- develop an understanding of smooth functions, smooth maps, diffeomorphisms, and tangent spaces.
- explain the Inverse function theorem, immersions and submersions.
- develop the fundamental concepts of 2-manifolds and distinguish between orientable and non-orientable surfaces.
- explore the properties of compact and connected surfaces.

Learning Outcomes

This course will enable the students to:

- identify and construct examples of topological manifolds, smooth structures and manifolds with and without boundary.
- demonstrate understanding of diffeomorphisms and tangent spaces.
- apply the Inverse function theorem, immersions and submersions.
- define key concepts such as 2-manifolds, orientability, compactness, connectedness and boundary of a surface.
- differentiate between orientable and non-orientable surfaces using examples such as the sphere, torus, Möbius strip and Klein bottle.

Syllabus

Unit – 1 **(12 hours)**
Topological manifolds, Topological properties of manifolds, Smooth structures, Examples of smooth manifolds, Manifolds with boundary.

Unit – 2 **(11 hours)**
Smooth functions and smooth maps, Lie groups, Diffeomorphisms.

Unit – 3 **(10 hours)**
Derivatives and tangents, Inverse function theorem, Immersions and submersions.

Unit – 4 **(12 hours)**
Complexes, Connected sum of two surfaces, Non-orientable surfaces (2- Manifolds), Compact and connected surfaces, Classification of compact and connected surfaces with and without boundary.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] V. Guillemin and Alan Pollack, *Differential Topology*, Prentice-Hall, 1974.
[2] L. C. Kinsey, *Topology of Surfaces*, Springer Verlag, 1997.
[3] J. M. Lee, *Introduction to Smooth Manifolds*, Second Edition, Springer, 2013.

Suggested Readings

(i) L. Conlon, *Differentiable Manifolds*, Second Edition, Birkhäuser Advanced Texts, 2001.
(ii) M. Spivak, *A Comprehensive Introduction to Differential Geometry*, Volume 1, Third Edition, Publish or Perish, Huston, Texas, 1999.
(iii) L. W. Tu, *Introduction to Manifolds*, Second Edition, Springer, 2011.
(iv) F. W. Warner, *Foundations of Differentiable Manifolds and Lie Group*, Springer-Verlag, 1983.

DISCIPLINE SPECIFIC ELECTIVE: GENERAL MEASURE THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: General Measure Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Measure Theory and Topology

Learning Objectives

The primary objective of this course is to introduce:

- real valued and complex valued measures.
- decomposition of measure spaces and of measures.
- extension of a premeasure to a measure, Lebesgue measure on Euclidean spaces.
- representation of measures and functionals in terms of integrals.
- product measures.

Learning Outcomes

This course will enable the students to:

- appreciate signed measures and complex measures, mutual singularity of measures, Hahn and Jordan decompositions, Lebesgue decomposition, Radon–Nikodym theorem.
- verify conditions under which a set function defined on a collection of subsets of a set has an extension to a measure on a sigma-algebra.
- apply Riesz representation theorem for bounded linear functionals on L^p-spaces.
- understand product measure and the results of Fubini and Tonelli, and express the Lebesgue measure on Euclidean spaces as a product measure.
- apply Riesz–Markov representation theorem for the bounded linear functionals on the space of continuous functions.

Syllabus

Unit – 1 **(13 hours)**

Signed measures, Hahn and Jordan decompositions, Mutually singular measures, Radon–Nikodym theorem, Lebesgue decomposition, Complex measure.

Unit – 2 **(10 hours)**

The Carathéodory extension theorem, Lebesgue measure on $\mathbb{R}^n$, Regularity and translation invariance of Lebesgue measure on $\mathbb{R}^n$.

Unit – 3 **(10 hours)**

Riesz representation theorem for the dual of L^p-spaces, Product measures, Fubini's theorem, Tonelli's theorem.

Unit – 4 **(12 hours)**

Locally compact Hausdorff spaces and construction of Radon measure, Riesz–Markov representation theorem for positive linear functionals on $C_c(X)$, Riesz representation theorem for the dual of $C(X)$.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] H. L. Royden and P. M. Fitzpatrick, *Real Analysis*, Fourth Edition, Pearson, 2015.
[2] M. E. Taylor, *Measure Theory and Integration*, American Mathematical Society, 2006.

Suggested Readings

(i) G. B. Folland, *Real Analysis: Modern Techniques and Their Applications*, Second Edition, Wiley, New York, 1999.
(ii) P. R. Halmos, *Measure Theory*, Springer Science + Business Media, LLC, 2014.
(iii) E. M. Stein and R. Shakarchi, *Real Analysis: Measure Theory, Integration and Hilbert Spaces*, New Age International Publishers, New Delhi, 2010.

DISCIPLINE SPECIFIC ELECTIVE: THEORY OF NON-COMMUTATIVE RINGS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Theory of Non-commutative Rings	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Groups and Rings

Learning Objectives

The primary objective of this course is to:

- give students an understanding of Wedderburn–Artin theory of semisimple rings.
- develop Jacobson's general theory of radicals, prime and semiprime rings, and primitive and semiprimitive rings.
- introduce the structure of primitive rings as a generalisation of the Wedderburn–Artin theorem on Artinian simple rings.

Learning Outcomes

This course will enable the students to:

- know about an extensive variety of rings, including free rings, Weyl algebra, Hilbert twist and triangular ring.
- understand the module theoretic definition of semisimple rings and how it leads to the Wedderburn–Artin structure theorem on their complete classification.
- know Jacobson's general theory of radicals, semiprime rings, prime, primitive and semiprimitive rings and their structures.
- understand the significance of the fundamental result 'Density Theorem' and its consequences on the structure of primitive rings.

Syllabus

Unit – 1 **(11 hours)**

Simple rings, Reduced rings, Dedekind-finite rings, Algebra, Quaternions, Free k-rings, Rings with generators and relations, Weyl algebra, Formal power series ring, Hilbert's twist ring, Differential polynomial rings, Derivation and inner derivation on a ring, Triangular rings, Characterization of one-sided and two-sided ideals in such rings.

Unit – 2 **(11 hours)**

Noetherian and Artinian rings, Examples of one-sided Noetherian and Artinian triangular rings, Twisted polynomial ring and Quotient of free $\mathbb{Z}$-ring, Semisimple rings, Structure of semisimple rings: Wedderburn–Artin's theorem.

Unit – 3 **(10 hours)**

Structure theorem of simple left Artinian rings, Jacobson radical, J-semisimple rings, Nil and nilpotent ideals, Connection between semisimple and J-semisimple rings, Hopkins–Levitzki theorem, Nakayama's lemma.

Unit – 4 **(13 hours)**
Prime radical, Characterisation of prime and semiprime ideals, Prime and semiprime rings, Structure theorem of primitive rings, Density theorem.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] T.-Y. Lam, *A First Course in Noncommutative Rings*, Springer, 2001.

Suggested Readings
(i) I. N. Herstein, *Noncommutative Rings*, The Mathematical Association of America, 2005.
(ii) T. W. Hungerford, *Algebra*, Springer-Verlag, New York, 1981.
(iii) L. H. Rowen, *Ring Theory*, Student Edition, Academic Press, 1991.

Generic Elective (GE) Courses

GENERIC ELECTIVE – 4 (i): NONSMOOTH OPTIMIZATION

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
GE-4 (i): Nonsmooth Optimization	4	3	1	0	Class XII pass with Mathematics	Basics of Nonlinear Optimization

Learning Objectives

The primary objective of this course is to:

- understand the tools to deal with nonsmooth convex functions.
- study conjugate duality in terms of conjugate functions for constrained nonlinear optimization problems.
- introduce numerical techniques to solve constrained nonlinear optimization problems.

Learning Outcomes

This course will enable the students to learn:

- the notions of subgradients and subdifferentials for nonsmooth convex functions.
- the use of conjugate functions to develop the theory of conjugate duality.
- about numerical methods like gradient descent method, gradient projection method, Newton's method and conjugate gradient method.
- penalty approach technique to solve constrained nonlinear optimization problems.

Syllabus

Unit – 1 **(11 hours)**

Extended real valued functions, Proper convex functions, Closure of convex functions, Differential derivatives, Subgradients and subdifferentials.

Unit – 2 **(12 hours)**

Conjugate functions, Biconjugate functions, Perturbation functions, Closure of convex functions, Directional derivatives, Subgradients and subdifferentials.

Unit – 3 **(12 hours)**

Gradient descent method, Gradient projection method, Newton's method, Conjugate gradient method.

Unit – 4 **(10 hours)**

Penalty function methods, Exterior penalty function, Interior penalty functions.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. Avriel, *Nonlinear Programming: Analysis and Methods*, Dover Publications, 2003.
[2] O. Güler, *Foundations of Optimization*, Springer, 2010.

Suggested Readings

(i) A. Bagirov, N. Karitsa and M. M. Makela, *Introduction to Nonsmooth Optimization: Theory, Practice and Software*, Springer, 2014.
(ii) M. S. Bazaraa, H. D. Sherali and C. M. Shetty, *Nonlinear Programming: Theory and Algorithms,* Third Edition, John Wiley & Sons, 2013.
(iii) D. G. Luenberger and Y. Ye, *Linear and Nonlinear Programming*, Springer, 2008.

GENERIC ELECTIVE – 4 (ii): PROBABILITY THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
GE-4 (ii): Probability Theory	4	3	1	0	Class XII pass with Mathematics	Basics of Integration

Learning Objectives

The primary objective of this course is to introduce:

- probability space as a measure space and random variables as measurable functions.
- expectation and moments of random variables.
- notion of convergence in probability.
- conditioning on sub-σ-algebra.

Learning Outcomes

This course will enable the students to learn:

- about probability or uncertainty in abstract setting.
- moments and expectation of random variables which help to understand applications of probability in industry.
- how to apply the idea of convergence in probability.
- weak law and strong law of large numbers and their applications.

Syllabus

Unit – 1 **(11 hours)**

Probability: σ-algebra, Constructing probability triples, The extension theorem, Random variables, Independence of events, Continuity of probabilities, Limit events, The Borel–Cantelli Lemma.

Unit – 2 **(10 hours)**

Expected values: Simple, general non-negative and arbitrary random variables, Moment generating functions, Markov's inequality, Chebyshev's inequality.

Unit – 3 **(12 hours)**

Convergence of random variables: Convergence almost surely, Convergence in probability, Weak law of large numbers, Strong law of large numbers.

Unit – 4 **(12 hours)**

Distributions of random variables: Examples of distributions, Characteristic functions, The central limit theorem, Conditional probability, Conditioning on random variable, Conditioning on a sub-σ-algebra, Conditional variance.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. S. Rosenthal, *A First Look at Rigorous Probability Theory*, Second Edition, World Scientific, Singapore, 2006.

Suggested Readings

(i) W. Feller, *An Introduction to Probability Theory and its Applications*, Volume 1, Third Edition, Wiley, 2008.
(ii) J. E. Michael and J. S. Rosenthal, *Probability and Statistics: The Science of Uncertainty*, Second Edition, W. H. Freeman & Co Ltd., 2009.
(iii) S. Ross, *A First Course in Probability*, Tenth Edition, Pearson Education, 2022.
(iv) D. W. Stroock, *Probability Theory, An Analytic View*, Cambridge University Press, 2024.

Syllabi of Courses in Semester-III of M.Sc. Mathematics under Structure-3 (Research)

Discipline Specific Core (DSC) Course

DISCIPLINE SPECIFIC CORE: MATRIX GROUPS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSC: Matrix Groups	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Calculus and Linear Algebra

Learning Objectives

The primary objective of this course is to:

- introduce the notion of matrix groups which serves as a bridge between algebra, geometry and analysis.
- use exponential and logarithm maps to connect one-parameter subgroups with Lie algebras.
- connect Clifford algebra constructions to orthogonal groups, spin groups and isomorphism questions.

Learning Outcomes

This course will enable the students to:

- distinguish between general linear, orthogonal and special orthogonal groups with explicit examples.
- construct and compute reflection and rotation matrices in $\mathbb{R}^n$and analyze their properties.
- comprehend maximal tori, their coverings and conjugacy results in compact matrix groups.
- appreciate the applications of matrix groups in algebraic geometry, complex analysis, group and ring theory, number theory, quantum physics, Einstein's special relativity, Heisenberg's uncertainty principle, quark theory, Fourier series, combinatorics and many more areas.

Syllabus

Unit – 1 **(10 hours)**

The general linear groups, The orthogonal groups, The isomorphism question, Reflection in $\mathbb{R}^n$, Curves in a vector space, Smooth homeomorphisms, The special orthogonal groups.

Unit – 2 **(11 hours)**

Orthogonal matrices and isometries, Exponential and Logarithm of a matrix, One-parameter subgroups, Lie Algebras, $SO(3)$ and $Sp(1)$.

Unit – 3 **(12 hours)**

Maximal tori, Covering by maximal tori, Reflections in $\mathbb{R}^n$, Monogenic groups.

Unit – 4 **(12 hours)**
Conjugacy of maximal tori, Clifford algebras, $Pin(k)$, $Spin(k)$ and isomorphisms.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] M. L. Curtis, *Matrix Groups*, Springer, 1984.

Suggested Readings
(i) A. Baker, *Matrix Groups: An Introduction to Lie Group Theory*, Springer Undergraduate Mathematics Series, 2001.
(ii) K. Tapp, *Matrix Groups for Undergraduates*, American Mathematical Society, 2016.

Discipline Specific Elective (DSE) Courses

DISCIPLINE SPECIFIC ELECTIVE: ADVANCED COMPLEX ANALYSIS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Advanced Complex Analysis	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Complex Analysis

Learning Objectives

The primary objective of this course is to:

- explore the metric space structure of the spaces of continuous and analytic functions, through Arzela–Ascoli, Hurwitz and Montel theorems.
- investigate various characterizations of simply connected regions, with a special focus on the Riemann mapping theorem.
- use Runge's and Mittag–Leffler's theorems to approximate and interpolate analytic and meromorphic functions.
- analyze the range of analytic functions using Bloch's/ Landau's constants and Picard's theorem.

Learning Outcomes

This course will enable the students to:

- apply various variants of maximum modulus theorem, encompassing Hadamard's three circles theorem and Phragmen–Lindelöf theorem.
- comprehend the notions of normality, compactness, equicontinuity and local boundedness for the spaces of continuous and analytic functions.
- construct and factorize entire functions using infinite products, including special functions like gamma and zeta.
- analyze the harmonic functions on a disk using Poisson kernel, which in turn, solves the Dirichlet problem for a unit disk.

Syllabus

Unit – 1 **(12 hours)**

Convex functions and Hadamard's three circles theorem, Maximum-Modulus theorem (third version), Phragmen–Lindelöf theorem, Spaces of continuous functions, Normal and equicontinuous families, Arzela–Ascoli theorem.

Unit – 2 **(11 hours)**

Spaces of analytic functions, Hurwitz's theorem, Montel's theorem, Riemann mapping theorem, Infinite products, Weierstrass factorization theorem, Factorization of sine function.

Unit – 3 **(10 hours)**
Gamma and Riemann zeta function, Runge's theorem, Characterizations of simple connectedness, Mittag–Leffler's theorem.

Unit – 4 **(12 hours)**
Harmonic functions, Mean value property, Maximum and minimum principles, Harmonic function on a disk, Harnack's theorem, Range of an analytic function, Bloch's theorem, Bloch's and Landau's constants, Picard's theorem.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] J. B. Conway, *Functions of One Complex Variable*, Second Edition, Narosa Publishing House, New Delhi, 2002.

Suggested Readings
(i) L. V. Ahlfors, *Complex Analysis*, Mc Graw Hill Co., Indian Edition, 2017.
(ii) L. Hahn and B. Epstein, *Classical Complex Analysis*, Jones and Bartlett, 1996.
(iii) E. M. Stein and R. Shakarchi, *Complex Analysis*, Princeton University Press, 2003.
(iv) D. Ullrich, *Complex Made Simple*, Volume 97, American Mathematical Society, 2008.

DISCIPLINE SPECIFIC ELECTIVE: ADVANCED FUNCTIONAL ANALYSIS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Advanced Functional Analysis	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis and Topology

Learning Objectives
The primary objective of this course is to:
- define and explain the structure of a topological vector space and its fundamental properties.
- differentiate between normed, metrizable, locally convex and Hausdorff topological vector spaces.
- introduce the foundational theorems of functional analysis, including the Hahn–Banach, Banach–Steinhaus, Open mapping, and Closed graph theorems in the context of locally convex spaces.
- explain some applications of Banach–Alaoglu theorem and Krein–Milman theorem.

Learning Outcomes
This course will enable the students to:
- appreciate types of topological vector spaces and their separation properties.
- understand quotient spaces, weak topology and weak*-topology.
- analyze concepts of continuity, boundedness, and convergence for linear operators and functionals on topological vector spaces.
- understand the notion of local convexity and the role of seminorms in defining locally convex topologies.

Syllabus

Unit – 1 **(12 hours)**
Topological vector spaces, Types of Topological vector spaces, Separation properties, Linear mappings, Finite dimensional spaces, Metrization, Boundedness and continuity, Seminorms and local convexity, Normability.

Unit – 2 **(11 hours)**
Quotient spaces, Seminorms and quotient spaces, Examples, Baire category theorem, Banach–Steinhaus theorem, The open mapping theorem and the closed graph theorem on topological vector spaces.

Unit – 3 **(11 hours)**
Hahn–Banach separation theorem on topological vector spaces, Continuous extension theorem, Weak topologies, Weak topology and convexity, Weak topology and metrizability, Weak*-topology of a dual space, Compact convex sets, Banach–Alaoglu theorem and applications, Goldstine theorem.

Unit – 4 **(11 hours)**
Extreme points, Krein–Milman theorem, Convex hull of compact sets, Applications of Krein–Milman theorem: Stone–Weierstrass theorem, Markov–Kakutani fixed point theorem.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.
[2] W. Rudin, *Functional Analysis*, Second Edition, Tata Mc Graw-Hill, 2011.

Suggested Readings
(i) V. I. Bogachev and O. G. Smolyanov, *Topological Vector Spaces and Their Applications*, Springer, 2017.
(ii) S. Kantorovitz, *Introduction to Modern Analysis*, Oxford Graduate Texts in Mathematics, Oxford University Press, 2006.
(iii) J. Voigt, *A Course on Topological Vector Spaces*, Birkhäuser, 2020.

DISCIPLINE SPECIFIC ELECTIVE: ALGEBRAIC CODING THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Algebraic Coding Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra, Groups and Rings

Learning Objectives
The primary objective of this course is to:
- provide an introduction to algebraic coding theory, particularly linear codes.
- discuss bounds on the parameters along with cyclic codes.
- describe some well-known codes, such as Reed–Muller and Golay codes.
- explore the algebraic structure of Cyclic and Quadratic residue codes over fields and rings.

Learning Outcomes
This course will enable the students to:
- get an insight into the matrix representation of a code, as well as encoding and decoding.
- understand Hamming, MDS and Reed–Muller codes.
- describe cyclic codes and their generator polynomial.
- learn about special cyclic codes, such as Quadratic residue codes, and their properties over the ring $\mathbb{Z}_4$.

Syllabus

Unit – 1 **(10 hours)**
Error detecting and error correcting codes, Maximum likelihood decoding, Hamming distance, Linear codes, Hamming weight, Generator matrix, Parity check matrix, Equivalence of linear codes, Encoding and decoding of linear codes, Syndrome decoding.

Unit – 2 **(11 hours)**
Bounds on codes, Sphere covering bound, Hamming bound, Perfect codes, Binary Hamming codes, Binary Golay codes, Singleton bound and MDS codes. Propagation rules, Reed–Muller codes.

Unit – 3 **(12 hours)**
Cyclic codes, Cyclic codes as ideals, Generator polynomial of cyclic codes, Generator and parity-check matrices of cyclic codes**,** Decoding of cyclic codes, Burst error correcting codes.

Unit – 4 **(12 hours)**
Quadratic residue codes: QR codes over fields of characteristic 2 and 3, Cyclic codes and their generating polynomial over $\mathbb{Z}_4$, QR codes over $\mathbb{Z}_4$.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] S. Ling and C. Xing, *Coding Theory: A First Course*, Cambridge University Press, 2004.
[2] W. C. Huffman and V. Pless, *Fundamentals of Error Correcting Codes*, Cambridge University Press, 2010.

Suggested Readings

(i) R. Hill, *A First Course in Coding Theory*, Oxford University Press, 1986.
(ii) F. J. Mac William and N. J. A. Sloane, *Theory of Error Correcting Codes, Part I & II,* Elsevier/North-Holland, Amsterdam, 1977.

DISCIPLINE SPECIFIC ELECTIVE: ALGEBRAIC TOPOLOGY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Algebraic Topology	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- understand the concepts of homotopic maps, homotopy type, retracts, and deformation retracts, and their role in algebraic topology.
- compute fundamental groups of basic topological spaces.
- acquire knowledge of covering projections, lifting properties, Borsuk–Ulam theorem, and classification techniques of covering spaces.
- learn free groups and free products to understand and apply the Seifert–Van Kampen theorem.

Learning Outcomes

This course will enable the students to:

- distinguish between spaces with the same homotopy type.
- compute fundamental groups of standard spaces such as n-sphere $\mathbb{S}^n$ and punctured planes.
- apply the concept of fundamental groups to prove Brouwer's fixed-point theorem and the Fundamental theorem of Algebra.
- explain the lifting theorems and their implications in topology.
- classify covering spaces for given base spaces.
- apply Seifert–Van Kampen theorem to compute fundamental groups of glued spaces.

Syllabus

Unit – 1 **(11 hours)**
Homotopic maps, Homotopy type, Retract and deformation retract.

Unit – 2 **(12 hours)**
Fundamental group, Calculation of fundamental groups of n-sphere $\mathbb{S}^n$ and punctured plane, Brouwer's fixed-point theorem, Fundamental theorem of Algebra.

Unit – 3 **(12 hours)**
Covering projections, Lifting theorems, Borsuk–Ulam theorem, Classification of covering spaces.

Unit – 4 **(10 hours)**
Free products, Free groups, Seifert–Van Kampen theorem and its applications.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. R. Munkres, *Elements of Algebraic Topology*, Addison-Wesley Publishing Company, 1984.
[2] T. B. Singh, *Introduction to Topology*, Springer Nature Singapore, 2019.

Suggested Readings

(i) G. E. Bredon, *Geometry and Topology*, Springer, 2014.
(ii) A. Hatcher, *Algebraic Topology*, Cambridge University Press, 2002.
(iii) W. S. Massey, *A Basic Course in Algebraic Topology*, World Publishing Corporation, 2009.
(iv) J. J. Rotman, *An Introduction to Algebraic Topology*, Springer, 2011.
(v) E. H. Spanier, *Algebraic Topology*, Springer-Verlag, 1989.

DISCIPLINE SPECIFIC ELECTIVE: COMMUTATIVE ALGEBRA

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Commutative Algebra	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra and Field Theory

Learning Objectives

The objective of this course is to:

- develop a solid understanding of the structure of commutative rings, ideals, their radicals, extension, contraction etc.
- study important constructions such as total quotient rings, localizations.
- develop basic foundation in other areas of mathematics such as algebraic geometry, algebraic number theory.

Learning Outcomes

This course will enable the students to:

- know the localization of rings at a prime ideal that is an algebraic analogue of the geometric notion concentrating attention near a point.
- know more closely the polynomial rings, power series rings in one or more variables over a commutative ring and their prime spectrum.
- define, identify, and elaborate integral closure of rings, valuations rings, discrete valuation rings, structure theorem of Artin rings.

Syllabus

Unit – 1 **(12 hours)**

Radical of an ideal, Prime avoidance lemma, Chinese remainder theorem, Extension and contraction of ideals, Jacobson radical of a ring, Nakayama lemma, Tensor product of modules.

Unit – 2 **(13 hours)**

Rings and modules of fractions, Localization, Local properties, Primary decomposition, First and second uniqueness theorem of primary decomposition, Associated prime ideals of decomposable ideals.

Unit – 3 **(10 hours)**

Integral ring extensions, Going up theorem, Going down theorem, Integrally closed domains, Valuation rings, Hilbert's Nullstellensatz theorem.

Unit – 4 **(10 hours)**

Noetherian rings, Primary decomposition in Noetherian rings, Artin rings, Structure theorem for Artin rings, Discrete valuation rings.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] M. F. Atiyah and I. G. MacDonald, *Introduction to Commutative Algebra*, CRC Press, Taylor & Francis, 2018.

Suggested Readings
(i) D. Eisenbud, *Commutative Algebra with a View Towards Algebraic Geometry*, Springer, 2004.
(ii) R. Y. Sharp, *Steps in Commutative Algebra*, Cambridge University Press, 2000.
(iii) B. Singh, *Basic Commutative Algebra*, World Scientific, 2011.
(iv) O. Zariski and P. Samuel, *Commutative Algebra*, Volume I & II, Springer, 1975.

DISCIPLINE SPECIFIC ELECTIVE: DIFFERENTIAL GEOMETRY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Differential Geometry	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra, Multivariate Calculus and Differential Equations

Learning Objectives

The primary objective of this course is to introduce:

- surfaces and parametrized surfaces.
- orientation on connected surfaces.
- geodesics on surfaces.
- Weingarten maps on oriented surfaces.
- arc length and curvature of oriented plane curves.
- curvatures of oriented surfaces.

Learning Outcomes

This course will enable the students to:

- understand the concepts of level sets and graphs of functions, smooth vector fields, tangent spaces of level sets.
- appreciate surfaces and parametrized surfaces, Gauss map, geodesics and parallel transport on oriented surfaces.
- know what the Weingarten map of an oriented surface is, realize it as shape operator and use it to compute curvature of oriented plane curves.
- find global parametrization and hence arc length of an oriented plane curve.
- compute various types of curvatures of surfaces.

Syllabus

Unit – 1 **(10 hours)**

Level sets in $\mathbb{R}^{n+1}$ and graphs of functions, Smooth vector fields and existence and uniqueness of their integral curves, Tangent spaces of level sets at regular points, Surfaces in $\mathbb{R}^{n+1}$ as inverse images of regular values of smooth functions, Necessary condition for extrema of functions on surfaces-Lagrange multipliers, Existence of a normal vector field on a connected surface, Orientation, Gauss map.

Unit – 2 **(13 hours)**

The notion of a geodesic on a surface, Existence and uniqueness of a geodesic on a surface through a given point with a given velocity vector thereof, Covariant derivative of a smooth vector field, Parallel vector field along a curve, Existence and uniqueness of a parallel vector field along a curve with a given initial vector, Weingarten map of a surface at a point, Local parametrization and curvature of a plane curve.

Unit – 3 **(10 hours)**
Global parametrization and arc length of an oriented plane curve, Differential 1-forms, Line integral of 1-forms over parametrized curves.

Unit – 4 **(12 hours)**
Parametrized surfaces with examples, Curvature of surfaces, Normal curvature of a surface at a point in a given direction, Principal curvatures, First and second fundamental forms, Gauss-Kronecker curvature and mean curvature.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] A. Pressley, *Elementary Differential Geometry*, Springer-Verlag London Limited, 2012.
[2] J. A. Thorpe, *Elementary Topics in Differential Geometry*, Springer (India) Pvt. Limited, 2004.

Suggested Readings
(i) W. Kuhnel, *Differential Geometry: Curves-Surfaces-Manifolds*, Third Edition, American Mathematical Society, 2015.
(ii) B. O' Neill, *Elementary Differential Geometry*, Second Edition, Academic Press INC., Academic Press, New York, 2006.

DISCIPLINE SPECIFIC ELECTIVE: DYNAMICAL SYSTEMS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Dynamical Systems	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology and Ordinary Differential Equations

Learning Objectives

The primary objective of this course is to:

- understand discrete and continuous systems with case studies to study nonlinear systems of ordinary differential equations and dynamical systems.
- understand the concepts, models and techniques to realize the real-world problems and stability of the systems along with the chaotic dynamic behaviour of models by understanding bifurcations.

Learning Outcomes

This course will enable the students to learn:

- formulation of mathematical models with the stability analysis near the equilibrium points.
- how the concept of phase portraits helps to analyse mathematical model graphically.
- the qualitative behaviour of the solution set of a given system of differential equations including the invariant sets and limiting behaviour of the dynamical system or flow defined by the system of differential equations.
- how different bifurcations explain the chaotic behaviour of the system.

Syllabus

Unit – 1 **(13 hours)**

Linear systems: Jordan forms, Stability theory; Nonlinear systems: Fundamental existence-uniqueness theorem, Dependence on initial conditions and parameters, Flow of a differential equation, Linearization, Stable manifold theorem, Hartman–Grobman theorem.

Unit – 2 **(10 hours)**

Stability and Lyapunov functions, Saddle points, Nodes, Foci, Centers and nonhyperbolic critical points, Center manifold theorem.

Unit – 3 **(12 hours)**

Limit sets and attractors, Periodic orbits and limit cycles, Poincaré map, Stable manifold theorem for periodic orbits, Poincare-Bendixson theorem.

Unit – 4 **(10 hours)**

Bifurcations at nonhyperbolic equilibrium points, Saddle node, Transcritical and Pitchfork bifurcations, Hopf bifurcation.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. W. Hirsch, S. Smale and R. L. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, Third Edition, Academic Press, 2013.
[2] L. Perko, *Differential Equations and Dynamical Systems*, Third Edition, Springer Verlag, 2001.

Suggested Readings

(i) R. L. Devaney, *A First Course in Chaotic Dynamical Systems: Theory and Experiment*, CRC Press, Taylor & Francis, 2018.
(ii) S. H. Strogatz, *Nonlinear Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 2018.
(iii) S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, TAM Volume 2, Springer-Verlag, NY, 1990.

DISCIPLINE SPECIFIC ELECTIVE: FINITE ELEMENT METHODS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Finite Element Methods	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Differential Equations

Learning Objectives

The primary objective of this course is to:

- introduce basic aspects of finite element methods (FEM) including domain discretization, polynomial interpolation, application of boundary conditions, assembly of global arrays, and solution of the resulting algebraic systems.
- discuss the use of finite element methods in solving engineering problems in the domain of solid mechanics, fluid mechanics, heat transfer and electromagnetism.

Learning Outcomes

This course will enable the students to:

- use integral statement to deduce finite element approximations for the underlying linear partial differential equations.
- write special-purpose finite element programs within a procedural programming environment.
- use finite element methods to solve engineering problems in solids mechanics, fluid mechanics, heat transfer, and electromagnetism.
- assess the accuracy and reliability of finite element solutions and troubleshoot problems arising from errors in a given finite element analysis.

Syllabus

Unit – 1 **(12 hours)**

Basic concepts of weak formulation, Variational formulation of a one dimensional model equation, Basis function and finite element solutions, Collocation method, Ritz method, Least square method, Standard Galerkin method, FEM for model problem, Error estimate for FEM for model equation, Convergence analysis.

Unit – 2 **(11 hours)**

Various shapes of finite element, Higher order basis functions, Finite element methods for elliptic problems: Variational methods, Standard Galerkin method, Error estimate for FEM for elliptic problem, FEM for Poisson equation.

Unit – 3 **(12 hours)**

Finite element methods for parabolic problems: One dimensional model problems, Semi-discretization in space, Error estimates, Discretization in space and time, Galerkin method, Finite element methods for hyperbolic problems: Standard Galerkin method, Standard Galerkin method with strongly and weakly imposed boundary conditions.

Unit – 4 **(10 hours)**

Applications of the FEM to second order BVPs in one dimension, Applications of the FEM to linear elliptic, parabolic and hyperbolic equations.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] G. Evans, J. Blackledge and P. Yardley, *Numerical Methods for Partial Differential Equations*, Springer-Verlag, London, 2000.
[2] C. Johnson, *Numerical Solutions of Partial Differential Equations by Finite Element Methods*, *Cambridge University Press*, Cambridge, 1987.
[3] J. Whiteley, *Finite Element Methods - A Practical Guide*, Springer, 2016.

Suggested Readings

(i) Z. Chen, *Finite Element Methods and Their Applications*, Springer-Verlag, New York, 2005.
(ii) V. Thomee, *Galerkin Finite Element Methods for Parabolic Problems*, Second Edition, Springer-Verlag, Berlin, 2006.

DISCIPLINE SPECIFIC ELECTIVE: NUMERICAL METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Numerical Methods for Ordinary Differential Equations	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Ordinary Differential Equations

Learning Objectives
The primary objective of this course is to:
- develop the basic theory underlying the numerical solution of differential equations.
- introduce the concepts of consistency, stability and convergence of finite difference methods.
- execute the numerical schemes for the solution of differential equations.

Learning Outcomes
This course will enable the students to:
- gain a thorough understanding of the fundamental concepts involved in the construction and analysis of finite difference schemes for solving ordinary differential equations (ODEs).
- apply various numerical methods based on finite difference approaches to obtain approximate solutions for both initial value problems (IVPs) and boundary value problems (BVPs).
- develop the ability to select appropriate finite difference methods for specific types of problems and effectively apply them to real world applications.

Syllabus

Unit – 1 **(11 hours)**
Initial value problems: Existence and uniqueness of solution, Finite difference equation, Truncation error, Single step methods for first order IVPs and system of IVPs- Family of explicit and implicit Runge–Kutta methods, Taylor series methods, Derivation, Truncation error, Consistency, Stability and convergence analysis.

Unit – 2 **(12 hours)**
IVPs for the system of ODEs, Consistency, Zero stability and convergence of linear multistep methods, Routh–Hurwitz criterion, Order and error constant, First Dahlquist Barrier, Local truncation error and global truncation error, Error bounds, Local error, Linear stability theory, Higher order differential equations.

Unit – 3 **(12 hours)**
Derivation of explicit and implicit multistep methods for IVPs and system of IVPs, Truncation error, Stability and convergence analysis of family of Nystrom method, Adams–Bashforth method,

Adams–Moulton method, Milne–Simpson method, Predictor corrector method, and Modified predictor corrector method, Hybrid method, Multistep methods for second order IVPs.

Unit – 4 **(10 hours)**
Linear BVPs for second order ordinary differential equations, Shooting method, Finite difference method, Collocation method, Derivative boundary conditions, Nonlinear two-point BVPs, Higher order finite difference methods, Stability, Truncation error and convergence analysis.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] M. K. Jain, S. R. K. Iyenger and R. K. Jain, *Numerical Methods for Scientific and Engineering Computations*, Seventh Edition, New Age International Publisher, 2019.
[2] J. D. Lambert, *Numerical Methods for Ordinary Differential Systems: The Initial Value Problem*, John Wiley & Sons, 1991.

Suggested Readings
(i) K. E. Atkinson, W. Han and D. E. Stewart, *Numerical Solution of Ordinary Differential Equations*, John Wiley & Sons, 2009.
(ii) J. C. Butcher, *The Numerical Analysis of Ordinary Differential Equations*, Second Edition, Wiley, New York, 2008.
(iii) L. Collatz, *The Numerical Treatment of Differential Equations*, Springer-Verlag, 1966.

DISCIPLINE SPECIFIC ELECTIVE: REPRESENTATION OF FINITE GROUPS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Representation of Finite Groups	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra and Group Theory

Learning Objectives
The primary objective of this course is to:
- represent finite groups as groups of matrices (via homomorphisms) and apply the tools of linear algebra to study the group structure.
- introduce the notion of Group algebra, which plays an essential role in classifying representations of groups.
- to discuss some applications of representations of finite groups, such as the Burnside's theorem.

Learning Outcomes
This course will enable the students to:
- define and construct examples of group representations, *FG*-modules, group algebras.
- grasp key concepts and tools of representation theory and establish a link between *FG*-modules and group representations.
- prove and apply Maschke's theorem and Schur's lemma to describe all irreducible representations of finite groups over the field of complex numbers.
- apply the theory of characters and group representations to gain insight into group structure, such as normal subgroups, and the solubility of groups.

Syllabus

Unit – 1 **(11 hours)**
Representation of groups, *FG*-modules and *FG*-submodules, and reducibility, Permutation modules, *FG*-modules and equivalent representations, Reducible and irreducible *FG*-modules, Group algebra of *G*, Regular *FG*-module and regular representations, *FG*-homomorphisms, Direct sum of *FG*-modules.

Unit – 2 **(11 hours)**
Maschke's theorem for *FG*-modules and consequences. Schur's lemma and its converse, Application of Schur's lemma, Irreducible modules and group algebra, Structure of group algebra and space of *CG*-homomorphisms.

Unit – 3 **(10 hours)**
Characters and their properties, Permutation and regular characters, Inner product, Number of irreducible characters, Orthogonality relations and finding normal subgroups.

Unit – 4 **(13 hours)**
Algebraic numbers, Algebraic integers and their properties, Character values, The Burnside's *(p,q)*-theorem and solubility of some particular groups.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] G. James and M. Liebeck, *Representations and Characters of Groups*, Second Edition, Cambridge University Press, 2005.

Suggested Readings
(i) C. W. Curtis and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras*, American Mathematical Society, 2006.
(ii) W. Fulton and J. Harris, *Representation Theory - A First Course*, Springer-Verlag, 2004.
(iii) I. M. Issacs, *Character Theory of Finite Groups*, American Mathematical Society reprint, 2006.

DISCIPLINE SPECIFIC ELECTIVE: THEORY OF BOUNDED OPERATORS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Theory of Bounded Operators	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis

Learning Objectives
The primary objective of this course is to:
- introduce some classes of bounded linear operators which play a central role in both pure and applied mathematics.
- study the properties and spectral theory of these operators.

Learning Outcomes
This course will enable the students to understand:
- the spectrum and sub-divisions of spectrum of standard operators like shifts and multiplication.
- the spectral theorem for some classes of bounded linear operators.
- the concepts of compactness, self-adjointness and positivity of bounded linear operators.
- trace class and Hilbert–Schmidt operators.

Syllabus

Unit – 1 **(11 hours)**
Properties of spectrum and resolvent of bounded operators, Subdivision of the spectrum including point, approximate and compression spectrum.

Unit – 2 **(10 hours)**
Operators on Hilbert spaces, Adjoint operator, Projections and idempotents, Operations with projections, Invariant and reducing subspaces.

Unit – 3 **(14 hours)**
Compact operators on Hilbert spaces, Diagonalisation of compact self-adjoint operators, Spectral theorem and functional calculus for Compact normal operators, Positive operators, Compact operators on Banach spaces, Spectral theory of compact operators.

Unit – 4 **(10 hours)**
Polar decomposition, Singular values, Trace class operators, Trace norm and Hilbert Schmidt operators.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] R. Bhatia, *Notes on Functional Analysis*, Hindustan Book Agency, 2009.
[2] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.

Suggested Readings

(i) E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, 2006.
(ii) B. Simon, *Operator Theory: A Comprehensive Course in Analysis*, Part 4, American Mathematical Society, 2015.
(iii) S. R. Garcia, J. Mashregi and W. T. Ross, *Operator Theory by Example*, Oxford University Press, 2023.

DISCIPLINE SPECIFIC ELECTIVE: TOPOLOGICAL DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Topological Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- provide a strong background of topological dynamical systems including their applications.
- develop some useful and interesting dynamical properties like expansivity, shadowing and topological stability with supporting examples and results from symbolic and topological dynamics.
- introduce the celebrated Sarkovskii's theorem.

Learning Outcomes

This course will enable the students to:

- construct interesting examples of dynamical systems and topological conjugacy.
- visualize stable sets, omega sets and alpha limit sets.
- understand the applications of Sarkovskii's theorem.
- use subshifts of finite type to characterize irreducible matrices.
- prove key results on expansivity and shadowing regarding existence/non-existence, product, subspace and their different characterizations etc.
- find the class of topologically stable homeomorphisms.

Syllabus

Unit – 1 **(10 hours)**

Definition and examples (including real life examples) of dynamical systems, Orbits, Types of orbits, Topological conjugacy and orbits, Phase portrait-graphical analysis of orbit, Periodic points and stable sets, Omega and alpha limit sets and their properties.

Unit – 2 **(10 hours)**

Sarkovskii's theorem, Shift spaces and subshift, Subshift of finite type, Subshift represented by a matrix, Characterizations of irreducible matrices.

Unit – 3 **(13 hours)**

Definition and examples of expansive homeomorphisms, Properties of expansive homeomorphisms, Non-existence of expansive homeomorphism on the unit interval and unit circle, Generators and weak generators, Generators and expansive homeomorphisms.

Unit – 4 **(12 hours)**

Converging semi-orbits for expansive homeomorphisms, Definition, examples and properties of maps having shadowing property, Topological Anosov homeomorphisms and topological stability.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] N. Aoki and K. Hiraide, *Topological Theory of Dynamical Systems: Recent Advances*, North Holland Publications, 1994.
[2] M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge University Press, 2004.

Suggested Readings

(i) D. C. Hanselman and B. Little field, *Mastering MATLAB*, Pearson, 2012.
(ii) D. Lind and B. Marcus, *An Introduction to Symbolic Dynamics and Coding*, Cambridge University Press, 1996.
(iii) C. Robinson, *Dynamical Systems*, *Stability, Symbolic Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 1998.
(iv) J. de. Vries, *Elements of Topological Dynamics*, Springer, 1993.

Research Methods/ Tools/ Writing Courses

ADVANCED RESEARCH METHODOLOGY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Advanced Research Methodology	2	1	0	1	Same as for entry to M.Sc. Mathematics	NIL

Learning Objectives

The primary objective of this course is to introduce:

- potential ethical problems in research design, data collection, analysis, and reporting.
- important issues like plagiarism, fabrication, falsification, informed consent and confidentiality.
- ways in which conflicts of interest and research misconduct arise.
- the rich contribution of India in the field of Mathematics.

Learning Outcomes

This course will enable the students to:

- appreciate the importance of ethical conduct for the credibility and societal trust in science.
- foster a culture of responsibility, respect, and fairness in research environments.
- use national and international ethical standards in planning and conducting research.
- inculcate the habit of self-reading and acquire the in-depth knowledge of history of the core discipline.

Syllabus

Unit – 1 **(8 hours)**

Research Ethics: Ethics with respect to science and research, Intellectual honesty and research integrity; Scientific misconducts: Falsification, Fabrication and Plagiarism (FFP).

Unit – 2 **(7 hours)**

History of Mathematics/ Indian Mathematics, Exploring web, Exploring web resources: MAA, AMS, SIAM, arXiv, ResearchGate; Journal metrics: Impact factor of journal as per JCR, MCQ, SNIP, SJR, Google Scholar metric; Reviews/Databases: MathSciNet, zbMath, Web of Science, Scopus.

Practical (30 hours)

- Self-reading
- Seminar

on broad research area of the core discipline.

Essential Readings

[1] G. G. Joseph, *Indian Mathematics: Engaging with the World from Ancient to Modern Times*, World Scientific Publishing Europe Ltd., 2016.
[2] Committee on Publication Ethics- COPE (https://publicationethics.org/)
[3] University Grants Commission (Promotion of Academic Integrity and Prevention of Plagiarism in Higher Educational Institutions) Regulations 2018 (The Gazette of India: Extraordinary, Part-iii-Sec.4)

TOOLS FOR RESEARCH

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Tools for Research	2	0	0	2	Same as for entry to M.Sc. Mathematics	NIL

Learning Objectives

The primary objective of this course is to introduce:

- some advanced tools of LaTeX.
- research article/dissertation writing.
- beamer as tool for mathematical talk.
- the art of making mathematical poster.
- modern mathematical software to enhance research skills.

Learning Outcomes

This course will enable the students to:

- write research article/dissertation.
- prepare a mathematical talk/poster.
- use software to perform research activities.

Syllabus

Unit – 1 **(30 hours)**

Practical: Preparing a research article/dissertation, Preparing a mathematical talk and poster using beamer.

Unit – 2 **(30 hours)**

Practical: Learning and using Mathematical Software like MATLAB, Mathematica and Scilab.

Essential Readings

[1] M. Goossens, F. Mittelbach, S. Rahtz, D. Roegel and H. Voss, *The LaTeX Graphics Companion*, Addison-Wesley, 2008.
[2] N. J. Higham, *Handbook of Writing for the Mathematical Sciences*, SIAM, 1998
[3] L. Lamport, *LaTeX, a Document Preparation System*, Pearson, 2008.
[4] P. Wellin, S. Kemin and R. Gaylord, *An Introduction to Programming with Mathematica*, Third Edition, Cambridge University Press, UK, 2005.

Syllabi of Courses in Semester-IV of M.Sc. Mathematics under Structure-3 (Research)

Discipline Specific Elective (DSE) Courses

DISCIPLINE SPECIFIC ELECTIVE: ADVANCED FLUID DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Advanced Fluid Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Calculus and Partial Differential Equations

Learning Objectives

The primary objective of this course is to:

- prepare a foundation for advanced studies in compressible flow, boundary layer theory and magnetohydrodynamics.
- develop concepts, models, and techniques that enable problem-solving in compressible flow, boundary layer theory and magnetohydrodynamics.
- equip students with concepts and techniques to conduct research in the above mentioned domains.

Learning Outcomes

This course will enable the students to:

- learn conservation laws, first and second laws of thermodynamics, internal energy and entropy, different forms of energy equations and dimensional analysis.
- know about compressibility in real fluids, wave motion, sound waves, hyperbolic and dispersive waves, shock waves, their formation, properties and elementary analysis.
- know the concepts of boundary layer, boundary layer equations and their solutions, measurements of boundary layer thickness.
- understand the interaction between hydrodynamic processes and electromagnetic phenomena.
- formulate the basic equations of motion in inviscid and viscous conducting fluid flow and explain Alfven's theorem and magnetohydrodynamic (MHD) waves and MHD shocks.

Syllabus

Unit – 1 **(11 hours)**

Flow characteristics, Conservation laws, Equation of state of a substance, First and second law of thermodynamics, Internal energy and entropy, Energy equation, Nondimensionalizing the basic equations of incompressible viscous fluid flow, Nondimensional numbers.

Unit – 2 **(12 hours)**

Compressibility effects in real fluids, Equations of motion, Sound wave, Hyperbolic and dispersive waves, Isentropic gas flow, Flow through a nozzle, Method of characteristics, Shock jump conditions, Non-linear plane waves, Shock waves and their elementary analysis, Similarity solutions.

Unit – 3 **(11 hours)**

Boundary layer concept, Estimation of boundary layer thickness and friction forces, Prandtl's boundary layer equations, Boundary layer along a flat plate, Boundary layer thickness, General properties of the boundary layer equations, Similar solutions, Momentum and energy integral equations for the boundary layer.

Unit – 4 **(11 hours)**

Maxwell's electromagnetic field equations, Magnetohydrodynamic (MHD) approximations, Magnetic field equation, Magnetic Reynolds number, Magnetic body force, Equations of Motions of conducting fluid, Alfven's theorem, MHD waves, MHD shock waves.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] F. Chorlton, *Text Book of Fluid Dynamics*, CBS Publisher, 2005.
[2] H. Schlichting and K. Gersten, *Boundary Layer Theory*, Ninth Edition, Springer, 2017.
[3] G. B. Witham, *Linear and Nonlinear Waves*, John Wiley & Sons, 1999.

Suggested Readings

(i) K. R. Cramer and S. I. Pai, *Magnetofluid Dynamics for Engineers and Applied Physics*, McGraw Hill Book Co., New York, 1973.
(ii) Y. Shao-Wen, *Foundations of Fluid Mechanics*, PHI, New Delhi, 1960.

DISCIPLINE SPECIFIC ELECTIVE: BANACH AND C*-ALGEBRAS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Banach and C*-Algebras	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis and Topology

Learning Objectives

The primary objective of this course is to introduce:

- Banach algebras and C*-algebras.
- various ways to construct new operator algebras using given ones.
- spectrum of elements in Banach algebras and to study its properties.
- Gelfand representations of commutative Banach algebras and of C*-algebras.

Learning Outcomes

This course will enable the students to:

- familiarize with the representations of operator algebras.
- realize commutative Banach algebras and abelian C*-algebras as space of continuous functions on locally compact groups.
- understand the powerful tool of functional calculus.
- identify any C*-algebra as closed *-subalgebra of space of bounded linear operators on a Hilbert space.

Syllabus

Unit – 1 **(11 hours)**

Elementary properties and examples of Banach algebras, Ideals and quotients, Invertible elements, Spectrum and spectral radius, Spectral radius formula, Spectral mapping theorem (for polynomials), Gelfand–Mazur theorem.

Unit – 2 **(11 hours)**

Multiplicative linear functionals, Commutative Banach algebra, w*-topology, Gelfand transform of an element, Structure space, Gelfand representation.

Unit – 3 **(12 hours)**

Elementary properties and examples of C*-algebras, Unitization, Gelfand–Naimark representation of commutative C*-algebras, Continuous functional calculus, Spectral mapping theorem for normal elements, Positive elements of C*-algebras.

Unit – 4 **(11 hours)**

Ideals in C*-algebras, Approximate units, Quotients, Positive linear functionals, Gelfand–Naimark–Segal representation of C*-algebras.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.
[2] G. J. Murphy, *C*-algebras and Operator Theory*, Academic Press Inc., 1990.

Suggested Readings

(i) J. B. Conway, *A Course in Operator Theory, Graduate Texts in Mathematics*, Springer, 2007.
(ii) J. Dixmier, *C*-algebras*, North-Holland Publishing Company, 1977.
(iii) R. G. Douglas, *Banach Algebra Techniques in Operator Theory, Graduate Texts in Mathematics*, Springer, 1998.
(iv) E. Kaniuth, *A Course on Commutative Banach Algebras*, Graduate Texts in Mathematics, Springer, 2009.
(v) S. Kantorovitz, *Introduction to Modern Analysis*, Oxford Graduate Texts in Mathematics, Oxford University Press, 2006.
(vi) M. Takesaki, *Theory of Operator Algebras I*, Springer, 2002.

DISCIPLINE SPECIFIC ELECTIVE: CHAOS THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Chaos Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- introduce some useful and interesting notions like Topological Transitivity and Sensitive dependence on initial conditions.
- study different types of chaos including Devaney's chaos and finding their interrelationships.
- know classical result that period three implies chaos on intervals.
- relate chaos and decomposition theorems.
- study Topological entropy through open covers and also Bowen's definition of entropy, equivalence of these two definitions on compact metric spaces.
- study various interesting results related to topological entropy.

Learning Outcomes

This course will enable the students to:

- construct interesting examples of Topological transitive maps, Topological mixing maps etc.
- know classical examples of Devaney's chaotic maps like tent map, shift maps, logistic maps.
- study and compare different types of chaos.
- find relation between transitivity and chaos on intervals.
- relate chaos theory and classical decomposition theorems.
- study very useful notion of Topological entropy including its properties.
- calculate entropy of any homeomorphism of closed unit interval and of unit circle.

Syllabus

Unit – 1 **(12 hours)**

Topological Transitivity, Locally eventually onto maps, Topological mixing, Sensitive dependence on initial conditions, Devaney's definition of chaos, Transitivity and limit sets for continuous interval maps.

Unit – 2 **(11 hours)**

Characterizing topological mixing in terms of topological transitivity for continuous interval maps, Topological Weakly Mixing, Totally Transitive maps, Relation between transitivity and chaos on intervals, Logistic maps and shift maps as chaotic maps.

Unit – 3 **(12 hours)**

Various other definitions of chaos and their interrelationships. Period three implies chaos, Chaos and decomposition theorems including Bowen's decomposition theorem, Topological Entropy:

Definition using open covers, Examples and properties, Bowen's definition of topological entropy, Equivalence of two definitions, Topological version of Kolmogorov–Sinai theorem.

Unit – 4 **(10 hours)**
Topological Entropy of maps on a compact metric space, Topological Entropy of product maps, of iterations of a map, Topological entropy of an expansive homeomorphism on a compact metric space, of the two-sided shift, of any homeomorphism of the unit interval and of the unit circle.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] N. Aoki and K. Hiraide, *Topological Theory of Dynamical Systems: Recent Advances*, North Holland Publications, 1994.
[2] R. L. Devaney, *A First Course in Chaotic Dynamical Systems*, CRC Press, 2018.
[3] S. Ruette, *Chaos for Continuous Interval Maps: A Survey of Relationship Between Various Kinds of Chaos*, 2018.
[4] Peter Walters, *An Introduction to Ergodic Theory*, Springer, 2000.

Suggested Readings
(i) L. Alsedà, J. Llibre and M. Misiurewicz, *Combinatorial Dynamics and Entropy in Dimension One*, Advanced Series in Nonlinear Dynamics, World Scientific, 2000.
(ii) L. S. Block and W. A. Coppel, *Dynamics in One Dimension*, Springer, 2014.
(iii) M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge University Press, 2015.

DISCIPLINE SPECIFIC ELECTIVE: CHARACTER THEORY OF FINITE GROUPS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Character Theory of Finite Groups	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra and Group Theory

Learning Objectives

The primary objective of this course is to:

- introduce characters of finite groups, class functions, and find the number of irreducible characters.
- find character tables of S_5, using characters of tensor products of CG-modules and powers of characters.
- define restricted characters and prove Clifford's theorem and its application to find the character table of A_5.
- explore some arithmetic properties of character values and introduce real characters.

Learning Outcomes

This course will enable the students to:

- define and construct examples of characters, prove some fundamental properties of characters, and calculate character tables of some small groups and symmetric groups, etc.
- construct new characters from given characters and understand the notion of induced and restricted characters.
- prove the Frobenius reciprocity theorem and the Frobenius–Schur count of involutions.

Syllabus

Unit – 1 **(10 hours)**

Group characters and their properties, Inner product of characters, Class functions and number of irreducible characters, Character tables and some orthogonality relations, Normal subgroups and lifted characters, Linear characters, Character tables of D_6, S_4, A_4.

Unit – 2 **(11 hours)**

Character of tensor products of CG-modules, Powers of characters, Decomposition of power of a character, Character table of symmetric group S_5, Character table of direct product of groups, Restricted characters, Constituents of a restricted character, Clifford's theorem, Restriction of symmetric groups to alternate groups in general normal subgroups of index two, Character table of A_5.

Unit – 3 **(12 hours)**

Induced CG-modules and their characters: Homomorphisms, Transitivity of induction, Frobenius reciprocity theorem, Values of induced characters. Algebraic integers, Some properties of degrees of irreducible characters and arithmetic properties of character values.

Unit – 4 **(12 hours)**

Real representations, Real conjugacy classes and real characters, Characters which can be realized over the reals, RG-modules and CG-modules, G-invariant symmetric bilinear form, Indicator function, The Frobenius–Schur count of involutions.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] G. James and M. Liebeck, *Representations and Characters of Groups*, Second Edition, Cambridge University Press, 2005.

Suggested Readings

(i) I. M. Issacs, *Character Theory of Finite Groups*, American Mathematical Society reprint, 2006.
(ii) W. Ledermann, *Introduction to Group Characters*, Second Edition, Cambridge University Press, 1987.

DISCIPLINE SPECIFIC ELECTIVE: COMPUTATIONAL FLUID DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Computational Fluid Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Partial Differential Equation (Undergraduate level)

Learning Objectives

The primary objective of this course is to teach:

- various numerical schemes on finite difference and finite volume methods for solving PDEs.
- discretization errors and grid dependence.
- some real-world applications of PDEs and fluid dynamics.
- discretization of governing equations of diffusion, convection-diffusion, fluid flow and thereby computing the numerical solutions using the flow variables using algorithms.

Learning Outcomes

This course will enable the students to learn:

- techniques for solving the PDEs along with some initial and boundary conditions by using the finite difference and finite volume methods.
- the basic conservation principles of mass, momentum, energy, discretization of governing equations.
- discretization techniques.
- some popular algorithms like SIMPLE and SIMPLER used to obtain the solutions of steady and unsteady flow problems by finite volume methods.

Syllabus

Unit – 1 **(12 hours)**

Basics of discretization using finite differences, Various single and multi-step explicit and implicit finite difference schemes for 1-D and 2-D parabolic and hyperbolic initial boundary value problems, Alternating Direction Implicit schemes (ADI) for 2-D parabolic and hyperbolic equations, Order of accuracy, Consistency, Stability and convergence of a finite difference scheme, Courant Friedrich Lewy condition.

Unit – 2 **(12 hours)**

Finite difference schemes for second and fourth order 2-D elliptic boundary value problem and applications, Finite volume method for diffusion and convection-diffusion equations, Discretization of one and two-dimensional steady state diffusion and convection-diffusion equations, Central difference, Upwind, Exponential, Hybrid, Power-law and QUICK schemes and their properties.

Unit – 3 **(11 hours)**

Flow field calculation, Pressure-velocity coupling, Vorticity-stream function approach, Primitive variables, Staggered grid, Pressure and velocity corrections, Pressure correction equation, SIMPLE and SIMPLER algorithms.

Unit – 4 **(10 hours)**
Finite volume methods for unsteady flows, Discretization of one-dimensional transient heat conduction, Explicit, fully implicit and Crank–Nicolson schemes, Implementation of boundary conditions.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] J. C. Strikweda, *Finite Difference Schemes and Partial Differential Equations*, Second Edition, SIAM, 2004.
[2] H. K. Versteeg and W. Malalasekera, *An Introduction to Computational Fluid Dynamics: The Finite Volume Method*, Second Edition, Pearson, 2008.

Suggested Readings
(i) J. D. Anderson, *Computational Fluid Dynamics*, McGraw-Hill, 1995.
(ii) S. V. Patankar, *Numerical Heat Transfer and Fluid Flow*, CRC Press, Taylor and Francis, Indian Edition, 2017.
(iii) R. H. Pletcher, J. C. Tannehill and D. A. Anderson, *Computational Fluid Mechanics and Heat Transfer*, CRC Press, Taylor and Francis, 2013.
(iv) J. W. Thomas, *Numerical Partial Differential Equations: Finite Difference Methods*, Springer, 2013.

DISCIPLINE SPECIFIC ELECTIVE: DIFFERENTIAL TOPOLOGY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Differential Topology	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- introduce the concepts of topological manifolds, smooth structures, smooth manifolds, and manifolds with boundary.
- develop an understanding of smooth functions, smooth maps, diffeomorphisms, and tangent spaces.
- explain the Inverse function theorem, immersions and submersions.
- develop the fundamental concepts of 2-manifolds and distinguish between orientable and non-orientable surfaces.
- explore the properties of compact and connected surfaces.

Learning Outcomes

This course will enable the students to:

- identify and construct examples of topological manifolds, smooth structures and manifolds with and without boundary.
- demonstrate understanding of diffeomorphisms and tangent spaces.
- apply the Inverse function theorem, immersions and submersions.
- define key concepts such as 2-manifolds, orientability, compactness, connectedness and boundary of a surface.
- differentiate between orientable and non-orientable surfaces using examples such as the sphere, torus, Möbius strip and Klein bottle.

Syllabus

Unit – 1 **(12 hours)**

Topological manifolds, Topological properties of manifolds, Smooth structures, Examples of smooth manifolds, Manifolds with boundary.

Unit – 2 **(11 hours)**

Smooth functions and smooth maps, Lie groups, Diffeomorphisms.

Unit – 3 **(10 hours)**

Derivatives and tangents, Inverse function theorem, Immersions and submersions.

Unit – 4 **(12 hours)**

Complexes, Connected sum of two surfaces, Non-orientable surfaces (2- Manifolds), Compact and connected surfaces, Classification of compact and connected surfaces with and without boundary.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] V. Guillemin and Alan Pollack, *Differential Topology*, Prentice-Hall, 1974.
[2] L. C. Kinsey, *Topology of Surfaces*, Springer Verlag, 1997.
[3] J. M. Lee, *Introduction to Smooth Manifolds*, Second Edition, Springer, 2013.

Suggested Readings

(i) L. Conlon, *Differentiable Manifolds*, Second Edition, Birkhäuser Advanced Texts, 2001.
(ii) M. Spivak, *A Comprehensive Introduction to Differential Geometry*, Volume 1, Third Edition, Publish or Perish, Huston, Texas, 1999.
(iii) L. W. Tu, *Introduction to Manifolds*, Second Edition, Springer, 2011.
(iv) F. W. Warner, *Foundations of Differentiable Manifolds and Lie Group*, Springer-Verlag, 1983.

DISCIPLINE SPECIFIC ELECTIVE: GENERAL MEASURE THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: General Measure Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Measure Theory and Topology

Learning Objectives
The primary objective of this course is to introduce:
- real valued and complex valued measures.
- decomposition of measure spaces and of measures.
- extension of a premeasure to a measure, Lebesgue measure on Euclidean spaces.
- representation of measures and functionals in terms of integrals.
- product measures.

Learning Outcomes
This course will enable the students to:
- appreciate signed measures and complex measures, mutual singularity of measures, Hahn and Jordan decompositions, Lebesgue decomposition, Radon–Nikodym theorem.
- verify conditions under which a set function defined on a collection of subsets of a set has an extension to a measure on a sigma-algebra.
- apply Riesz representation theorem for bounded linear functionals on L^p-spaces.
- understand product measure and the results of Fubini and Tonelli, and express the Lebesgue measure on Euclidean spaces as a product measure.
- apply Riesz–Markov representation theorem for the bounded linear functionals on the space of continuous functions.

Syllabus

Unit – 1 **(13 hours)**
Signed measures, Hahn and Jordan decompositions, Mutually singular measures, Radon–Nikodym theorem, Lebesgue decomposition, Complex measure.

Unit – 2 **(10 hours)**
The Carathéodory extension theorem, Lebesgue measure on $\mathbb{R}^n$, Regularity and translation invariance of Lebesgue measure on $\mathbb{R}^n$.

Unit – 3 **(10 hours)**
Riesz representation theorem for the dual of L^p-spaces, Product measures, Fubini’s theorem, Tonelli’s theorem.

Unit – 4 **(12 hours)**
Locally compact Hausdorff spaces and construction of Radon measure, Riesz–Markov representation theorem for positive linear functionals on $C_c(X)$, Riesz representation theorem for the dual of $C(X)$.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] H. L. Royden and P. M. Fitzpatrick, *Real Analysis*, Fourth Edition, Pearson, 2015.
[2] M. E. Taylor, *Measure Theory and Integration*, American Mathematical Society, 2006.

Suggested Readings

(i) G. B. Folland, *Real Analysis: Modern Techniques and Their Applications*, Second Edition, Wiley, New York, 1999.
(ii) P. R. Halmos, *Measure Theory*, Springer Science + Business Media, LLC, 2014.
(iii) E. M. Stein and R. Shakarchi, *Real Analysis: Measure Theory, Integration and Hilbert Spaces*, New Age International Publishers, New Delhi, 2010.

DISCIPLINE SPECIFIC ELECTIVE: NONSMOOTH OPTIMIZATION

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Nonsmooth Optimization	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Nonlinear Optimization

Learning Objectives
The primary objective of this course is to:
- understand the tools to deal with nonsmooth convex functions.
- study conjugate duality in terms of conjugate functions for constrained nonlinear optimization problems.
- introduce numerical techniques to solve constrained nonlinear optimization problems.

Learning Outcomes
This course will enable the students to learn:
- the notions of subgradients and subdifferentials for nonsmooth convex functions.
- the use of conjugate functions to develop the theory of conjugate duality.
- about numerical methods like gradient descent method, gradient projection method, Newton's method and conjugate gradient method.
- penalty approach technique to solve constrained nonlinear optimization problems.

Syllabus

Unit – 1 **(11 hours)**
Extended real valued functions, Proper convex functions, Closure of convex functions, Differential derivatives, Subgradients and subdifferentials.

Unit – 2 **(12 hours)**
Conjugate functions, Biconjugate functions, Perturbation functions, Closure of convex functions, Directional derivatives, Subgradients and subdifferentials.

Unit – 3 **(12 hours)**
Gradient descent method, Gradient projection method, Newton's method, Conjugate gradient method.

Unit – 4 **(10 hours)**
Penalty function methods, Exterior penalty function, Interior penalty functions.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] M. Avriel, *Nonlinear Programming: Analysis and Methods*, Dover Publications, 2003.

[2] O. Güler, *Foundations of Optimization*, Springer, 2010.

Suggested Readings

(i) A. Bagirov, N. Karitsa and M. M. Makela, *Introduction to Nonsmooth Optimization: Theory, Practice and Software*, Springer, 2014.
(ii) M. S. Bazaraa, H. D. Sherali and C. M. Shetty, *Nonlinear Programming: Theory and Algorithms,* Third Edition, John Wiley & Sons, 2013.
(iii) D. G. Luenberger and Y. Ye, *Linear and Nonlinear Programming*, Springer, 2008.

DISCIPLINE SPECIFIC ELECTIVE: PROBABILITY THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Probability Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Measure Theory

Learning Objectives

The primary objective of this course is to introduce:

- probability space as a measure space and random variables as measurable functions.
- expectation and moments of random variables.
- notion of convergence in probability.
- conditioning on sub-σ-algebra.

Learning Outcomes

This course will enable the students to learn:

- about probability or uncertainty in abstract setting.
- moments and expectation of random variables which help to understand applications of probability in industry.
- how to apply the idea of convergence in probability.
- weak law and strong law of large numbers and their applications.

Syllabus

Unit – 1 **(11 hours)**

Probability: σ-algebra, Constructing probability triples, The extension theorem, Random variables, Independence of events, Continuity of probabilities, Limit events, The Borel–Cantelli Lemma.

Unit – 2 **(10 hours)**

Expected values: Simple, general non-negative and arbitrary random variables, Moment generating functions, Markov's inequality, Chebyshev's inequality.

Unit – 3 **(12 hours)**

Convergence of random variables: Convergence almost surely, Convergence in probability, Weak law of large numbers, Strong law of large numbers.

Unit – 4 **(12 hours)**

Distributions of random variables: Examples of distributions, Characteristic functions, The central limit theorem, Conditional probability, Conditioning on random variable, Conditioning on a sub-σ-algebra, Conditional variance.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. S. Rosenthal, *A First Look at Rigorous Probability Theory*, Second Edition, World Scientific, Singapore, 2006.

Suggested Readings

(i) W. Feller, *An Introduction to Probability Theory and its Applications*, Volume 1, Third Edition, Wiley, 2008.
(ii) J. E. Michael and J. S. Rosenthal, *Probability and Statistics: The Science of Uncertainty*, Second Edition, W. H. Freeman & Co Ltd., 2009.
(iii) S. Ross, *A First Course in Probability*, Tenth Edition, Pearson Education, 2022.
(iv) D. W. Stroock, *Probability Theory, An Analytic View*, Cambridge University Press, 2024.

DISCIPLINE SPECIFIC ELECTIVE: SIMPLICIAL HOMOLOGY THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Simplicial Homology Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- introduce the foundations of simplicial complexes and homology theory.
- develop an understanding of chain mappings and induced homomorphisms.
- apply these concepts to establish fundamental results in topology such as Euler–Poincaré theorem, Brouwer's and Lefschetz fixed-point theorems.

Learning Outcomes

This course will enable the students to:

- identify hyperplanes, simplexes and finite simplicial complexes as subsets of a Euclidean space.
- learn the idea of compact triangulable spaces as geometric carriers of finite simplicial complexes (polyhedra).
- learn the use of homological algebra to associate simplicial homology groups and illustrate it by computing simplicial homology groups of some well-known compact polyhedral.
- prove important applications of simplicial homology theory like invariance of dimension, Euler's formula, Lefschetz and Brouwer's fixed point theorems.

Syllabus

Unit – 1 **(11 hours)**

Geometric simplexes, Geometric complexes and polyhedra, Simplicial maps, Simplicial approximation of continuous maps between two polyhedral.

Unit – 2 **(12 hours)**

Orientation of geometric complexes, Chain complexes, Simplicial homology groups, Structure of homology groups, Relative homology groups, Computation of homology groups, Homology groups of n-sphere.

Unit – 3 **(12 hours)**

Chain mappings, Chain derivation, Chain homotopy, Contiguous maps, Homomorphism induced by continuous maps between two polyhedra, Functorial property of induced homomorphisms, Topological and homotopy invariance of homology groups.

Unit – 4 **(10 hours)**

Euler–Poincaré theorem and Euler's formula, Invariance of dimension, Brouwer's fixed point theorem, Degree of self-mappings of $\mathbb{S}^n$, Brouwer's degree theorem, Existence of eigen values, Lefschetz fixed point theorem.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] F. H. Croom, *Basic Concepts of Algebraic Topology*, Springer, 1978.

Suggested Readings

(i) M. K. Agoston, *Algebraic Topology: A First Course*, Marcel Dekker, 1976.
(ii) M. A. Armstrong, *Basic Topology*, Springer, 1983.
(iii) S. Deo, *Algebraic Topology - A Primer*, Second Edition, Hindustan Book Agency, 2018.

DISCIPLINE SPECIFIC ELECTIVE: THEORY OF NON-COMMUTATIVE RINGS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Theory of Non-commutative Rings	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Groups and Rings

Learning Objectives

The primary objective of this course is to:

- give students an understanding of Wedderburn–Artin theory of semisimple rings.
- develop Jacobson's general theory of radicals, prime and semiprime rings, and primitive and semiprimitive rings.
- introduce the structure of primitive rings as a generalisation of the Wedderburn–Artin theorem on Artinian simple rings.

Learning Outcomes

This course will enable the students to:

- know about an extensive variety of rings, including free rings, Weyl algebra, Hilbert twist and triangular ring.
- understand the module theoretic definition of semisimple rings and how it leads to the Wedderburn–Artin structure theorem on their complete classification.
- know Jacobson's general theory of radicals, semiprime rings, prime, primitive and semiprimitive rings and their structures.
- understand the significance of the fundamental result 'Density Theorem' and its consequences on the structure of primitive rings.

Syllabus

Unit – 1 **(11 hours)**

Simple rings, Reduced rings, Dedekind-finite rings, Algebra, Quaternions, Free k-rings, Rings with generators and relations, Weyl algebra, Formal power series ring, Hilbert's twist ring, Differential polynomial rings, Derivation and inner derivation on a ring, Triangular rings, Characterization of one-sided and two-sided ideals in such rings.

Unit – 2 **(11 hours)**

Noetherian and Artinian rings, Examples of one-sided Noetherian and Artinian triangular rings, Twisted polynomial ring and Quotient of free $\mathbb{Z}$-ring, Semisimple rings, Structure of semisimple rings: Wedderburn–Artin's theorem.

Unit – 3 **(10 hours)**

Structure theorem of simple left Artinian rings, Jacobson radical, J-semisimple rings, Nil and nilpotent ideals, Connection between semisimple and J-semisimple rings, Hopkins–Levitzki theorem, Nakayama's lemma.

Unit – 4 **(13 hours)**

Prime radical, Characterisation of prime and semiprime ideals, Prime and semiprime rings, Structure theorem of primitive rings, Density theorem.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] T.-Y. Lam, *A First Course in Noncommutative Rings*, Springer, 2001.

Suggested Readings

(i) I. N. Herstein, *Noncommutative Rings*, The Mathematical Association of America, 2005.
(ii) T. W. Hungerford, *Algebra*, Springer-Verlag, New York, 1981.
(iii) L. H. Rowen, *Ring Theory*, Student Edition, Academic Press, 1991.

DISCIPLINE SPECIFIC ELECTIVE: THEORY OF UNBOUNDED OPERATORS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Theory of Unbounded Operators	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis and Bounded Operators

Learning Objectives

The primary objective of this course is to:

- introduce the notion of unbounded operators.
- develop the theory of operator semigroups and understand their role in applications, particularly for solving differential equations.

Learning Outcomes

This course will enable the students to:

- identify closed and closable linear operators on Banach spaces.
- compute adjoints of unbounded linear operators.
- understand spectral properties of some unbounded operators.
- comprehend the role unbounded operators and semigroups play in applications, particularly in studying solutions of differential equations.

Syllabus

Unit – 1 **(10 hours)**

Unbounded linear operators, Hilbert adjoints, Hellinger–Toeplitz theorem, Hermitian, symmetric and self-adjoint linear operators, Closed linear operators, Closable operators and their closures on Banach spaces.

Unit – 2 **(12 hours)**

Cayley transform, Deficiency indices, Spectral properties of self-adjoint operators, Multiplication and differentiation operators and their spectra.

Unit – 3 **(11 hours)**

Analytic properties of exponential functions, Matrix Semigroups, Uniformly continuous semigroups, Semigroups on Hilbert spaces, Strongly continuous semigroups.

Unit – 4 **(12 hours)**

Generators of semigroups and their resolvents, Hille–Yosida theorem (for contraction semigroup), Dissipative operators and their properties, Lumer–Phillips theorem, Generators of Group, Stone's theorem.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] K. J. Engle and R. Nagel, *One-parameter Semigroups for Linear Evolution Equations*, Springer-Verlag, New York, 2000.
[2] E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, 2006.

Suggested Readings

(i) S. Goldberg, *Unbounded Linear Operators: Theory and Applications*, Dover Publications, 2006.
(ii) E. Hille and R. S. Phillips, *Functional Analysis and Semi-groups*. American Mathematical Society, Providence, RI, 1957.
(iii) A. Pazy, *Semigroups of Linear Operators and Applications to Partial Differential Equations*, Springer, 1983.
(iv) M. Schechter, *Principles of Functional Analysis*, Second Edition, American Mathematical Society, 2001.
(v) J. Weidmann, *Linear Operators in Hilbert Spaces, Graduate Texts in Mathematics*, Springer, New York, 1980.

Research Methods/ Tools/ Writing Courses

TECHNIQUES OF RESEARCH WRITING

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Techniques of Research Writing	**2**	**0**	**0**	**2**	**Same as for entry to M.Sc. Mathematics**	**NIL**

Learning Objectives
The primary objective of this course is to:
- equip the students with the techniques for writing mathematics properly.
- provide knowledge about differentiating between good and bad mathematics writing.
- prepare the students with basic skills required for writing a research article in mathematics.

Learning Outcomes
This course will enable the students to:
- understand the difference between good and bad mathematical writing.
- know the techniques of writing a good research article in mathematics.

Syllabus

Unit – 1 **(30 hours)**
Practical: Learning basic skills of writing mathematics.

Unit – 2 **(30 hours)**
Practical: Writing a report on any topic of core discipline.

Essential Readings
[1] N. J. Higham, *Handbook for writing for the Mathematical Sciences*, SIAM, 1998.
[2] N. E. Steenrod, P. R. Halmos, M. M. Schiffer and J. A. Dieudonné, *How to Write Mathematics*, American Mathematical Society, 1973.

Syllabi of Courses in Semester-I of One-year M.Sc. Mathematics under Structure-1 (Only Course work)

Discipline Specific Core (DSC) Courses

DISCIPLINE SPECIFIC CORE – 1: FLUID DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSC-1: Fluid Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Calculus and Partial Differential Equations

Learning Objectives

The objective of this course is to:

- prepare a mathematical foundation to study the motion of fluids.
- develop concepts, models, and techniques to solve problems of fluid flow.
- develop the ability to conduct advanced studies and research in the broad field of fluid dynamics.

Learning Outcomes

After studying this course, the student will be able to:

- understand the concept of fluids, their classification, flow lines, models and approaches to study fluid flow.
- formulate mass and momentum conservation principles and obtain their solution for non-viscous flow.
- know potential flow, Bernoulli's equation, Kelvin's minimum energy and circulation theorems.
- understand two- and three-dimensional motion, complex potential, circle theorem, Blasius theorem, Weiss's and Butler's sphere theorems.
- apply the concept of stress and strain in viscous flow to derive Navier–Stokes equation of motion and energy equation.

Syllabus

Unit – 1 **(10 hours)**

Classification of fluids, Continuum model, Eulerian and Lagrangian approach of description, Differentiation following the fluid motion, Flow lines, vorticity and circulation, Conservation of mass: Equation of continuity, Boundary surface.

Unit – 2 **(12 hours)**

Forces in fluid motion, Conservation of momentum: Euler's equation of motion, Theory of irrotational motion: Integration of Euler's equation under different conditions, Bernoulli's equation, Impulsive motion, Kelvin's minimum energy and circulation theorems, Potential theorem.

Unit – 3 **(13 hours)**

Two-dimensional motion: Complex potential, Line sources, sinks, doublets and vortices, Two-dimensional image system, Milne–Thomson circle theorem, Images with respect to a plane and cylinder, Blasius theorem. Three-dimensional flows, Weiss's sphere theorem, Images with respect to sphere, Axi-symmetric flow, Stokes stream function, Butler's sphere theorem, Flow past spheres and cylinders.

Unit – 4 **(10 hours)**

Stress and strain analysis, Newton's law of viscosity, Laminar flow, Navier–Stokes equation of motion, Steady flow between parallel planes and Poiseuille flow, Constitutive equation, Energy equation.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] F. Chorlton, *Text Book of Fluid Dynamics*, CBS Publisher, 2005.
[2] R. W. Fox, P. J. Pritchard and A. T. McDonald, *Introduction to Fluid Mechanics*, Seventh Edition, John Wiley & Sons, 2009.
[3] P. K. Kundu, I. M. Cohen and D. R. Dowling, *Fluid Mechanics*, Sixth Edition, Academic Press, 2016.

Suggested Readings

(i) L. M. Milne-Thomson, *Theoretical Hydrodynamics*, The Macmillan company, USA, 1969.
(ii) D. E. Rutherford, *Fluid Dynamics*, Oliver and Boyd Ltd., 1978.

DISCIPLINE SPECIFIC CORE – 2: MEASURE AND INTEGRATION

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSC-2: Measure and Integration	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Real Analysis and Riemann Integration

Learning Objectives

The primary objective of this course is to:

- extend the notion of length of an interval with the introduction of the concept of Lebesgue outer measure for any subset of real line.
- investigate the properties of Lebesgue measurable sets and functions.
- familiarize students with the Lebesgue integration of functions and its comparison with Riemann integration.
- generalize the concepts of measure and integration to an abstract space.

Learning Outcomes

This course will enable the students to:

- verify whether a given subset of $\mathbb{R}$ or a real valued function is measurable.
- understand the requirement and the concept of the Lebesgue integral (a generalization of the Riemann integration) along with its properties.
- understand the statements and proofs of the fundamental integral convergence theorems and demonstrate their applications.
- carry out a comprehensive study of functions of bounded variation and their utility in understanding differentiation and integration.
- apply Hölder and Minkowski inequalities in L^p-spaces and understand completeness of L^p-spaces.

Syllabus

Unit – 1 **(14 hours)**

Lebesgue outer measure, Measurable sets, Lebesgue measure, Borel sets, Regularity, Measurable functions, Borel and Lebesgue measurability, Non-measurable sets.

Unit – 2 **(13 hours)**

Integration of nonnegative functions, General integral, Integration of series, Riemann and Lebesgue integrals.

Unit – 3 **(8 hours)**

Functions of bounded variation, Lebesgue's differentiation theorem, Differentiation and integration, Absolute continuity of functions.

Unit – 4 **(10 hours)**
Measures and outer measures, Measure spaces, Integration with respect to a measure, L^p-spaces, Hölder's and Minkowski's inequalities, Completeness of L^p-spaces.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] G. de Barra, *Measure Theory and Integration*, Ellis Horwood Ltd., Chichester, John Wiley & Sons, Inc., New York, 1981 (Indian Reprint, 2014).

Suggested Readings
(i) M. Capinski and P. E. Kopp, *Measure, Integral and Probability*, Springer, 2005.
(ii) E. Hewitt and K. Stromberg, *Real and Abstract Analysis: A Modern Treatment of the Theory of Functions of a Real Variable*, Springer, Berlin, 1975.
(iii) H. L. Royden and P.M. Fitzpatrick, *Real Analysis*, Fourth Edition, Pearson, 2015.

Discipline Specific Elective (DSE) Courses

DISCIPLINE SPECIFIC ELECTIVE – 1 (i): COMMUTATIVE ALGEBRA

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-1 (i): Commutative Algebra	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra and Field Theory

Learning Objectives
The objective of this course is to:

- develop a solid understanding of the structure of commutative rings, ideals, their radicals, extension, contraction etc.
- study important constructions such as total quotient rings, localizations.
- develop basic foundation in other areas of mathematics such as algebraic geometry, algebraic number theory.

Learning Outcomes
This course will enable the students to:

- know the localization of rings at a prime ideal that is an algebraic analogue of the geometric notion concentrating attention near a point.
- know more closely the polynomial rings, power series rings in one or more variables over a commutative ring and their prime spectrum.
- define, identify, and elaborate integral closure of rings, valuations rings, discrete valuation rings, structure theorem of Artin rings.

Syllabus

Unit – 1 **(12 hours)**
Radical of an ideal, Prime avoidance lemma, Chinese remainder theorem, Extension and contraction of ideals, Jacobson radical of a ring, Nakayama lemma, Tensor product of modules.

Unit – 2 **(13 hours)**
Rings and modules of fractions, Localization, Local properties, Primary decomposition, First and second uniqueness theorem of primary decomposition, Associated prime ideals of decomposable ideals.

Unit – 3 **(10 hours)**
Integral ring extensions, Going up theorem, Going down theorem, Integrally closed domains, Valuation rings, Hilbert's Nullstellensatz theorem.

Unit – 4 **(10 hours)**
Noetherian rings, Primary decomposition in Noetherian rings, Artin rings, Structure theorem for Artin rings, Discrete valuation rings.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. F. Atiyah and I. G. MacDonald, *Introduction to Commutative Algebra*, CRC Press, Taylor & Francis, 2018.

Suggested Readings

(i) D. Eisenbud, *Commutative Algebra with a View Towards Algebraic Geometry*, Springer, 2004.
(ii) R. Y. Sharp, *Steps in Commutative Algebra*, Cambridge University Press, 2000.
(iii) B. Singh, *Basic Commutative Algebra*, World Scientific, 2011.
(iv) O. Zariski and P. Samuel, *Commutative Algebra*, Volume I & II, Springer, 1975.

DISCIPLINE SPECIFIC ELECTIVE – 1 (ii): DYNAMICAL SYSTEMS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-1 (ii): Dynamical Systems	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology and Ordinary Differential Equations

Learning Objectives
The primary objective of this course is to:
- understand discrete and continuous systems with case studies to study nonlinear systems of ordinary differential equations and dynamical systems.
- understand the concepts, models and techniques to realize the real-world problems and stability of the systems along with the chaotic dynamic behaviour of models by understanding bifurcations.

Learning Outcomes
This course will enable the students to learn:
- formulation of mathematical models with the stability analysis near the equilibrium points.
- how the concept of phase portraits helps to analyse mathematical model graphically.
- the qualitative behaviour of the solution set of a given system of differential equations including the invariant sets and limiting behaviour of the dynamical system or flow defined by the system of differential equations.
- how different bifurcations explain the chaotic behaviour of the system.

Syllabus

Unit – 1 **(13 hours)**
Linear systems: Jordan forms, Stability theory; Nonlinear systems: Fundamental existence-uniqueness theorem, Dependence on initial conditions and parameters, Flow of a differential equation, Linearization, Stable manifold theorem, Hartman–Grobman theorem.

Unit – 2 **(10 hours)**
Stability and Lyapunov functions, Saddle points, Nodes, Foci, Centers and nonhyperbolic critical points, Center manifold theorem.

Unit – 3 **(12 hours)**
Limit sets and attractors, Periodic orbits and limit cycles, Poincaré map, Stable manifold theorem for periodic orbits, Poincare-Bendixson theorem.

Unit – 4 **(10 hours)**
Bifurcations at nonhyperbolic equilibrium points, Saddle node, Transcritical and Pitchfork bifurcations, Hopf bifurcation.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. W. Hirsch, S. Smale and R. L. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, Third Edition, Academic Press, 2013.
[2] L. Perko, *Differential Equations and Dynamical Systems*, Third Edition, Springer Verlag, 2001.

Suggested Readings

(i) R. L. Devaney, *A First Course in Chaotic Dynamical Systems: Theory and Experiment*, CRC Press, Taylor & Francis, 2018.
(ii) S. H. Strogatz, *Nonlinear Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 2018.
(iii) S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, TAM Volume 2, Springer-Verlag, NY, 1990.

DISCIPLINE SPECIFIC ELECTIVE – 1 (iii): INTRODUCTION TO TOPOLOGY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-1 (iii): Introduction to Topology	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Metric Spaces

Learning Objectives

The primary objective of this course is to introduce:

- basic principles of point-set topology, including bases and subbases for a topology.
- continuity, homeomorphisms, and different types of topologies, such as product and box topologies.
- key notions of connectedness and local connectedness.
- compactness and its significance in topological spaces.

Learning Outcomes

This course will enable the students to:

- analyze subsets of topological spaces by determining their interior, closure, boundary, and limit points, as well as identifying bases and subbases.
- identify continuous functions between topological spaces, analyze mappings into product spaces, and compare topological properties of different spaces.
- evaluate the connectedness and path connectedness of the product of an arbitrary family of spaces.
- understand key classifications of topological spaces, including Hausdorff spaces, first and second countable spaces, and separable spaces.
- explore advanced concepts such as limit point compactness and Tychonoff's theorem.

Syllabus

Unit – 1 **(10 hours)**

Topological spaces, Basis, Order topology, Subspace topology, Metric topology, Closed set and limit points, Hausdorff spaces.

Unit – 2 **(12 hours)**

Continuous functions, Homeomorphism, The box and product topologies, Metrizability of products of metric spaces, Connected and path connected spaces.

Unit – 3 **(12 hours)**

Locally connected and locally path connected spaces, Connectedness of product of spaces, First and second countable spaces, Separable spaces.

Unit – 4 **(11 hours)**

Compact spaces, The Tychonoff theorem, Limit point compactness, Sequential compactness.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. R. Munkres, *Topology*, Updated Second Edition, Pearson, 2021.
[2] T. B. Singh, *Introduction to Topology*, Springer Nature, 2019.

Suggested Readings

(i) G. E. Bredon, *Topology and Geometry*, Springer, 2014.
(ii) J. Dugundji, *Topology*, Allyn and Bacon Inc., Boston, 1978.
(iii) J. L. Kelley, *General Topology*, Dover Publications, 2017.
(iv) G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill Education, 2017.
(v) L. A. Steen and J. A. Seebach, *Counterexamples in Topology*, Dover Publications, 2013.
(vi) S. Willard, *General Topology*, Dover Publications, 2004.

DISCIPLINE SPECIFIC ELECTIVE – 1 (iv): THEORY OF BOUNDED OPERATORS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-1 (iv): Theory of Bounded Operators	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis

Learning Objectives

The primary objective of this course is to:

- introduce some classes of bounded linear operators which play a central role in both pure and applied mathematics.
- study the properties and spectral theory of these operators.

Learning Outcomes

This course will enable the students to understand:

- the spectrum and sub-divisions of spectrum of standard operators like shifts and multiplication.
- the spectral theorem for some classes of bounded linear operators.
- the concepts of compactness, self-adjointness and positivity of bounded linear operators.
- trace class and Hilbert–Schmidt operators.

Syllabus

Unit – 1 **(11 hours)**

Properties of spectrum and resolvent of bounded operators, Subdivision of the spectrum including point, approximate and compression spectrum.

Unit – 2 **(10 hours)**

Operators on Hilbert spaces, Adjoint operator, Projections and idempotents, Operations with projections, Invariant and reducing subspaces.

Unit – 3 **(14 hours)**

Compact operators on Hilbert spaces, Diagonalisation of compact self-adjoint operators, Spectral theorem and functional calculus for Compact normal operators, Positive operators, Compact operators on Banach spaces, Spectral theory of compact operators.

Unit – 4 **(10 hours)**

Polar decomposition, Singular values, Trace class operators, Trace norm and Hilbert Schmidt operators.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] R. Bhatia, *Notes on Functional Analysis*, Hindustan Book Agency, 2009.
[2] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.

Suggested Readings

(i) E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, 2006.
(ii) B. Simon, *Operator Theory: A Comprehensive Course in Analysis*, Part 4, American Mathematical Society, 2015.
(iii) S. R. Garcia, J. Mashregi and W. T. Ross, *Operator Theory by Example*, Oxford University Press, 2023.

DISCIPLINE SPECIFIC ELECTIVE – 2 (i): NUMERICAL METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-2 (i): Numerical Methods for Ordinary Differential Equations	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Ordinary Differential Equations

Learning Objectives
The primary objective of this course is to:
- develop the basic theory underlying the numerical solution of differential equations.
- introduce the concepts of consistency, stability and convergence of finite difference methods.
- execute the numerical schemes for the solution of differential equations.

Learning Outcomes
This course will enable the students to:
- gain a thorough understanding of the fundamental concepts involved in the construction and analysis of finite difference schemes for solving ordinary differential equations (ODEs).
- apply various numerical methods based on finite difference approaches to obtain approximate solutions for both initial value problems (IVPs) and boundary value problems (BVPs).
- develop the ability to select appropriate finite difference methods for specific types of problems and effectively apply them to real world applications.

Syllabus

Unit – 1 **(11 hours)**
Initial value problems: Existence and uniqueness of solution, Finite difference equation, Truncation error, Single step methods for first order IVPs and system of IVPs- Family of explicit and implicit Runge–Kutta methods, Taylor series methods, Derivation, Truncation error, Consistency, Stability and convergence analysis.

Unit – 2 **(12 hours)**
IVPs for the system of ODEs, Consistency, Zero stability and convergence of linear multistep methods, Routh–Hurwitz criterion, Order and error constant, First Dahlquist Barrier, Local truncation error and global truncation error, Error bounds, Local error, Linear stability theory, Higher order differential equations.

Unit – 3 **(12 hours)**
Derivation of explicit and implicit multistep methods for IVPs and system of IVPs, Truncation error, Stability and convergence analysis of family of Nystrom method, Adams–Bashforth method,

Adams–Moulton method, Milne–Simpson method, Predictor corrector method, and Modified predictor corrector method, Hybrid method, Multistep methods for second order IVPs.

Unit – 4 **(10 hours)**

Linear BVPs for second order ordinary differential equations, Shooting method, Finite difference method, Collocation method, Derivative boundary conditions, Nonlinear two-point BVPs, Higher order finite difference methods, Stability, Truncation error and convergence analysis.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. K. Jain, S. R. K. Iyenger and R. K. Jain, *Numerical Methods for Scientific and Engineering Computations*, Seventh Edition, New Age International Publisher, 2019.

[2] J. D. Lambert, *Numerical Methods for Ordinary Differential Systems: The Initial Value Problem*, John Wiley & Sons, 1991.

Suggested Readings

(i) K. E. Atkinson, W. Han and D. E. Stewart, *Numerical Solution of Ordinary Differential Equations*, John Wiley & Sons, 2009.

(ii) J. C. Butcher, *The Numerical Analysis of Ordinary Differential Equations*, Second Edition, Wiley, New York, 2008.

(iii) L. Collatz, *The Numerical Treatment of Differential Equations*, Springer-Verlag, 1966.

DISCIPLINE SPECIFIC ELECTIVE – 2 (ii): ORDINARY DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-2 (ii): Ordinary Differential Equations	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Differential Equations and Calculus

Learning Objectives

The objective of this course is to study:

- existence, uniqueness, and continuity of solutions of initial value problems (IVPs)
- homogeneous and non-homogeneous linear systems
- stability of solutions for systems of ordinary differential equations.
- eigenvalues and eigenfunctions of Sturm-Liouville systems and Green's functions
- applications of theory of ordinary differential equations in real world problems.

Learning Outcomes

After studying this course, the student will be able to:

- know about the existence, uniqueness, and continuity of solutions of IVPs.
- apply the matrix method of solution for linear systems of differential equations.
- analyze the stability of solutions for systems of ordinary differential equations.
- understand Green's functions and their applications in the solution of boundary value problems (BVPs).
- comprehend the properties of eigenvalues and eigenfunctions of Sturm-Liouville systems.

Syllabus

Unit – 1 **(12 hours)**

Well-posed problems, Existence, uniqueness, and continuity theorems for the solution of IVPs of the first order, Picard's method, Existence and uniqueness of solution for systems and higher order IVPs , Global existence theorem.

Unit – 2 **(9 hours)**

Homogeneous and non-homogeneous linear systems, Linear systems with constant coefficients and their solution by matrix method, Linear equations with periodic coefficients.

Unit – 3 **(12 hours)**

Stability of autonomous system of differential equations, Critical points of an autonomous system and their classification. Stability of linear systems with constant coefficients, Linear plane autonomous system and phase portrait analysis, Perturbed systems, Method of Lyapunov for nonlinear systems, Limit cycles, Poincare-Bendixson's theorem and its applications.

Unit – 4 **(12 hours)**
Sturm separation and comparison theorems, Adjoint forms and Lagrange's identity, Two-point boundary value problems, Green's functions, Construction of Green's functions, Sturm-Liouville systems, eigenvalues and eigenfunctions.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] E. A. Coddington, *An Introduction to Ordinary Differential Equations*, Dover Publications, 2012.
[2] T. Myint-U, *Ordinary Differential Equations*, Elsevier, North-Holland, 1978.
[3] S. L. Ross, *Differential Equations*, Second Edition, John Wiley & Sons, India, 2007.

Suggested Readings
(i) L. Perko, *Differential Equations and Dynamical Systems,* Springer, 2001.
(ii) G. F. Simmons, *Differential Equations with Applications and Historical Notes*, Third Edition, CRC Press, 2017.

DISCIPLINE SPECIFIC ELECTIVE – 2 (iii): REPRESENTATION OF FINITE GROUPS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-2 (iii): Representation of Finite Groups	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra and Group Theory

Learning Objectives
The primary objective of this course is to:
- represent finite groups as groups of matrices (via homomorphisms) and apply the tools of linear algebra to study the group structure.
- introduce the notion of Group algebra, which plays an essential role in classifying representations of groups.
- to discuss some applications of representations of finite groups, such as the Burnside's theorem.

Learning Outcomes
This course will enable the students to:
- define and construct examples of group representations, *FG*-modules, group algebras.
- grasp key concepts and tools of representation theory and establish a link between *FG*-modules and group representations.
- prove and apply Maschke's theorem and Schur's lemma to describe all irreducible representations of finite groups over the field of complex numbers.
- apply the theory of characters and group representations to gain insight into group structure, such as normal subgroups, and the solubility of groups.

Syllabus

Unit – 1 **(11 hours)**
Representation of groups, *FG*-modules and *FG*-submodules, and reducibility, Permutation modules, *FG*-modules and equivalent representations, Reducible and irreducible *FG*-modules, Group algebra of *G*, Regular *FG*-module and regular representations, *FG*-homomorphisms, Direct sum of *FG*-modules.

Unit – 2 **(11 hours)**
Maschke's theorem for *FG*-modules and consequences. Schur's lemma and its converse, Application of Schur's lemma, Irreducible modules and group algebra, Structure of group algebra and space of *CG*-homomorphisms.

Unit – 3 **(10 hours)**
Characters and their properties, Permutation and regular characters, Inner product, Number of irreducible characters, Orthogonality relations and finding normal subgroups.

Unit – 4 **(13 hours)**

Algebraic numbers, Algebraic integers and their properties, Character values, The Burnside's *(p,q)*-theorem and solubility of some particular groups.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] G. James and M. Liebeck, *Representations and Characters of Groups*, Second Edition, Cambridge University Press, 2005.

Suggested Readings

(i) C. W. Curtis and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras*, American Mathematical Society, 2006.
(ii) W. Fulton and J. Harris, *Representation Theory - A First Course*, Springer-Verlag, 2004.
(iii) I. M. Issacs, *Character Theory of Finite Groups*, American Mathematical Society reprint, 2006.

DISCIPLINE SPECIFIC ELECTIVE – 2 (iv): TOPOLOGICAL DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-2 (iv): Topological Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- provide a strong background of topological dynamical systems including their applications.
- develop some useful and interesting dynamical properties like expansivity, shadowing and topological stability with supporting examples and results from symbolic and topological dynamics.
- introduce the celebrated Sarkovskii's theorem.

Learning Outcomes

This course will enable the students to:

- construct interesting examples of dynamical systems and topological conjugacy.
- visualize stable sets, omega sets and alpha limit sets.
- understand the applications of Sarkovskii's theorem.
- use subshifts of finite type to characterize irreducible matrices.
- prove key results on expansivity and shadowing regarding existence/non-existence, product, subspace and their different characterizations etc.
- find the class of topologically stable homeomorphisms.

Syllabus

Unit – 1 **(10 hours)**

Definition and examples (including real life examples) of dynamical systems, Orbits, Types of orbits, Topological conjugacy and orbits, Phase portrait-graphical analysis of orbit, Periodic points and stable sets, Omega and alpha limit sets and their properties.

Unit – 2 **(10 hours)**

Sarkovskii's theorem, Shift spaces and subshift, Subshift of finite type, Subshift represented by a matrix, Characterizations of irreducible matrices.

Unit – 3 **(13 hours)**

Definition and examples of expansive homeomorphisms, Properties of expansive homeomorphisms, Non-existence of expansive homeomorphism on the unit interval and unit circle, Generators and weak generators, Generators and expansive homeomorphisms.

Unit – 4 **(12 hours)**

Converging semi-orbits for expansive homeomorphisms, Definition, examples and properties of maps having shadowing property, Topological Anosov homeomorphisms and topological stability.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] N. Aoki and K. Hiraide, *Topological Theory of Dynamical Systems: Recent Advances*, North Holland Publications, 1994.
[2] M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge University Press, 2004.

Suggested Readings

(i) D. C. Hanselman and B. Little field, *Mastering MATLAB*, Pearson, 2012.
(ii) D. Lind and B. Marcus, *An Introduction to Symbolic Dynamics and Coding*, Cambridge University Press, 1996.
(iii) C. Robinson, *Dynamical Systems*, *Stability, Symbolic Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 1998.
(iv) J. de. Vries, *Elements of Topological Dynamics*, Springer, 1993.

DISCIPLINE SPECIFIC ELECTIVE – 3 (i): ADVANCED FUNCTIONAL ANALYSIS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-3 (i): Advanced Functional Analysis	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis and Topology

Learning Objectives

The primary objective of this course is to:

- define and explain the structure of a topological vector space and its fundamental properties.
- differentiate between normed, metrizable, locally convex and Hausdorff topological vector spaces.
- introduce the foundational theorems of functional analysis, including the Hahn–Banach, Banach–Steinhaus, Open mapping, and Closed graph theorems in the context of locally convex spaces.
- explain some applications of Banach–Alaoglu theorem and Krein–Milman theorem.

Learning Outcomes

This course will enable the students to:

- appreciate types of topological vector spaces and their separation properties.
- understand quotient spaces, weak topology and weak*-topology.
- analyze concepts of continuity, boundedness, and convergence for linear operators and functionals on topological vector spaces.
- understand the notion of local convexity and the role of seminorms in defining locally convex topologies.

Syllabus

Unit – 1 **(12 hours)**

Topological vector spaces, Types of Topological vector spaces, Separation properties, Linear mappings, Finite dimensional spaces, Metrization, Boundedness and continuity, Seminorms and local convexity, Normability.

Unit – 2 **(11 hours)**

Quotient spaces, Seminorms and quotient spaces, Examples, Baire category theorem, Banach–Steinhaus theorem, The open mapping theorem and the closed graph theorem on topological vector spaces.

Unit – 3 **(11 hours)**

Hahn–Banach separation theorem on topological vector spaces, Continuous extension theorem, Weak topologies, Weak topology and convexity, Weak topology and metrizability, Weak*-topology of a dual space, Compact convex sets, Banach–Alaoglu theorem and applications, Goldstine theorem.

Unit – 4 **(11 hours)**

Extreme points, Krein–Milman theorem, Convex hull of compact sets, Applications of Krein–Milman theorem: Stone–Weierstrass theorem, Markov–Kakutani fixed point theorem.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.
[2] W. Rudin, *Functional Analysis*, Second Edition, Tata Mc Graw-Hill, 2011.

Suggested Readings

(i) V. I. Bogachev and O. G. Smolyanov, *Topological Vector Spaces and Their Applications*, Springer, 2017.
(ii) S. Kantorovitz, *Introduction to Modern Analysis*, Oxford Graduate Texts in Mathematics, Oxford University Press, 2006.
(iii) J. Voigt, *A Course on Topological Vector Spaces*, Birkhäuser, 2020.

DISCIPLINE SPECIFIC ELECTIVE – 3 (ii): ALGEBRAIC CODING THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-3 (ii): Algebraic Coding Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra, Groups and Rings

Learning Objectives
The primary objective of this course is to:
- provide an introduction to algebraic coding theory, particularly linear codes.
- discuss bounds on the parameters along with cyclic codes.
- describe some well-known codes, such as Reed–Muller and Golay codes.
- explore the algebraic structure of Cyclic and Quadratic residue codes over fields and rings.

Learning Outcomes
This course will enable the students to:
- get an insight into the matrix representation of a code, as well as encoding and decoding.
- understand Hamming, MDS and Reed–Muller codes.
- describe cyclic codes and their generator polynomial.
- learn about special cyclic codes, such as Quadratic residue codes, and their properties over the ring $\mathbb{Z}_4$.

Syllabus

Unit – 1 **(10 hours)**
Error detecting and error correcting codes, Maximum likelihood decoding, Hamming distance, Linear codes, Hamming weight, Generator matrix, Parity check matrix, Equivalence of linear codes, Encoding and decoding of linear codes, Syndrome decoding.

Unit – 2 **(11 hours)**
Bounds on codes, Sphere covering bound, Hamming bound, Perfect codes, Binary Hamming codes, Binary Golay codes, Singleton bound and MDS codes. Propagation rules, Reed–Muller codes.

Unit – 3 **(12 hours)**
Cyclic codes, Cyclic codes as ideals, Generator polynomial of cyclic codes, Generator and parity-check matrices of cyclic codes**,** Decoding of cyclic codes, Burst error correcting codes.

Unit – 4 **(12 hours)**
Quadratic residue codes: QR codes over fields of characteristic 2 and 3, Cyclic codes and their generating polynomial over $\mathbb{Z}_4$, QR codes over $\mathbb{Z}_4$.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] S. Ling and C. Xing, *Coding Theory: A First Course*, Cambridge University Press, 2004.
[2] W. C. Huffman and V. Pless, *Fundamentals of Error Correcting Codes*, Cambridge University Press, 2010.

Suggested Readings

(i) R. Hill, *A First Course in Coding Theory*, Oxford University Press, 1986.
(ii) F. J. Mac William and N. J. A. Sloane, *Theory of Error Correcting Codes, Part I & II,* Elsevier/North-Holland, Amsterdam, 1977.

DISCIPLINE SPECIFIC ELECTIVE – 3 (iii): DIFFERENTIAL GEOMETRY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-3 (iii): Differential Geometry	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra, Multivariate Calculus and Differential Equations

Learning Objectives
The primary objective of this course is to introduce:
- surfaces and parametrized surfaces.
- orientation on connected surfaces.
- geodesics on surfaces.
- Weingarten maps on oriented surfaces.
- arc length and curvature of oriented plane curves.
- curvatures of oriented surfaces.

Learning Outcomes
This course will enable the students to:
- understand the concepts of level sets and graphs of functions, smooth vector fields, tangent spaces of level sets.
- appreciate surfaces and parametrized surfaces, Gauss map, geodesics and parallel transport on oriented surfaces.
- know what the Weingarten map of an oriented surface is, realize it as shape operator and use it to compute curvature of oriented plane curves.
- find global parametrization and hence arc length of an oriented plane curve.
- compute various types of curvatures of surfaces.

Syllabus

Unit – 1 **(10 hours)**
Level sets in $\mathbb{R}^{n+1}$ and graphs of functions, Smooth vector fields and existence and uniqueness of their integral curves, Tangent spaces of level sets at regular points, Surfaces in $\mathbb{R}^{n+1}$ as inverse images of regular values of smooth functions, Necessary condition for extrema of functions on surfaces-Lagrange multipliers, Existence of a normal vector field on a connected surface, Orientation, Gauss map.

Unit – 2 **(13 hours)**
The notion of a geodesic on a surface, Existence and uniqueness of a geodesic on a surface through a given point with a given velocity vector thereof, Covariant derivative of a smooth vector field, Parallel vector field along a curve, Existence and uniqueness of a parallel vector field along a curve with a given initial vector, Weingarten map of a surface at a point, Local parametrization and curvature of a plane curve.

Unit – 3 **(10 hours)**

Global parametrization and arc length of an oriented plane curve, Differential 1-forms, Line integral of 1-forms over parametrized curves.

Unit – 4 **(12 hours)**

Parametrized surfaces with examples, Curvature of surfaces, Normal curvature of a surface at a point in a given direction, Principal curvatures, First and second fundamental forms, Gauss-Kronecker curvature and mean curvature.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] A. Pressley, *Elementary Differential Geometry*, Springer-Verlag London Limited, 2012.
[2] J. A. Thorpe, *Elementary Topics in Differential Geometry*, Springer (India) Pvt. Limited, 2004.

Suggested Readings

(i) W. Kuhnel, *Differential Geometry: Curves-Surfaces-Manifolds*, Third Edition, American Mathematical Society, 2015.
(ii) B. O' Neill, *Elementary Differential Geometry*, Second Edition, Academic Press INC., Academic Press, New York, 2006.

DISCIPLINE SPECIFIC ELECTIVE – 3 (iv): FINITE ELEMENT METHODS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-3 (iv): Finite Element Methods	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Differential Equations

Learning Objectives

The primary objective of this course is to:

- introduce basic aspects of finite element methods (FEM) including domain discretization, polynomial interpolation, application of boundary conditions, assembly of global arrays, and solution of the resulting algebraic systems.
- discuss the use of finite element methods in solving engineering problems in the domain of solid mechanics, fluid mechanics, heat transfer and electromagnetism.

Learning Outcomes

This course will enable the students to:

- use integral statement to deduce finite element approximations for the underlying linear partial differential equations.
- write special-purpose finite element programs within a procedural programming environment.
- use finite element methods to solve engineering problems in solids mechanics, fluid mechanics, heat transfer, and electromagnetism.
- assess the accuracy and reliability of finite element solutions and troubleshoot problems arising from errors in a given finite element analysis.

Syllabus

Unit – 1 **(12 hours)**

Basic concepts of weak formulation, Variational formulation of a one dimensional model equation, Basis function and finite element solutions, Collocation method, Ritz method, Least square method, Standard Galerkin method, FEM for model problem, Error estimate for FEM for model equation, Convergence analysis.

Unit – 2 **(11 hours)**

Various shapes of finite element, Higher order basis functions, Finite element methods for elliptic problems: Variational methods, Standard Galerkin method, Error estimate for FEM for elliptic problem, FEM for Poisson equation.

Unit – 3 **(12 hours)**

Finite element methods for parabolic problems: One dimensional model problems, Semi-discretization in space, Error estimates, Discretization in space and time, Galerkin method, Finite element methods for hyperbolic problems: Standard Galerkin method, Standard Galerkin method with strongly and weakly imposed boundary conditions.

Unit – 4 **(10 hours)**

Applications of the FEM to second order BVPs in one dimension, Applications of the FEM to linear elliptic, parabolic and hyperbolic equations.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] G. Evans, J. Blackledge and P. Yardley, *Numerical Methods for Partial Differential Equations*, Springer-Verlag, London, 2000.

[2] C. Johnson, *Numerical Solutions of Partial Differential Equations by Finite Element Methods*, *Cambridge University Press*, Cambridge, 1987.

[3] J. Whiteley, *Finite Element Methods - A Practical Guide*, Springer, 2016.

Skill-Based Course (SBC)

DEVELOPING MATHEMATICAL IDEAS

Course Title	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Developing Mathematical Ideas	**2**	**0**	**0**	**2**	**Same as for entry to M.Sc. Mathematics**	**NIL**

Learning Objectives

This course will train students

- to develop skills to create new mathematical ideas independently.
- to present these ideas adeptly.

Learning Outcomes

Students will be able to

- hone their analytical skills and their ability to think critically.
- learn to work collaboratively.
- acquire skills that help them to create and develop new mathematical ideas.

Methodology

We plan to form groups of students, say of 5-6 each, who will be assigned a piece of mathematical work (article/ paper published in reputed journals/ periodicals/book chapters). The designated groups will be required to read and understand this mathematical work under the supervision of a faculty member. They will be further encouraged to pose meaningful questions/problems in the context of the mathematics they have read and possibly offer solutions. Presentations will be conducted for these groups as a part of their assessment process.

Appropriate material for the study will be provided by the department/faculty. Papers/articles for example, may be chosen from resources like *Involve* (Link: https://msp.org/involve/), *SIAM Undergraduate Research Online* (Link: https://www.siam.org/publications/siuro), *The American Mathematical Monthly* (Mathematical Association of America, Taylor and Francis, Link: https://www.tandfonline.com/journals/uamm20), *Mathematics Magazine* (Mathematical Association of America, Taylor and Francis, Link: https://www.tandfonline.com/journals/umma20), *The Mathematics Student* (Indian Mathematical Society, Link: https://www.indianmathsoc.org/MS.html) and *The Mathematical Intelligencer* (Springer Nature, Link: https://link.springer.com/journal/283). Advanced topics beyond the prescribed syllabus from textbooks/ research monographs may also be chosen.

Generic Elective (GE) Courses

GENERIC ELECTIVE – 1 (i): DYNAMICAL SYSTEMS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
GE-1 (i): Dynamical Systems	4	3	1	0	Class XII pass with Mathematics	Basics of Topology and Ordinary Differential Equations

Learning Objectives

The primary objective of this course is to:

- understand discrete and continuous systems with case studies to study nonlinear systems of ordinary differential equations and dynamical systems.
- understand the concepts, models and techniques to realize the real-world problems and stability of the systems along with the chaotic dynamic behaviour of models by understanding bifurcations.

Learning Outcomes

This course will enable the students to learn:

- formulation of mathematical models with the stability analysis near the equilibrium points.
- how the concept of phase portraits helps to analyse mathematical model graphically.
- the qualitative behaviour of the solution set of a given system of differential equations including the invariant sets and limiting behaviour of the dynamical system or flow defined by the system of differential equations.
- how different bifurcations explain the chaotic behaviour of the system.

Syllabus

Unit – 1 **(13 hours)**

Linear systems: Jordan forms, Stability theory; Nonlinear systems: Fundamental existence-uniqueness theorem, Dependence on initial conditions and parameters, Flow of a differential equation, Linearization, Stable manifold theorem, Hartman–Grobman theorem.

Unit – 2 **(10 hours)**

Stability and Lyapunov functions, Saddle points, Nodes, Foci, Centers and nonhyperbolic critical points, Center manifold theorem.

Unit – 3 **(12 hours)**

Limit sets and attractors, Periodic orbits and limit cycles, Poincaré map, Stable manifold theorem for periodic orbits, Poincare-Bendixson theorem.

Unit – 4 **(10 hours)**

Bifurcations at nonhyperbolic equilibrium points, Saddle node, Transcritical and Pitchfork bifurcations, Hopf bifurcation.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. W. Hirsch, S. Smale and R. L. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, Third Edition, Academic Press, 2013.
[2] L. Perko, *Differential Equations and Dynamical Systems*, Third Edition, Springer Verlag, 2001.

Suggested Readings

(i) R. L. Devaney, *A First Course in Chaotic Dynamical Systems: Theory and Experiment*, CRC Press, Taylor & Francis, 2018.
(ii) S. H. Strogatz, *Nonlinear Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 2018.
(iii) S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, TAM Volume 2, Springer-Verlag, NY, 1990.

GENERIC ELECTIVE – 1 (ii): NUMERICAL METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
GE-1 (ii): Numerical Methods for Ordinary Differential Equations	4	3	1	0	Class XII pass with Mathematics	Basics of Ordinary Differential Equations

Learning Objectives

The primary objective of this course is to:

- develop the basic theory underlying the numerical solution of differential equations.
- introduce the concepts of consistency, stability and convergence of finite difference methods.
- execute the numerical schemes for the solution of differential equations.

Learning Outcomes

This course will enable the students to:

- gain a thorough understanding of the fundamental concepts involved in the construction and analysis of finite difference schemes for solving ordinary differential equations (ODEs).
- apply various numerical methods based on finite difference approaches to obtain approximate solutions for both initial value problems (IVPs) and boundary value problems (BVPs).
- develop the ability to select appropriate finite difference methods for specific types of problems and effectively apply them to real world applications.

Syllabus

Unit – 1 **(11 hours)**

Initial value problems: Existence and uniqueness of solution, Finite difference equation, Truncation error, Single step methods for first order IVPs and system of IVPs- Family of explicit and implicit Runge–Kutta methods, Taylor series methods, Derivation, Truncation error, Consistency, Stability and convergence analysis.

Unit – 2 **(12 hours)**

IVPs for the system of ODEs, Consistency, Zero stability and convergence of linear multistep methods, Routh–Hurwitz criterion, Order and error constant, First Dahlquist Barrier, Local truncation error and global truncation error, Error bounds, Local error, Linear stability theory, Higher order differential equations.

Unit – 3 **(12 hours)**

Derivation of explicit and implicit multistep methods for IVPs and system of IVPs, Truncation error, Stability and convergence analysis of family of Nystrom method, Adams–Bashforth method,

Adams–Moulton method, Milne–Simpson method, Predictor corrector method, and Modified predictor corrector method, Hybrid method, Multistep methods for second order IVPs.

Unit – 4 **(10 hours)**
Linear BVPs for second order ordinary differential equations, Shooting method, Finite difference method, Collocation method, Derivative boundary conditions, Nonlinear two-point BVPs, Higher order finite difference methods, Stability, Truncation error and convergence analysis.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] M. K. Jain, S. R. K. Iyenger and R. K. Jain, *Numerical Methods for Scientific and Engineering Computations*, Seventh Edition, New Age International Publisher, 2019.
[2] J. D. Lambert, *Numerical Methods for Ordinary Differential Systems: The Initial Value Problem*, John Wiley & Sons, 1991.

Suggested Readings
(i) K. E. Atkinson, W. Han and D. E. Stewart, *Numerical Solution of Ordinary Differential Equations*, John Wiley & Sons, 2009.
(ii) J. C. Butcher, *The Numerical Analysis of Ordinary Differential Equations*, Second Edition, Wiley, New York, 2008.
(iii) L. Collatz, *The Numerical Treatment of Differential Equations*, Springer-Verlag, 1966.

Syllabi of Courses in Semester-II of One-year M.Sc. Mathematics under Structure-1 (Only Course work)

Discipline Specific Core (DSC) Courses

DISCIPLINE SPECIFIC CORE – 3: PARTIAL DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSC-3: Partial Differential Equations	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Multivariate Calculus and Differential Equations

Learning Objectives

The main objective of this course is to introduce:

- well-posedness, fundamental solutions, existence and uniqueness of solutions for Laplace equation, Poisson equation and heat equation.
- solution for wave equation by spherical means.
- characteristics, complete integrals, envelopes and conservation laws for first-order nonlinear partial differential equations.
- classical solution techniques such as Green's function, similarity solutions and transform methods.

Learning Outcomes

This course will enable the students to:

- understand Laplace equation, Poisson equation, and Heat equation, their fundamental solutions, uniqueness principles, mean value properties, and Green's function.
- apply the method of spherical means to solve homogeneous and nonhomogeneous wave equations.
- use characteristics to solve nonlinear partial differential equations, construct complete integrals and envelopes, and understand conservation laws.
- implement various techniques such as similarity solutions and transform methods to derive solutions of different types of partial differential equations.

Syllabus

Unit – 1 **(12 hours)**

Well-posed problems, Classical solution, Laplace equation, Poisson equation, Fundamental solution, Strong maximum principle and uniqueness of solution, Mean value formulas, Representation formula, Green's function, Poisson's formula.

Unit – 2 **(10 hours)**

Heat equation, Fundamental solution for homogeneous and nonhomogeneous initial-value problems, Mean value formula, Strong maximum principle and uniqueness of solution, Local estimates for the solution.

Unit – 3 **(13 hours)**

Wave equation: Solution of homogeneous and nonhomogeneous problems by spherical means,

Nonlinear first order partial differential equations: Complete integrals and envelopes, Characteristics, Introduction to conservation laws.

Unit – 4 **(10 hours)**
Other solution methods: Similarity solutions, Fourier transform and Laplace transform, Cole–Hopf transformation, Potential function, Hodograph and Legendre transform.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] L. C. Evans, *Partial Differential Equations*, American Mathematical Society, Providence, RI, 1998.
[2] F. John, *Partial Differential Equations*, Fourth Edition, Springer-Verlag, New York, 1982.

Suggested Readings
(i) P. R. Garabedian, *Partial Differential Equations*, John Wiley & Sons, Inc., New York- London-Sydney, 1964.
(ii) A. K. Nandakumaran and P. S. Datti, *Partial Differential Equations: Classical Theory with a Modern Touch*, Cambridge University Press, 2020.

DISCIPLINE SPECIFIC CORE – 4: ANALYSIS OF SEVERAL VARIABLES

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSC-4: Analysis of Several Variables	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Calculus, Real Analysis including Riemann Integration

Learning Objectives

The primary objective of this course is to:

- introduce differentiation of vector valued functions on $\mathbb{R}^n$ and their properties.
- familiarize students with integration of functions over rectangles and bounded sets in $\mathbb{R}^n$.
- extend integration of functions to unbounded sets in $\mathbb{R}^n$.
- study change of variables and its applications.

Learning Outcomes

This course will enable the students to:

- check differentiability of vector valued functions on $\mathbb{R}^n$, understand the relation between directional derivative and differentiability, apply chain rule, mean value theorems, inverse and implicit function theorems.
- understand higher order derivatives and be able to apply Taylor's formulas with integral remainder, Lagrange's remainder and Peano's remainder.
- master the concepts of integration over rectangles and bounded sets in $\mathbb{R}^n$.
- generalize the integration theory to unbounded sets in $\mathbb{R}^n$.
- grasp the effect of change of variables in integration.

Syllabus

Unit– 1 **(12 hours)**

The differentiability of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$, Partial derivatives and differentiability, Directional derivatives and differentiability, Chain rule, Mean value theorems, Inverse function theorem and Implicit function theorem.

Unit– 2 **(11 hours)**

Derivatives of higher order, Taylor's formulas with integral remainder, Lagrange's remainder and Peano's remainder, Integral over a rectangle, Existence of the integral.

Unit– 3 **(10 hours)**

Evaluation of the integral, Fubini's theorem, Integral over a bounded set.

Unit– 4 **(12 hours)**

Rectifiable sets, Improper integrals, Change of variable theorem, Applications of change of variables.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] M. Giaquinta and G. Modica, *Mathematical Analysis: An Introduction to Functions of Several Variables*, Birkhäuser, 2009.
[2] J. R. Munkres, *Analysis on Manifolds*, CRC Press, Taylor & Francis, 2018.

Suggested Readings
(i) W. Rudin, *Principles of Mathematical Analysis*, Third Edition, Mc Graw Hill, 1986.
(ii) M. Spivak, *Calculus on Manifolds: A Modern Approach to Classical Theorems of Advanced Calculus*, Taylor & Francis, 2018.

Discipline Specific Elective (DSE) Courses

DISCIPLINE SPECIFIC ELECTIVE – 4 (i): ADVANCED FLUID DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-4 (i): Advanced Fluid Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Calculus and Partial Differential Equations

Learning Objectives

The primary objective of this course is to:

- prepare a foundation for advanced studies in compressible flow, boundary layer theory and magnetohydrodynamics.
- develop concepts, models, and techniques that enable problem-solving in compressible flow, boundary layer theory and magnetohydrodynamics.
- equip students with concepts and techniques to conduct research in the above mentioned domains.

Learning Outcomes

This course will enable the students to:

- learn conservation laws, first and second laws of thermodynamics, internal energy and entropy, different forms of energy equations and dimensional analysis.
- know about compressibility in real fluids, wave motion, sound waves, hyperbolic and dispersive waves, shock waves, their formation, properties and elementary analysis.
- know the concepts of boundary layer, boundary layer equations and their solutions, measurements of boundary layer thickness.
- understand the interaction between hydrodynamic processes and electromagnetic phenomena.
- formulate the basic equations of motion in inviscid and viscous conducting fluid flow and explain Alfven’s theorem and magnetohydrodynamic (MHD) waves and MHD shocks.

Syllabus

Unit – 1 **(11 hours)**

Flow characteristics, Conservation laws, Equation of state of a substance, First and second law of thermodynamics, Internal energy and entropy, Energy equation, Nondimensionalizing the basic equations of incompressible viscous fluid flow, Nondimensional numbers.

Unit – 2 **(12 hours)**

Compressibility effects in real fluids, Equations of motion, Sound wave, Hyperbolic and dispersive waves, Isentropic gas flow, Flow through a nozzle, Method of characteristics, Shock jump conditions, Non-linear plane waves, Shock waves and their elementary analysis, Similarity solutions.

Unit – 3 **(11 hours)**

Boundary layer concept, Estimation of boundary layer thickness and friction forces, Prandtl's boundary layer equations, Boundary layer along a flat plate, Boundary layer thickness, General properties of the boundary layer equations, Similar solutions, Momentum and energy integral equations for the boundary layer.

Unit – 4 **(11 hours)**

Maxwell's electromagnetic field equations, Magnetohydrodynamic (MHD) approximations, Magnetic field equation, Magnetic Reynolds number, Magnetic body force, Equations of Motions of conducting fluid, Alfven's theorem, MHD waves, MHD shock waves.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] F. Chorlton, *Text Book of Fluid Dynamics*, CBS Publisher, 2005.
[2] H. Schlichting and K. Gersten, *Boundary Layer Theory*, Ninth Edition, Springer, 2017.
[3] G. B. Witham, *Linear and Nonlinear Waves*, John Wiley & Sons, 1999.

Suggested Readings

(i) K. R. Cramer and S. I. Pai, *Magnetofluid Dynamics for Engineers and Applied Physics*, McGraw Hill Book Co., New York, 1973.
(ii) Y. Shao-Wen, *Foundations of Fluid Mechanics*, PHI, New Delhi, 1960.

DISCIPLINE SPECIFIC ELECTIVE – 4 (ii): MODULE THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-4 (ii): Module Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Groups and Rings

Learning Objectives

The primary objective of this course is to:

- introduce a new algebraic structure, namely, module which is a generalization of a vector space when the underlying field is replaced by an arbitrary ring. The study of modules over a ring also provides an insight into the structure of the ring.
- study free modules, finitely generated modules, projective and injective modules.
- classify the finitely generated modules over a principal ideal domain (PID).

Learning Outcomes

This course will enable the students to:

- identify and construct examples of modules, and apply homomorphism theorems on the same.
- define and characterize Noetherian, Artinian module, and apply the structure theorem of finitely generated modules over PID.
- distinguish between projective, injective, free, and semi simple modules.
- prove universal property of tensor product of modules, and Hilbert basis theorem.

Syllabus

Unit – 1 **(13 hours)**

Basic concepts of module theory, Fundamental theorems of homomorphism, Direct product and direct sum of modules, Exact sequences, Split exact sequences.

Unit – 2 **(10 hours)**

Free modules, Projective and injective modules, Dual basis lemma, Baer's criterion, Divisible modules.

Unit – 3 **(12 hours)**

Tensor product of modules, Chain conditions, Hilbert basis theorem.

Unit – 4 **(10 hours)**

Modules over PID's, Semi simple modules.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. F. Athiyah and I. G. Macdonald, *Introduction to Commutative Algebra*, Addison Wesley, 1969.
[2] P. M. Cohn, *Basic Algebra*, Springer International Edition, 2003.
[3] P. M. Cohn, *Classic Algebra*, John Wiley & Sons Ltd., 2000.

Suggested Readings

(i) D. S. Dummit and R. M. Foote, *Abstract Algebra*, Wiley India Pvt. Ltd., 2011.
(ii) N. Jacobson, *Basic Algebra*, Volume II, Dover Publications Inc., 2009.
(iii) T. W. Hungerford, *Algebra*, Springer-Verlag, 1981.

DISCIPLINE SPECIFIC ELECTIVE – 4 (iii): PROBABILITY THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-4 (iii): Probability Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Measure Theory

Learning Objectives

The primary objective of this course is to introduce:

- probability space as a measure space and random variables as measurable functions.
- expectation and moments of random variables.
- notion of convergence in probability.
- conditioning on sub-σ-algebra.

Learning Outcomes

This course will enable the students to learn:

- about probability or uncertainty in abstract setting.
- moments and expectation of random variables which help to understand applications of probability in industry.
- how to apply the idea of convergence in probability.
- weak law and strong law of large numbers and their applications.

Syllabus

Unit – 1 **(11 hours)**

Probability: σ-algebra, Constructing probability triples, The extension theorem, Random variables, Independence of events, Continuity of probabilities, Limit events, The Borel–Cantelli Lemma.

Unit – 2 **(10 hours)**

Expected values: Simple, general non-negative and arbitrary random variables, Moment generating functions, Markov's inequality, Chebyshev's inequality.

Unit – 3 **(12 hours)**

Convergence of random variables: Convergence almost surely, Convergence in probability, Weak law of large numbers, Strong law of large numbers.

Unit – 4 **(12 hours)**

Distributions of random variables: Examples of distributions, Characteristic functions, The central limit theorem, Conditional probability, Conditioning on random variable, Conditioning on a sub-σ-algebra, Conditional variance.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. S. Rosenthal, *A First Look at Rigorous Probability Theory*, Second Edition, World Scientific, Singapore, 2006.

Suggested Readings

(i) W. Feller, *An Introduction to Probability Theory and its Applications*, Volume 1, Third Edition, Wiley, 2008.
(ii) J. E. Michael and J. S. Rosenthal, *Probability and Statistics: The Science of Uncertainty*, Second Edition, W. H. Freeman & Co Ltd., 2009.
(iii) S. Ross, *A First Course in Probability*, Tenth Edition, Pearson Education, 2022.
(iv) D. W. Stroock, *Probability Theory, An Analytic View*, Cambridge University Press, 2024.

DISCIPLINE SPECIFIC ELECTIVE – 4 (iv): THEORY OF UNBOUNDED OPERATORS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-4 (iv): Theory of Unbounded Operators	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis and Bounded Operators

Learning Objectives
The primary objective of this course is to:
- introduce the notion of unbounded operators.
- develop the theory of operator semigroups and understand their role in applications, particularly for solving differential equations.

Learning Outcomes
This course will enable the students to:
- identify closed and closable linear operators on Banach spaces.
- compute adjoints of unbounded linear operators.
- understand spectral properties of some unbounded operators.
- comprehend the role unbounded operators and semigroups play in applications, particularly in studying solutions of differential equations.

Syllabus

Unit – 1 **(10 hours)**
Unbounded linear operators, Hilbert adjoints, Hellinger–Toeplitz theorem, Hermitian, symmetric and self-adjoint linear operators, Closed linear operators, Closable operators and their closures on Banach spaces.

Unit – 2 **(12 hours)**
Cayley transform, Deficiency indices, Spectral properties of self-adjoint operators, Multiplication and differentiation operators and their spectra.

Unit – 3 **(11 hours)**
Analytic properties of exponential functions, Matrix Semigroups, Uniformly continuous semigroups, Semigroups on Hilbert spaces, Strongly continuous semigroups.

Unit – 4 **(12 hours)**
Generators of semigroups and their resolvents, Hille–Yosida theorem (for contraction semigroup), Dissipative operators and their properties, Lumer–Phillips theorem, Generators of Group, Stone's theorem.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] K. J. Engle and R. Nagel, *One-parameter Semigroups for Linear Evolution Equations*, Springer-Verlag, New York, 2000.
[2] E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, 2006.

Suggested Readings

(i) S. Goldberg, *Unbounded Linear Operators: Theory and Applications*, Dover Publications, 2006.
(ii) E. Hille and R. S. Phillips, *Functional Analysis and Semi-groups*. American Mathematical Society, Providence, RI, 1957.
(iii) A. Pazy, *Semigroups of Linear Operators and Applications to Partial Differential Equations*, Springer, 1983.
(iv) M. Schechter, *Principles of Functional Analysis*, Second Edition, American Mathematical Society, 2001.
(v) J. Weidmann, *Linear Operators in Hilbert Spaces, Graduate Texts in Mathematics*, Springer, New York, 1980.

DISCIPLINE SPECIFIC ELECTIVE – 5 (i): BANACH AND C*-ALGEBRAS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-5 (i): Banach and C*-Algebras	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis and Topology

Learning Objectives

The primary objective of this course is to introduce:

- Banach algebras and C*-algebras.
- various ways to construct new operator algebras using given ones.
- spectrum of elements in Banach algebras and to study its properties.
- Gelfand representations of commutative Banach algebras and of C*-algebras.

Learning Outcomes

This course will enable the students to:

- familiarize with the representations of operator algebras.
- realize commutative Banach algebras and abelian C*-algebras as space of continuous functions on locally compact groups.
- understand the powerful tool of functional calculus.
- identify any C*-algebra as closed *-subalgebra of space of bounded linear operators on a Hilbert space.

Syllabus

Unit – 1 **(11 hours)**

Elementary properties and examples of Banach algebras, Ideals and quotients, Invertible elements, Spectrum and spectral radius, Spectral radius formula, Spectral mapping theorem (for polynomials), Gelfand–Mazur theorem.

Unit – 2 **(11 hours)**

Multiplicative linear functionals, Commutative Banach algebra, w*-topology, Gelfand transform of an element, Structure space, Gelfand representation.

Unit – 3 **(12 hours)**

Elementary properties and examples of C*-algebras, Unitization, Gelfand–Naimark representation of commutative C*-algebras, Continuous functional calculus, Spectral mapping theorem for normal elements, Positive elements of C*-algebras.

Unit – 4 **(11 hours)**

Ideals in C*-algebras, Approximate units, Quotients, Positive linear functionals, Gelfand–Naimark–Segal representation of C*-algebras.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.
[2] G. J. Murphy, *C*-algebras and Operator Theory*, Academic Press Inc., 1990.

Suggested Readings

(i) J. B. Conway, *A Course in Operator Theory, Graduate Texts in Mathematics*, Springer, 2007.
(ii) J. Dixmier, *C*-algebras*, North-Holland Publishing Company, 1977.
(iii) R. G. Douglas, *Banach Algebra Techniques in Operator Theory, Graduate Texts in Mathematics*, Springer, 1998.
(iv) E. Kaniuth, *A Course on Commutative Banach Algebras*, Graduate Texts in Mathematics, Springer, 2009.
(v) S. Kantorovitz, *Introduction to Modern Analysis*, Oxford Graduate Texts in Mathematics, Oxford University Press, 2006.
(vi) M. Takesaki, *Theory of Operator Algebras I*, Springer, 2002.

DISCIPLINE SPECIFIC ELECTIVE – 5 (ii): CHAOS THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-5 (ii): Chaos Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- introduce some useful and interesting notions like Topological Transitivity and Sensitive dependence on initial conditions.
- study different types of chaos including Devaney's chaos and finding their interrelationships.
- know classical result that period three implies chaos on intervals.
- relate chaos and decomposition theorems.
- study Topological entropy through open covers and also Bowen's definition of entropy, equivalence of these two definitions on compact metric spaces.
- study various interesting results related to topological entropy.

Learning Outcomes

This course will enable the students to:

- construct interesting examples of Topological transitive maps, Topological mixing maps etc.
- know classical examples of Devaney's chaotic maps like tent map, shift maps, logistic maps.
- study and compare different types of chaos.
- find relation between transitivity and chaos on intervals.
- relate chaos theory and classical decomposition theorems.
- study very useful notion of Topological entropy including its properties.
- calculate entropy of any homeomorphism of closed unit interval and of unit circle.

Syllabus

Unit – 1 **(12 hours)**

Topological Transitivity, Locally eventually onto maps, Topological mixing, Sensitive dependence on initial conditions, Devaney's definition of chaos, Transitivity and limit sets for continuous interval maps.

Unit – 2 **(11 hours)**

Characterizing topological mixing in terms of topological transitivity for continuous interval maps, Topological Weakly Mixing, Totally Transitive maps, Relation between transitivity and chaos on intervals, Logistic maps and shift maps as chaotic maps.

Unit – 3 **(12 hours)**

Various other definitions of chaos and their interrelationships. Period three implies chaos, Chaos and decomposition theorems including Bowen's decomposition theorem, Topological Entropy: Definition using open covers, Examples and properties, Bowen's definition of topological entropy,

Equivalence of two definitions, Topological version of Kolmogorov–Sinai theorem.

Unit – 4 **(10 hours)**
Topological Entropy of maps on a compact metric space, Topological Entropy of product maps, of iterations of a map, Topological entropy of an expansive homeomorphism on a compact metric space, of the two-sided shift, of any homeomorphism of the unit interval and of the unit circle.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] N. Aoki and K. Hiraide, *Topological Theory of Dynamical Systems: Recent Advances*, North Holland Publications, 1994.
[2] R. L. Devaney, *A First Course in Chaotic Dynamical Systems*, CRC Press, 2018.
[3] S. Ruette, *Chaos for Continuous Interval Maps: A Survey of Relationship Between Various Kinds of Chaos*, 2018.
[4] Peter Walters, *An Introduction to Ergodic Theory*, Springer, 2000.

Suggested Readings
(i) L. Alsedà, J. Llibre and M. Misiurewicz, *Combinatorial Dynamics and Entropy in Dimension One*, Advanced Series in Nonlinear Dynamics, World Scientific, 2000.
(ii) L. S. Block and W. A. Coppel, *Dynamics in One Dimension*, Springer, 2014.
(iii) M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge University Press, 2015.

DISCIPLINE SPECIFIC ELECTIVE – 5 (iii): COMPLEX ANALYSIS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-5 (iii): Complex Analysis	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Real Analysis and Metric Spaces

Learning Objectives

The primary objective of this course is to:

- gain insights of well-known classical results in the field of complex analysis.
- investigate various properties of analytic functions, conformal mappings and Möbius transformations.
- derive various forms of Cauchy’s theorem, integral formulas and maximum principles.
- represent complex-valued functions as Taylor and Laurent series.

Learning Outcomes

This course will enable the students to:

- construct Möbius transformations using Symmetry and Orientation principles.
- foresee the usage of simply connected regions in the complex plane for the existence of primitives and branch of logarithm.
- understand the behavior of zeros of analytic functions and meromorphic functions through Argument principle and Rouche’s theorem.
- evaluate the real integrals involving rational and trigonometric functions by contour integration using Residue theorem.
- apply Schwarz’s lemma to characterize the conformal maps of the open unit disk onto itself.

Syllabus

Unit – 1 **(10 hours)**
Extended plane and its spherical representation, Analytic functions, Branch of logarithm, Conformal mappings, Möbius transformations.

Unit – 2 **(10 hours)**
Line integrals, Fundamental theorem of Calculus for line integrals, Power series representation of analytic functions, Zeros of analytic functions, Liouville’s theorem.

Unit – 3 **(12 hours)**
Index of a closed curve, Cauchy’s theorem and integral formula, Morera’s Theorem, Homotopic version of Cauchy’s theorem and simple connectivity, Counting Zeros, Open mapping theorem, Goursat’s theorem.

Unit – 4 **(13 hours)**
Classification of singularities, Laurent series, Casorati-Weierstrass theorem, Residue theorem with applications, Argument principle, Rouche’s theorem, Maximum principles, Schwarz lemma.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. B. Conway, *Functions of One Complex Variable*, Second Edition, Narosa Publishing House, New Delhi, 2002.

Suggested Readings

(i) L. V. Ahlfors, *Complex Analysis*, Mc Graw Hill Co., Indian Edition, 2017.
(ii) T. W. Gamelin, *Complex Analysis*, Springer New York, NY, 2001.
(iii) L. Hahn and B. Epstein, *Classical Complex Analysis*, Jones and Bartlett, 1996.
(iv) E. M. Stein and R. Shakarchi, *Complex Analysis*, Princeton University Press, 2003.

DISCIPLINE SPECIFIC ELECTIVE – 5 (iv): NONSMOOTH OPTIMIZATION

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-5 (iv): Nonsmooth Optimization	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Nonlinear Optimization

Learning Objectives
The primary objective of this course is to:
- understand the tools to deal with nonsmooth convex functions.
- study conjugate duality in terms of conjugate functions for constrained nonlinear optimization problems.
- introduce numerical techniques to solve constrained nonlinear optimization problems.

Learning Outcomes
This course will enable the students to learn:
- the notions of subgradients and subdifferentials for nonsmooth convex functions.
- the use of conjugate functions to develop the theory of conjugate duality.
- about numerical methods like gradient descent method, gradient projection method, Newton's method and conjugate gradient method.
- penalty approach technique to solve constrained nonlinear optimization problems.

Syllabus

Unit – 1 **(11 hours)**
Extended real valued functions, Proper convex functions, Closure of convex functions, Differential derivatives, Subgradients and subdifferentials.

Unit – 2 **(12 hours)**
Conjugate functions, Biconjugate functions, Perturbation functions, Closure of convex functions, Directional derivatives, Subgradients and subdifferentials.

Unit – 3 **(12 hours)**
Gradient descent method, Gradient projection method, Newton's method, Conjugate gradient method.

Unit – 4 **(10 hours)**
Penalty function methods, Exterior penalty function, Interior penalty functions.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] M. Avriel, *Nonlinear Programming: Analysis and Methods*, Dover Publications, 2003.

[2] O. Güler, *Foundations of Optimization*, Springer, 2010.

Suggested Readings

(i) A. Bagirov, N. Karitsa and M. M. Makela, *Introduction to Nonsmooth Optimization: Theory*, *Practice and Software*, Springer, 2014.
(ii) M. S. Bazaraa, H. D. Sherali and C. M. Shetty, *Nonlinear Programming: Theory and Algorithms,* Third Edition, John Wiley & Sons, 2013.
(iii) D. G. Luenberger and Y. Ye, *Linear and Nonlinear Programming*, Springer, 2008.

DISCIPLINE SPECIFIC ELECTIVE – 6 (i): COMPUTATIONAL FLUID DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-6 (i): Computational Fluid Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Partial Differential Equation (Undergraduate level)

Learning Objectives

The primary objective of this course is to teach:

- various numerical schemes on finite difference and finite volume methods for solving PDEs.
- discretization errors and grid dependence.
- some real-world applications of PDEs and fluid dynamics.
- discretization of governing equations of diffusion, convection-diffusion, fluid flow and thereby computing the numerical solutions using the flow variables using algorithms.

Learning Outcomes

This course will enable the students to learn:

- techniques for solving the PDEs along with some initial and boundary conditions by using the finite difference and finite volume methods.
- the basic conservation principles of mass, momentum, energy, discretization of governing equations.
- discretization techniques.
- some popular algorithms like SIMPLE and SIMPLER used to obtain the solutions of steady and unsteady flow problems by finite volume methods.

Syllabus

Unit – 1 **(12 hours)**

Basics of discretization using finite differences, Various single and multi-step explicit and implicit finite difference schemes for 1-D and 2-D parabolic and hyperbolic initial boundary value problems, Alternating Direction Implicit schemes (ADI) for 2-D parabolic and hyperbolic equations, Order of accuracy, Consistency, Stability and convergence of a finite difference scheme, Courant Friedrich Lewy condition.

Unit – 2 **(12 hours)**

Finite difference schemes for second and fourth order 2-D elliptic boundary value problem and applications, Finite volume method for diffusion and convection-diffusion equations, Discretization of one and two-dimensional steady state diffusion and convection-diffusion equations, Central difference, Upwind, Exponential, Hybrid, Power-law and QUICK schemes and their properties.

Unit – 3 **(11 hours)**
Flow field calculation, Pressure-velocity coupling, Vorticity-stream function approach, Primitive variables, Staggered grid, Pressure and velocity corrections, Pressure correction equation, SIMPLE and SIMPLER algorithms.

Unit – 4 **(10 hours)**
Finite volume methods for unsteady flows, Discretization of one-dimensional transient heat conduction, Explicit, fully implicit and Crank–Nicolson schemes, Implementation of boundary conditions.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] J. C. Strikweda, *Finite Difference Schemes and Partial Differential Equations*, Second Edition, SIAM, 2004.
[2] H. K. Versteeg and W. Malalasekera, *An Introduction to Computational Fluid Dynamics: The Finite Volume Method*, Second Edition, Pearson, 2008.

Suggested Readings
(i) J. D. Anderson, *Computational Fluid Dynamics*, McGraw-Hill, 1995.
(ii) S. V. Patankar, *Numerical Heat Transfer and Fluid Flow*, CRC Press, Taylor and Francis, Indian Edition, 2017.
(iii) R. H. Pletcher, J. C. Tannehill and D. A. Anderson, *Computational Fluid Mechanics and Heat Transfer*, CRC Press, Taylor and Francis, 2013.
(iv) J. W. Thomas, *Numerical Partial Differential Equations: Finite Difference Methods*, Springer, 2013.

DISCIPLINE SPECIFIC ELECTIVE – 6 (ii): DIFFERENTIAL TOPOLOGY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-6 (ii): Differential Topology	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- introduce the concepts of topological manifolds, smooth structures, smooth manifolds, and manifolds with boundary.
- develop an understanding of smooth functions, smooth maps, diffeomorphisms, and tangent spaces.
- explain the Inverse function theorem, immersions and submersions.
- develop the fundamental concepts of 2-manifolds and distinguish between orientable and non-orientable surfaces.
- explore the properties of compact and connected surfaces.

Learning Outcomes

This course will enable the students to:

- identify and construct examples of topological manifolds, smooth structures and manifolds with and without boundary.
- demonstrate understanding of diffeomorphisms and tangent spaces.
- apply the Inverse function theorem, immersions and submersions.
- define key concepts such as 2-manifolds, orientability, compactness, connectedness and boundary of a surface.
- differentiate between orientable and non-orientable surfaces using examples such as the sphere, torus, Möbius strip and Klein bottle.

Syllabus

Unit – 1 **(12 hours)**

Topological manifolds, Topological properties of manifolds, Smooth structures, Examples of smooth manifolds, Manifolds with boundary.

Unit – 2 **(11 hours)**

Smooth functions and smooth maps, Lie groups, Diffeomorphisms.

Unit – 3 **(10 hours)**

Derivatives and tangents, Inverse function theorem, Immersions and submersions.

Unit – 4 **(12 hours)**

Complexes, Connected sum of two surfaces, Non-orientable surfaces (2- Manifolds), Compact and connected surfaces, Classification of compact and connected surfaces with and without boundary.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] V. Guillemin and Alan Pollack, *Differential Topology*, Prentice-Hall, 1974.
[2] L. C. Kinsey, *Topology of Surfaces*, Springer Verlag, 1997.
[3] J. M. Lee, *Introduction to Smooth Manifolds*, Second Edition, Springer, 2013.

Suggested Readings

(i) L. Conlon, *Differentiable Manifolds*, Second Edition, Birkhäuser Advanced Texts, 2001.
(ii) M. Spivak, *A Comprehensive Introduction to Differential Geometry*, Volume 1, Third Edition, Publish or Perish, Huston, Texas, 1999.
(iii) L. W. Tu, *Introduction to Manifolds*, Second Edition, Springer, 2011.
(iv) F. W. Warner, *Foundations of Differentiable Manifolds and Lie Group*, Springer-Verlag, 1983.

DISCIPLINE SPECIFIC ELECTIVE – 6 (iii): GENERAL MEASURE THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-6 (iii): General Measure Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Measure Theory and Topology

Learning Objectives

The primary objective of this course is to introduce:

- real valued and complex valued measures.
- decomposition of measure spaces and of measures.
- extension of a premeasure to a measure, Lebesgue measure on Euclidean spaces.
- representation of measures and functionals in terms of integrals.
- product measures.

Learning Outcomes

This course will enable the students to:

- appreciate signed measures and complex measures, mutual singularity of measures, Hahn and Jordan decompositions, Lebesgue decomposition, Radon–Nikodym theorem.
- verify conditions under which a set function defined on a collection of subsets of a set has an extension to a measure on a sigma-algebra.
- apply Riesz representation theorem for bounded linear functionals on L^p-spaces.
- understand product measure and the results of Fubini and Tonelli, and express the Lebesgue measure on Euclidean spaces as a product measure.
- apply Riesz–Markov representation theorem for the bounded linear functionals on the space of continuous functions.

Syllabus

Unit – 1 **(13 hours)**

Signed measures, Hahn and Jordan decompositions, Mutually singular measures, Radon–Nikodym theorem, Lebesgue decomposition, Complex measure.

Unit – 2 **(10 hours)**

The Carathéodory extension theorem, Lebesgue measure on $\mathbb{R}^n$, Regularity and translation invariance of Lebesgue measure on $\mathbb{R}^n$.

Unit – 3 **(10 hours)**

Riesz representation theorem for the dual of L^p-spaces, Product measures, Fubini's theorem, Tonelli's theorem.

Unit – 4 **(12 hours)**

Locally compact Hausdorff spaces and construction of Radon measure, Riesz–Markov representation theorem for positive linear functionals on $C_c(X)$, Riesz representation theorem for the dual of $C(X)$.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] H. L. Royden and P. M. Fitzpatrick, *Real Analysis*, Fourth Edition, Pearson, 2015.
[2] M. E. Taylor, *Measure Theory and Integration*, American Mathematical Society, 2006.

Suggested Readings

(i) G. B. Folland, *Real Analysis: Modern Techniques and Their Applications*, Second Edition, Wiley, New York, 1999.
(ii) P. R. Halmos, *Measure Theory*, Springer Science + Business Media, LLC, 2014.
(iii) E. M. Stein and R. Shakarchi, *Real Analysis: Measure Theory, Integration and Hilbert Spaces*, New Age International Publishers, New Delhi, 2010.

DISCIPLINE SPECIFIC ELECTIVE – 6 (iv): THEORY OF NON-COMMUTATIVE RINGS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE-6 (iv): Theory of Non-commutative Rings	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Groups and Rings

Learning Objectives

The primary objective of this course is to:

- give students an understanding of Wedderburn–Artin theory of semisimple rings.
- develop Jacobson's general theory of radicals, prime and semiprime rings, and primitive and semiprimitive rings.
- introduce the structure of primitive rings as a generalisation of the Wedderburn–Artin theorem on Artinian simple rings.

Learning Outcomes

This course will enable the students to:

- know about an extensive variety of rings, including free rings, Weyl algebra, Hilbert twist and triangular ring.
- understand the module theoretic definition of semisimple rings and how it leads to the Wedderburn–Artin structure theorem on their complete classification.
- know Jacobson's general theory of radicals, semiprime rings, prime, primitive and semiprimitive rings and their structures.
- understand the significance of the fundamental result 'Density Theorem' and its consequences on the structure of primitive rings.

Syllabus

Unit – 1 **(11 hours)**

Simple rings, Reduced rings, Dedekind-finite rings, Algebra, Quaternions, Free k-rings, Rings with generators and relations, Weyl algebra, Formal power series ring, Hilbert's twist ring, Differential polynomial rings, Derivation and inner derivation on a ring, Triangular rings, Characterization of one-sided and two-sided ideals in such rings.

Unit – 2 **(11 hours)**

Noetherian and Artinian rings, Examples of one-sided Noetherian and Artinian triangular rings, Twisted polynomial ring and Quotient of free $\mathbb{Z}$-ring, Semisimple rings, Structure of semisimple rings: Wedderburn–Artin's theorem.

Unit – 3 **(10 hours)**

Structure theorem of simple left Artinian rings, Jacobson radical, J-semisimple rings, Nil and nilpotent ideals, Connection between semisimple and J-semisimple rings, Hopkins–Levitzki theorem, Nakayama's lemma.

Unit – 4 **(13 hours)**
Prime radical, Characterisation of prime and semiprime ideals, Prime and semiprime rings, Structure theorem of primitive rings, Density theorem.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] T.-Y. Lam, *A First Course in Noncommutative Rings*, Springer, 2001.

Suggested Readings
(i) I. N. Herstein, *Noncommutative Rings*, The Mathematical Association of America, 2005.
(ii) T. W. Hungerford, *Algebra*, Springer-Verlag, New York, 1981.
(iii) L. H. Rowen, *Ring Theory*, Student Edition, Academic Press, 1991.

Skill-Based Course (SBC)

WORKSHOPS AND SEMINARS ON ADVANCED TOPICS

Course Title	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Workshops and Seminars on Advanced Topics	2	0	0	2	Same as for entry to M.Sc. Mathematics	NIL

Learning Objectives

In this course, we aim to train students to

- understand and assimilate mathematical ideas and techniques presented and discussed in workshops and seminars on varied advanced topics in mathematics.
- communicate mathematical ideas succinctly.

Learning Outcomes

This course will enable students to

- efficiently identify the core ideas in any mathematical discourse, particularly outside the classroom setting.
- write a concise summary of these ideas.

Methodology

The students will attend a requisite number of workshops and seminars organized by the Department through the semester. The workshops and seminars will be on advanced topics, building on the classroom courses offered by the Department. The students will be expected to prepare and submit summaries of a few of these (as mandated by the Department) and also give presentations for assessment.

Generic Elective (GE) Courses

GENERIC ELECTIVE – 2 (i): NONSMOOTH OPTIMIZATION

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
GE-2 (i): Nonsmooth Optimization	4	3	1	0	Class XII pass with Mathematics	Basics of Nonlinear Optimization

Learning Objectives

The primary objective of this course is to:

- understand the tools to deal with nonsmooth convex functions.
- study conjugate duality in terms of conjugate functions for constrained nonlinear optimization problems.
- introduce numerical techniques to solve constrained nonlinear optimization problems.

Learning Outcomes

This course will enable the students to learn:

- the notions of subgradients and subdifferentials for nonsmooth convex functions.
- the use of conjugate functions to develop the theory of conjugate duality.
- about numerical methods like gradient descent method, gradient projection method, Newton's method and conjugate gradient method.
- penalty approach technique to solve constrained nonlinear optimization problems.

Syllabus

Unit – 1 **(11 hours)**

Extended real valued functions, Proper convex functions, Closure of convex functions, Differential derivatives, Subgradients and subdifferentials.

Unit – 2 **(12 hours)**

Conjugate functions, Biconjugate functions, Perturbation functions, Closure of convex functions, Directional derivatives, Subgradients and subdifferentials.

Unit – 3 **(12 hours)**

Gradient descent method, Gradient projection method, Newton's method, Conjugate gradient method.

Unit – 4 **(10 hours)**

Penalty function methods, Exterior penalty function, Interior penalty functions.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. Avriel, *Nonlinear Programming: Analysis and Methods*, Dover Publications, 2003.
[2] O. Güler, *Foundations of Optimization*, Springer, 2010.

Suggested Readings

(i) A. Bagirov, N. Karitsa and M. M. Makela, *Introduction to Nonsmooth Optimization: Theory, Practice and Software*, Springer, 2014.
(ii) M. S. Bazaraa, H. D. Sherali and C. M. Shetty, *Nonlinear Programming: Theory and Algorithms,* Third Edition, John Wiley & Sons, 2013.
(iii) D. G. Luenberger and Y. Ye, *Linear and Nonlinear Programming*, Springer, 2008.

GENERIC ELECTIVE – 2 (ii): PROBABILITY THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
GE-2 (ii): Probability Theory	4	3	1	0	Class XII pass with Mathematics	Basics of Integration

Learning Objectives

The primary objective of this course is to introduce:

- probability space as a measure space and random variables as measurable functions.
- expectation and moments of random variables.
- notion of convergence in probability.
- conditioning on sub-σ-algebra.

Learning Outcomes

This course will enable the students to learn:

- about probability or uncertainty in abstract setting.
- moments and expectation of random variables which help to understand applications of probability in industry.
- how to apply the idea of convergence in probability.
- weak law and strong law of large numbers and their applications.

Syllabus

Unit – 1 **(11 hours)**

Probability: σ-algebra, Constructing probability triples, The extension theorem, Random variables, Independence of events, Continuity of probabilities, Limit events, The Borel–Cantelli Lemma.

Unit – 2 **(10 hours)**

Expected values: Simple, general non-negative and arbitrary random variables, Moment generating functions, Markov's inequality, Chebyshev's inequality.

Unit – 3 **(12 hours)**

Convergence of random variables: Convergence almost surely, Convergence in probability, Weak law of large numbers, Strong law of large numbers.

Unit – 4 **(12 hours)**

Distributions of random variables: Examples of distributions, Characteristic functions, The central limit theorem, Conditional probability, Conditioning on random variable, Conditioning on a sub-σ-algebra, Conditional variance.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. S. Rosenthal, *A First Look at Rigorous Probability Theory*, Second Edition, World Scientific, Singapore, 2006.

Suggested Readings

(i) W. Feller, *An Introduction to Probability Theory and its Applications*, Volume 1, Third Edition, Wiley, 2008.

(ii) J. E. Michael and J. S. Rosenthal, *Probability and Statistics: The Science of Uncertainty*, Second Edition, W. H. Freeman & Co Ltd., 2009.

(iii) S. Ross, *A First Course in Probability*, Tenth Edition, Pearson Education, 2022.

(iv) D. W. Stroock, *Probability Theory, An Analytic View*, Cambridge University Press, 2024.

Syllabi of Courses in Semester-I of One-year M.Sc. Mathematics under Structure-2 (Course work + Research)

Discipline Specific Core (DSC) Courses

DISCIPLINE SPECIFIC CORE – 1: FLUID DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSC-1: Fluid Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Calculus and Partial Differential Equations

Learning Objectives

The objective of this course is to:

- prepare a mathematical foundation to study the motion of fluids.
- develop concepts, models, and techniques to solve problems of fluid flow.
- develop the ability to conduct advanced studies and research in the broad field of fluid dynamics.

Learning Outcomes

After studying this course, the student will be able to:

- understand the concept of fluids, their classification, flow lines, models and approaches to study fluid flow.
- formulate mass and momentum conservation principles and obtain their solution for non-viscous flow.
- know potential flow, Bernoulli's equation, Kelvin's minimum energy and circulation theorems.
- understand two- and three-dimensional motion, complex potential, circle theorem, Blasius theorem, Weiss's and Butler's sphere theorems.
- apply the concept of stress and strain in viscous flow to derive Navier–Stokes equation of motion and energy equation.

Syllabus

Unit – 1 **(10 hours)**

Classification of fluids, Continuum model, Eulerian and Lagrangian approach of description, Differentiation following the fluid motion, Flow lines, vorticity and circulation, Conservation of mass: Equation of continuity, Boundary surface.

Unit – 2 **(12 hours)**

Forces in fluid motion, Conservation of momentum: Euler's equation of motion, Theory of irrotational motion: Integration of Euler's equation under different conditions, Bernoulli's equation, Impulsive motion, Kelvin's minimum energy and circulation theorems, Potential theorem.

Unit – 3 **(13 hours)**

Two-dimensional motion: Complex potential, Line sources, sinks, doublets and vortices, Two-dimensional image system, Milne–Thomson circle theorem, Images with respect to a plane and cylinder, Blasius theorem. Three-dimensional flows, Weiss's sphere theorem, Images with respect to sphere, Axi-symmetric flow, Stokes stream function, Butler's sphere theorem, Flow past spheres and cylinders.

Unit – 4 **(10 hours)**

Stress and strain analysis, Newton's law of viscosity, Laminar flow, Navier–Stokes equation of motion, Steady flow between parallel planes and Poiseuille flow, Constitutive equation, Energy equation.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] F. Chorlton, *Text Book of Fluid Dynamics*, CBS Publisher, 2005.
[2] R. W. Fox, P. J. Pritchard and A. T. McDonald, *Introduction to Fluid Mechanics*, Seventh Edition, John Wiley & Sons, 2009.
[3] P. K. Kundu, I. M. Cohen and D. R. Dowling, *Fluid Mechanics*, Sixth Edition, Academic Press, 2016.

Suggested Readings

(i) L. M. Milne-Thomson, *Theoretical Hydrodynamics*, The Macmillan company, USA, 1969.
(ii) D. E. Rutherford, *Fluid Dynamics*, Oliver and Boyd Ltd., 1978.

DISCIPLINE SPECIFIC CORE – 2: MEASURE AND INTEGRATION

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSC-2: Measure and Integration	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Real Analysis and Riemann Integration

Learning Objectives

The primary objective of this course is to:

- extend the notion of length of an interval with the introduction of the concept of Lebesgue outer measure for any subset of real line.
- investigate the properties of Lebesgue measurable sets and functions.
- familiarize students with the Lebesgue integration of functions and its comparison with Riemann integration.
- generalize the concepts of measure and integration to an abstract space.

Learning Outcomes

This course will enable the students to:

- verify whether a given subset of $\mathbb{R}$ or a real valued function is measurable.
- understand the requirement and the concept of the Lebesgue integral (a generalization of the Riemann integration) along with its properties.
- understand the statements and proofs of the fundamental integral convergence theorems and demonstrate their applications.
- carry out a comprehensive study of functions of bounded variation and their utility in understanding differentiation and integration.
- apply Hölder and Minkowski inequalities in L^p-spaces and understand completeness of L^p-spaces.

Syllabus

Unit – 1 **(14 hours)**

Lebesgue outer measure, Measurable sets, Lebesgue measure, Borel sets, Regularity, Measurable functions, Borel and Lebesgue measurability, Non-measurable sets.

Unit – 2 **(13 hours)**

Integration of nonnegative functions, General integral, Integration of series, Riemann and Lebesgue integrals.

Unit – 3 **(8 hours)**

Functions of bounded variation, Lebesgue's differentiation theorem, Differentiation and integration, Absolute continuity of functions.

Unit – 4 **(10 hours)**

Measures and outer measures, Measure spaces, Integration with respect to a measure, L^p-spaces, Hölder's and Minkowski's inequalities, Completeness of L^p-spaces.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] G. de Barra, *Measure Theory and Integration*, Ellis Horwood Ltd., Chichester, John Wiley & Sons, Inc., New York, 1981 (Indian Reprint, 2014).

Suggested Readings

(i) M. Capinski and P. E. Kopp, *Measure, Integral and Probability*, Springer, 2005.
(ii) E. Hewitt and K. Stromberg, *Real and Abstract Analysis: A Modern Treatment of the Theory of Functions of a Real Variable*, Springer, Berlin, 1975.
(iii) H. L. Royden and P.M. Fitzpatrick, *Real Analysis*, Fourth Edition, Pearson, 2015.

Discipline Specific Elective (DSE) Courses

DSE-1 and DSE-2

Group-1

DISCIPLINE SPECIFIC ELECTIVE: COMMUTATIVE ALGEBRA

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Commutative Algebra	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra and Field Theory

Learning Objectives

The objective of this course is to:

- develop a solid understanding of the structure of commutative rings, ideals, their radicals, extension, contraction etc.
- study important constructions such as total quotient rings, localizations.
- develop basic foundation in other areas of mathematics such as algebraic geometry, algebraic number theory.

Learning Outcomes

This course will enable the students to:

- know the localization of rings at a prime ideal that is an algebraic analogue of the geometric notion concentrating attention near a point.
- know more closely the polynomial rings, power series rings in one or more variables over a commutative ring and their prime spectrum.
- define, identify, and elaborate integral closure of rings, valuations rings, discrete valuation rings, structure theorem of Artin rings.

Syllabus

Unit – 1 **(12 hours)**

Radical of an ideal, Prime avoidance lemma, Chinese remainder theorem, Extension and contraction of ideals, Jacobson radical of a ring, Nakayama lemma, Tensor product of modules.

Unit – 2 **(13 hours)**

Rings and modules of fractions, Localization, Local properties, Primary decomposition, First and second uniqueness theorem of primary decomposition, Associated prime ideals of decomposable ideals.

Unit – 3 **(10 hours)**

Integral ring extensions, Going up theorem, Going down theorem, Integrally closed domains, Valuation rings, Hilbert's Nullstellensatz theorem.

Unit – 4 **(10 hours)**
Noetherian rings, Primary decomposition in Noetherian rings, Artin rings, Structure theorem for Artin rings, Discrete valuation rings.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] M. F. Atiyah and I. G. MacDonald, *Introduction to Commutative Algebra*, CRC Press, Taylor & Francis, 2018.

Suggested Readings
(i) D. Eisenbud, *Commutative Algebra with a View Towards Algebraic Geometry*, Springer, 2004.
(ii) R. Y. Sharp, *Steps in Commutative Algebra*, Cambridge University Press, 2000.
(iii) B. Singh, *Basic Commutative Algebra*, World Scientific, 2011.
(iv) O. Zariski and P. Samuel, *Commutative Algebra*, Volume I & II, Springer, 1975.

DISCIPLINE SPECIFIC ELECTIVE: DYNAMICAL SYSTEMS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Dynamical Systems	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology and Ordinary Differential Equations

Learning Objectives

The primary objective of this course is to:

- understand discrete and continuous systems with case studies to study nonlinear systems of ordinary differential equations and dynamical systems.
- understand the concepts, models and techniques to realize the real-world problems and stability of the systems along with the chaotic dynamic behaviour of models by understanding bifurcations.

Learning Outcomes

This course will enable the students to learn:

- formulation of mathematical models with the stability analysis near the equilibrium points.
- how the concept of phase portraits helps to analyse mathematical model graphically.
- the qualitative behaviour of the solution set of a given system of differential equations including the invariant sets and limiting behaviour of the dynamical system or flow defined by the system of differential equations.
- how different bifurcations explain the chaotic behaviour of the system.

Syllabus

Unit – 1 **(13 hours)**

Linear systems: Jordan forms, Stability theory; Nonlinear systems: Fundamental existence-uniqueness theorem, Dependence on initial conditions and parameters, Flow of a differential equation, Linearization, Stable manifold theorem, Hartman–Grobman theorem.

Unit – 2 **(10 hours)**

Stability and Lyapunov functions, Saddle points, Nodes, Foci, Centers and nonhyperbolic critical points, Center manifold theorem.

Unit – 3 **(12 hours)**

Limit sets and attractors, Periodic orbits and limit cycles, Poincaré map, Stable manifold theorem for periodic orbits, Poincare-Bendixson theorem.

Unit – 4 **(10 hours)**

Bifurcations at nonhyperbolic equilibrium points, Saddle node, Transcritical and Pitchfork bifurcations, Hopf bifurcation.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. W. Hirsch, S. Smale and R. L. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, Third Edition, Academic Press, 2013.
[2] L. Perko, *Differential Equations and Dynamical Systems*, Third Edition, Springer Verlag, 2001.

Suggested Readings

(i) R. L. Devaney, *A First Course in Chaotic Dynamical Systems: Theory and Experiment*, CRC Press, Taylor & Francis, 2018.
(ii) S. H. Strogatz, *Nonlinear Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 2018.
(iii) S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, TAM Volume 2, Springer-Verlag, NY, 1990.

DISCIPLINE SPECIFIC ELECTIVE: INTRODUCTION TO TOPOLOGY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Introduction to Topology	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Metric Spaces

Learning Objectives
The primary objective of this course is to introduce:
- basic principles of point-set topology, including bases and subbases for a topology.
- continuity, homeomorphisms, and different types of topologies, such as product and box topologies.
- key notions of connectedness and local connectedness.
- compactness and its significance in topological spaces.

Learning Outcomes
This course will enable the students to:
- analyze subsets of topological spaces by determining their interior, closure, boundary, and limit points, as well as identifying bases and subbases.
- identify continuous functions between topological spaces, analyze mappings into product spaces, and compare topological properties of different spaces.
- evaluate the connectedness and path connectedness of the product of an arbitrary family of spaces.
- understand key classifications of topological spaces, including Hausdorff spaces, first and second countable spaces, and separable spaces.
- explore advanced concepts such as limit point compactness and Tychonoff's theorem.

Syllabus

Unit – 1 **(10 hours)**
Topological spaces, Basis, Order topology, Subspace topology, Metric topology, Closed set and limit points, Hausdorff spaces.

Unit – 2 **(12 hours)**
Continuous functions, Homeomorphism, The box and product topologies, Metrizability of products of metric spaces, Connected and path connected spaces.

Unit – 3 **(12 hours)**
Locally connected and locally path connected spaces, Connectedness of product of spaces, First and second countable spaces, Separable spaces.

Unit – 4 **(11 hours)**
Compact spaces, The Tychonoff theorem, Limit point compactness, Sequential compactness.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. R. Munkres, *Topology*, Updated Second Edition, Pearson, 2021.
[2] T. B. Singh, *Introduction to Topology*, Springer Nature, 2019.

Suggested Readings

(i) G. E. Bredon, *Topology and Geometry*, Springer, 2014.
(ii) J. Dugundji, *Topology*, Allyn and Bacon Inc., Boston, 1978.
(iii) J. L. Kelley, *General Topology*, Dover Publications, 2017.
(iv) G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill Education, 2017.
(v) L. A. Steen and J. A. Seebach, *Counterexamples in Topology*, Dover Publications, 2013.
(vi) S. Willard, *General Topology*, Dover Publications, 2004.

DISCIPLINE SPECIFIC ELECTIVE: THEORY OF BOUNDED OPERATORS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Theory of Bounded Operators	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis

Learning Objectives
The primary objective of this course is to:
- introduce some classes of bounded linear operators which play a central role in both pure and applied mathematics.
- study the properties and spectral theory of these operators.

Learning Outcomes
This course will enable the students to understand:
- the spectrum and sub-divisions of spectrum of standard operators like shifts and multiplication.
- the spectral theorem for some classes of bounded linear operators.
- the concepts of compactness, self-adjointness and positivity of bounded linear operators.
- trace class and Hilbert–Schmidt operators.

Syllabus

Unit – 1 **(11 hours)**
Properties of spectrum and resolvent of bounded operators, Subdivision of the spectrum including point, approximate and compression spectrum.

Unit – 2 **(10 hours)**
Operators on Hilbert spaces, Adjoint operator, Projections and idempotents, Operations with projections, Invariant and reducing subspaces.

Unit – 3 **(14 hours)**
Compact operators on Hilbert spaces, Diagonalisation of compact self-adjoint operators, Spectral theorem and functional calculus for Compact normal operators, Positive operators, Compact operators on Banach spaces, Spectral theory of compact operators.

Unit – 4 **(10 hours)**
Polar decomposition, Singular values, Trace class operators, Trace norm and Hilbert Schmidt operators.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] R. Bhatia, *Notes on Functional Analysis*, Hindustan Book Agency, 2009.
[2] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.

Suggested Readings
(i) E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, 2006.
(ii) B. Simon, *Operator Theory: A Comprehensive Course in Analysis*, Part 4, American Mathematical Society, 2015.
(iii) S. R. Garcia, J. Mashregi and W. T. Ross, *Operator Theory by Example*, Oxford University Press, 2023.

Group-2

DISCIPLINE SPECIFIC ELECTIVE: NUMERICAL METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Numerical Methods for Ordinary Differential Equations	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Ordinary Differential Equations

Learning Objectives

The primary objective of this course is to:

- develop the basic theory underlying the numerical solution of differential equations.
- introduce the concepts of consistency, stability and convergence of finite difference methods.
- execute the numerical schemes for the solution of differential equations.

Learning Outcomes

This course will enable the students to:

- gain a thorough understanding of the fundamental concepts involved in the construction and analysis of finite difference schemes for solving ordinary differential equations (ODEs).
- apply various numerical methods based on finite difference approaches to obtain approximate solutions for both initial value problems (IVPs) and boundary value problems (BVPs).
- develop the ability to select appropriate finite difference methods for specific types of problems and effectively apply them to real world applications.

Syllabus

Unit – 1 **(11 hours)**

Initial value problems: Existence and uniqueness of solution, Finite difference equation, Truncation error, Single step methods for first order IVPs and system of IVPs- Family of explicit and implicit Runge–Kutta methods, Taylor series methods, Derivation, Truncation error, Consistency, Stability and convergence analysis.

Unit – 2 **(12 hours)**

IVPs for the system of ODEs, Consistency, Zero stability and convergence of linear multistep methods, Routh–Hurwitz criterion, Order and error constant, First Dahlquist Barrier, Local truncation error and global truncation error, Error bounds, Local error, Linear stability theory, Higher order differential equations.

Unit – 3 **(12 hours)**

Derivation of explicit and implicit multistep methods for IVPs and system of IVPs, Truncation error, Stability and convergence analysis of family of Nystrom method, Adams–Bashforth method, Adams–Moulton method, Milne–Simpson method, Predictor corrector method, and Modified predictor corrector method, Hybrid method, Multistep methods for second order IVPs.

Unit – 4 **(10 hours)**

Linear BVPs for second order ordinary differential equations, Shooting method, Finite difference method, Collocation method, Derivative boundary conditions, Nonlinear two-point BVPs, Higher order finite difference methods, Stability, Truncation error and convergence analysis.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. K. Jain, S. R. K. Iyenger and R. K. Jain, *Numerical Methods for Scientific and Engineering Computations*, Seventh Edition, New Age International Publisher, 2019.

[2] J. D. Lambert, *Numerical Methods for Ordinary Differential Systems: The Initial Value Problem*, John Wiley & Sons, 1991.

Suggested Readings

(i) K. E. Atkinson, W. Han and D. E. Stewart, *Numerical Solution of Ordinary Differential Equations*, John Wiley & Sons, 2009.

(ii) J. C. Butcher, *The Numerical Analysis of Ordinary Differential Equations*, Second Edition, Wiley, New York, 2008.

(iii) L. Collatz, *The Numerical Treatment of Differential Equations*, Springer-Verlag, 1966.

DISCIPLINE SPECIFIC ELECTIVE: ORDINARY DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Ordinary Differential Equations	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Differential Equations and Calculus

Learning Objectives

The objective of this course is to study:

- existence, uniqueness, and continuity of solutions of initial value problems (IVPs)
- homogeneous and non-homogeneous linear systems
- stability of solutions for systems of ordinary differential equations.
- eigenvalues and eigenfunctions of Sturm-Liouville systems and Green's functions
- applications of theory of ordinary differential equations in real world problems.

Learning Outcomes

After studying this course, the student will be able to:

- know about the existence, uniqueness, and continuity of solutions of IVPs.
- apply the matrix method of solution for linear systems of differential equations.
- analyze the stability of solutions for systems of ordinary differential equations.
- understand Green's functions and their applications in the solution of boundary value problems (BVPs).
- comprehend the properties of eigenvalues and eigenfunctions of Sturm-Liouville systems.

Syllabus

Unit – 1 **(12 hours)**

Well-posed problems, Existence, uniqueness, and continuity theorems for the solution of IVPs of the first order, Picard's method, Existence and uniqueness of solution for systems and higher order IVPs , Global existence theorem.

Unit – 2 **(9 hours)**

Homogeneous and non-homogeneous linear systems, Linear systems with constant coefficients and their solution by matrix method, Linear equations with periodic coefficients.

Unit – 3 **(12 hours)**

Stability of autonomous system of differential equations, Critical points of an autonomous system and their classification. Stability of linear systems with constant coefficients, Linear plane autonomous system and phase portrait analysis, Perturbed systems, Method of Lyapunov for nonlinear systems, Limit cycles, Poincare-Bendixson's theorem and its applications.

Unit – 4 **(12 hours)**

Sturm separation and comparison theorems, Adjoint forms and Lagrange's identity, Two-point boundary value problems, Green's functions, Construction of Green's functions, Sturm-Liouville systems, eigenvalues and eigenfunctions.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] E. A. Coddington, *An Introduction to Ordinary Differential Equations*, Dover Publications, 2012.
[2] T. Myint-U, *Ordinary Differential Equations*, Elsevier, North-Holland, 1978.
[3] S. L. Ross, *Differential Equation*s, Second Edition, John Wiley & Sons, India, 2007.

Suggested Readings

(i) L. Perko, *Differential Equations and Dynamical Systems,* Springer, 2001.
(ii) G. F. Simmons, *Differential Equations with Applications and Historical Notes*, Third Edition, CRC Press, 2017.

DISCIPLINE SPECIFIC ELECTIVE: REPRESENTATION OF FINITE GROUPS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Representation of Finite Groups	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra and Group Theory

Learning Objectives

The primary objective of this course is to:

- represent finite groups as groups of matrices (via homomorphisms) and apply the tools of linear algebra to study the group structure.
- introduce the notion of Group algebra, which plays an essential role in classifying representations of groups.
- to discuss some applications of representations of finite groups, such as the Burnside's theorem.

Learning Outcomes

This course will enable the students to:

- define and construct examples of group representations, *FG*-modules, group algebras.
- grasp key concepts and tools of representation theory and establish a link between *FG*-modules and group representations.
- prove and apply Maschke's theorem and Schur's lemma to describe all irreducible representations of finite groups over the field of complex numbers.
- apply the theory of characters and group representations to gain insight into group structure, such as normal subgroups, and the solubility of groups.

Syllabus

Unit – 1 **(11 hours)**

Representation of groups, *FG*-modules and *FG*-submodules, and reducibility, Permutation modules, *FG*-modules and equivalent representations, Reducible and irreducible *FG*-modules, Group algebra of *G*, Regular *FG*-module and regular representations, *FG*-homomorphisms, Direct sum of *FG*-modules.

Unit – 2 **(11 hours)**

Maschke's theorem for *FG*-modules and consequences. Schur's lemma and its converse, Application of Schur's lemma, Irreducible modules and group algebra, Structure of group algebra and space of *CG*-homomorphisms.

Unit – 3 **(10 hours)**

Characters and their properties, Permutation and regular characters, Inner product, Number of irreducible characters, Orthogonality relations and finding normal subgroups.

Unit – 4 **(13 hours)**
Algebraic numbers, Algebraic integers and their properties, Character values, The Burnside's *(p,q)*-theorem and solubility of some particular groups.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] G. James and M. Liebeck, *Representations and Characters of Groups*, Second Edition, Cambridge University Press, 2005.

Suggested Readings
(i) C. W. Curtis and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras*, American Mathematical Society, 2006.
(ii) W. Fulton and J. Harris, *Representation Theory - A First Course*, Springer-Verlag, 2004.
(iii) I. M. Issacs, *Character Theory of Finite Groups*, American Mathematical Society reprint, 2006.

DISCIPLINE SPECIFIC ELECTIVE: TOPOLOGICAL DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Topological Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- provide a strong background of topological dynamical systems including their applications.
- develop some useful and interesting dynamical properties like expansivity, shadowing and topological stability with supporting examples and results from symbolic and topological dynamics.
- introduce the celebrated Sarkovskii's theorem.

Learning Outcomes

This course will enable the students to:

- construct interesting examples of dynamical systems and topological conjugacy.
- visualize stable sets, omega sets and alpha limit sets.
- understand the applications of Sarkovskii's theorem.
- use subshifts of finite type to characterize irreducible matrices.
- prove key results on expansivity and shadowing regarding existence/non-existence, product, subspace and their different characterizations etc.
- find the class of topologically stable homeomorphisms.

Syllabus

Unit – 1 **(10 hours)**

Definition and examples (including real life examples) of dynamical systems, Orbits, Types of orbits, Topological conjugacy and orbits, Phase portrait-graphical analysis of orbit, Periodic points and stable sets, Omega and alpha limit sets and their properties.

Unit – 2 **(10 hours)**

Sarkovskii's theorem, Shift spaces and subshift, Subshift of finite type, Subshift represented by a matrix, Characterizations of irreducible matrices.

Unit – 3 **(13 hours)**

Definition and examples of expansive homeomorphisms, Properties of expansive homeomorphisms, Non-existence of expansive homeomorphism on the unit interval and unit circle, Generators and weak generators, Generators and expansive homeomorphisms.

Unit – 4 **(12 hours)**

Converging semi-orbits for expansive homeomorphisms, Definition, examples and properties of maps having shadowing property, Topological Anosov homeomorphisms and topological stability.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] N. Aoki and K. Hiraide, *Topological Theory of Dynamical Systems: Recent Advances*, North Holland Publications, 1994.

[2] M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge University Press, 2004.

Suggested Readings

(i) D. C. Hanselman and B. Little field, *Mastering MATLAB*, Pearson, 2012.

(ii) D. Lind and B. Marcus, *An Introduction to Symbolic Dynamics and Coding*, Cambridge University Press, 1996.

(iii) C. Robinson, *Dynamical Systems*, *Stability, Symbolic Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 1998.

(iv) J. de. Vries, *Elements of Topological Dynamics*, Springer, 1993.

Group-3

DISCIPLINE SPECIFIC ELECTIVE: ADVANCED FUNCTIONAL ANALYSIS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Advanced Functional Analysis	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis and Topology

Learning Objectives
The primary objective of this course is to:
- define and explain the structure of a topological vector space and its fundamental properties.
- differentiate between normed, metrizable, locally convex and Hausdorff topological vector spaces.
- introduce the foundational theorems of functional analysis, including the Hahn–Banach, Banach–Steinhaus, Open mapping, and Closed graph theorems in the context of locally convex spaces.
- explain some applications of Banach–Alaoglu theorem and Krein–Milman theorem.

Learning Outcomes
This course will enable the students to:
- appreciate types of topological vector spaces and their separation properties.
- understand quotient spaces, weak topology and weak*-topology.
- analyze concepts of continuity, boundedness, and convergence for linear operators and functionals on topological vector spaces.
- understand the notion of local convexity and the role of seminorms in defining locally convex topologies.

Syllabus

Unit – 1 **(12 hours)**
Topological vector spaces, Types of Topological vector spaces, Separation properties, Linear mappings, Finite dimensional spaces, Metrization, Boundedness and continuity, Seminorms and local convexity, Normability.

Unit – 2 **(11 hours)**
Quotient spaces, Seminorms and quotient spaces, Examples, Baire category theorem, Banach–Steinhaus theorem, The open mapping theorem and the closed graph theorem on topological vector spaces.

Unit – 3 **(11 hours)**
Hahn–Banach separation theorem on topological vector spaces, Continuous extension theorem,

Weak topologies, Weak topology and convexity, Weak topology and metrizability, Weak*-topology of a dual space, Compact convex sets, Banach–Alaoglu theorem and applications, Goldstine theorem.

Unit – 4 **(11 hours)**

Extreme points, Krein–Milman theorem, Convex hull of compact sets, Applications of Krein–Milman theorem: Stone–Weierstrass theorem, Markov–Kakutani fixed point theorem.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.
[2] W. Rudin, *Functional Analysis*, Second Edition, Tata Mc Graw-Hill, 2011.

Suggested Readings

(i) V. I. Bogachev and O. G. Smolyanov, *Topological Vector Spaces and Their Applications*, Springer, 2017.
(ii) S. Kantorovitz, *Introduction to Modern Analysis*, Oxford Graduate Texts in Mathematics, Oxford University Press, 2006.
(iii) J. Voigt, *A Course on Topological Vector Spaces*, Birkhäuser, 2020.

DISCIPLINE SPECIFIC ELECTIVE: ALGEBRAIC CODING THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Algebraic Coding Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra, Groups and Rings

Learning Objectives
The primary objective of this course is to:
- provide an introduction to algebraic coding theory, particularly linear codes.
- discuss bounds on the parameters along with cyclic codes.
- describe some well-known codes, such as Reed–Muller and Golay codes.
- explore the algebraic structure of Cyclic and Quadratic residue codes over fields and rings.

Learning Outcomes
This course will enable the students to:
- get an insight into the matrix representation of a code, as well as encoding and decoding.
- understand Hamming, MDS and Reed–Muller codes.
- describe cyclic codes and their generator polynomial.
- learn about special cyclic codes, such as Quadratic residue codes, and their properties over the ring $\mathbb{Z}_4$.

Syllabus

Unit – 1 **(10 hours)**
Error detecting and error correcting codes, Maximum likelihood decoding, Hamming distance, Linear codes, Hamming weight, Generator matrix, Parity check matrix, Equivalence of linear codes, Encoding and decoding of linear codes, Syndrome decoding.

Unit – 2 **(11 hours)**
Bounds on codes, Sphere covering bound, Hamming bound, Perfect codes, Binary Hamming codes, Binary Golay codes, Singleton bound and MDS codes. Propagation rules, Reed–Muller codes.

Unit – 3 **(12 hours)**
Cyclic codes, Cyclic codes as ideals, Generator polynomial of cyclic codes, Generator and parity-check matrices of cyclic codes, Decoding of cyclic codes, Burst error correcting codes.

Unit – 4 **(12 hours)**
Quadratic residue codes: QR codes over fields of characteristic 2 and 3, Cyclic codes and their generating polynomial over $\mathbb{Z}_4$, QR codes over $\mathbb{Z}_4$.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] S. Ling and C. Xing, *Coding Theory: A First Course*, Cambridge University Press, 2004.
[2] W. C. Huffman and V. Pless, *Fundamentals of Error Correcting Codes*, Cambridge University Press, 2010.

Suggested Readings

(i) R. Hill, *A First Course in Coding Theory*, Oxford University Press, 1986.
(ii) F. J. Mac William and N. J. A. Sloane, *Theory of Error Correcting Codes, Part I & II,* Elsevier/North-Holland, Amsterdam, 1977.

DISCIPLINE SPECIFIC ELECTIVE: DIFFERENTIAL GEOMETRY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Differential Geometry	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra, Multivariate Calculus and Differential Equations

Learning Objectives
The primary objective of this course is to introduce:
- surfaces and parametrized surfaces.
- orientation on connected surfaces.
- geodesics on surfaces.
- Weingarten maps on oriented surfaces.
- arc length and curvature of oriented plane curves.
- curvatures of oriented surfaces.

Learning Outcomes
This course will enable the students to:
- understand the concepts of level sets and graphs of functions, smooth vector fields, tangent spaces of level sets.
- appreciate surfaces and parametrized surfaces, Gauss map, geodesics and parallel transport on oriented surfaces.
- know what the Weingarten map of an oriented surface is, realize it as shape operator and use it to compute curvature of oriented plane curves.
- find global parametrization and hence arc length of an oriented plane curve.
- compute various types of curvatures of surfaces.

Syllabus

Unit – 1 **(10 hours)**
Level sets in $\mathbb{R}^{n+1}$ and graphs of functions, Smooth vector fields and existence and uniqueness of their integral curves, Tangent spaces of level sets at regular points, Surfaces in $\mathbb{R}^{n+1}$ as inverse images of regular values of smooth functions, Necessary condition for extrema of functions on surfaces-Lagrange multipliers, Existence of a normal vector field on a connected surface, Orientation, Gauss map.

Unit – 2 **(13 hours)**
The notion of a geodesic on a surface, Existence and uniqueness of a geodesic on a surface through a given point with a given velocity vector thereof, Covariant derivative of a smooth vector field, Parallel vector field along a curve, Existence and uniqueness of a parallel vector field along a curve with a given initial vector, Weingarten map of a surface at a point, Local parametrization and curvature of a plane curve.

Unit – 3 **(10 hours)**
Global parametrization and arc length of an oriented plane curve, Differential 1-forms, Line integral of 1-forms over parametrized curves.

Unit – 4 **(12 hours)**
Parametrized surfaces with examples, Curvature of surfaces, Normal curvature of a surface at a point in a given direction, Principal curvatures, First and second fundamental forms, Gauss-Kronecker curvature and mean curvature.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] A. Pressley, *Elementary Differential Geometry*, Springer-Verlag London Limited, 2012.
[2] J. A. Thorpe, *Elementary Topics in Differential Geometry*, Springer (India) Pvt. Limited, 2004.

Suggested Readings
(i) W. Kuhnel, *Differential Geometry: Curves-Surfaces-Manifolds*, Third Edition, American Mathematical Society, 2015.
(ii) B. O' Neill, *Elementary Differential Geometry*, Second Edition, Academic Press INC., Academic Press, New York, 2006.

DISCIPLINE SPECIFIC ELECTIVE: FINITE ELEMENT METHODS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Finite Element Methods	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Differential Equations

Learning Objectives

The primary objective of this course is to:

- introduce basic aspects of finite element methods (FEM) including domain discretization, polynomial interpolation, application of boundary conditions, assembly of global arrays, and solution of the resulting algebraic systems.
- discuss the use of finite element methods in solving engineering problems in the domain of solid mechanics, fluid mechanics, heat transfer and electromagnetism.

Learning Outcomes

This course will enable the students to:

- use integral statement to deduce finite element approximations for the underlying linear partial differential equations.
- write special-purpose finite element programs within a procedural programming environment.
- use finite element methods to solve engineering problems in solids mechanics, fluid mechanics, heat transfer, and electromagnetism.
- assess the accuracy and reliability of finite element solutions and troubleshoot problems arising from errors in a given finite element analysis.

Syllabus

Unit – 1 **(12 hours)**

Basic concepts of weak formulation, Variational formulation of a one dimensional model equation, Basis function and finite element solutions, Collocation method, Ritz method, Least square method, Standard Galerkin method, FEM for model problem, Error estimate for FEM for model equation, Convergence analysis.

Unit – 2 **(11 hours)**

Various shapes of finite element, Higher order basis functions, Finite element methods for elliptic problems: Variational methods, Standard Galerkin method, Error estimate for FEM for elliptic problem, FEM for Poisson equation.

Unit – 3 **(12 hours)**

Finite element methods for parabolic problems: One dimensional model problems, Semi-discretization in space, Error estimates, Discretization in space and time, Galerkin method, Finite element methods for hyperbolic problems: Standard Galerkin method, Standard Galerkin method with strongly and weakly imposed boundary conditions.

Unit – 4 **(10 hours)**

Applications of the FEM to second order BVPs in one dimension, Applications of the FEM to linear elliptic, parabolic and hyperbolic equations.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] G. Evans, J. Blackledge and P. Yardley, *Numerical Methods for Partial Differential Equations*, Springer-Verlag, London, 2000.

[2] C. Johnson, *Numerical Solutions of Partial Differential Equations by Finite Element Methods*, *Cambridge University Press*, Cambridge, 1987.

[3] J. Whiteley, *Finite Element Methods - A Practical Guide*, Springer, 2016.

Generic Elective (GE) Courses

GENERIC ELECTIVE – 1 (i): DYNAMICAL SYSTEMS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
GE-1 (i): Dynamical Systems	4	3	1	0	Class XII pass with Mathematics	Basics of Topology and Ordinary Differential Equations

Learning Objectives

The primary objective of this course is to:

- understand discrete and continuous systems with case studies to study nonlinear systems of ordinary differential equations and dynamical systems.
- understand the concepts, models and techniques to realize the real-world problems and stability of the systems along with the chaotic dynamic behaviour of models by understanding bifurcations.

Learning Outcomes

This course will enable the students to learn:

- formulation of mathematical models with the stability analysis near the equilibrium points.
- how the concept of phase portraits helps to analyse mathematical model graphically.
- the qualitative behaviour of the solution set of a given system of differential equations including the invariant sets and limiting behaviour of the dynamical system or flow defined by the system of differential equations.
- how different bifurcations explain the chaotic behaviour of the system.

Syllabus

Unit – 1 **(13 hours)**

Linear systems: Jordan forms, Stability theory; Nonlinear systems: Fundamental existence-uniqueness theorem, Dependence on initial conditions and parameters, Flow of a differential equation, Linearization, Stable manifold theorem, Hartman–Grobman theorem.

Unit – 2 **(10 hours)**

Stability and Lyapunov functions, Saddle points, Nodes, Foci, Centers and nonhyperbolic critical points, Center manifold theorem.

Unit – 3 **(12 hours)**

Limit sets and attractors, Periodic orbits and limit cycles, Poincaré map, Stable manifold theorem for periodic orbits, Poincare-Bendixson theorem.

Unit – 4 **(10 hours)**

Bifurcations at nonhyperbolic equilibrium points, Saddle node, Transcritical and Pitchfork bifurcations, Hopf bifurcation.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. W. Hirsch, S. Smale and R. L. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, Third Edition, Academic Press, 2013.
[2] L. Perko, *Differential Equations and Dynamical Systems*, Third Edition, Springer Verlag, 2001.

Suggested Readings

(i) R. L. Devaney, *A First Course in Chaotic Dynamical Systems: Theory and Experiment*, CRC Press, Taylor & Francis, 2018.
(ii) S. H. Strogatz, *Nonlinear Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 2018.
(iii) S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, TAM Volume 2, Springer-Verlag, NY, 1990.

GENERIC ELECTIVE – 1 (ii): NUMERICAL METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
GE-1 (ii): Numerical Methods for Ordinary Differential Equations	4	3	1	0	Class XII pass with Mathematics	Basics of Ordinary Differential Equations

Learning Objectives

The primary objective of this course is to:

- develop the basic theory underlying the numerical solution of differential equations.
- introduce the concepts of consistency, stability and convergence of finite difference methods.
- execute the numerical schemes for the solution of differential equations.

Learning Outcomes

This course will enable the students to:

- gain a thorough understanding of the fundamental concepts involved in the construction and analysis of finite difference schemes for solving ordinary differential equations (ODEs).
- apply various numerical methods based on finite difference approaches to obtain approximate solutions for both initial value problems (IVPs) and boundary value problems (BVPs).
- develop the ability to select appropriate finite difference methods for specific types of problems and effectively apply them to real world applications.

Syllabus

Unit – 1 **(11 hours)**

Initial value problems: Existence and uniqueness of solution, Finite difference equation, Truncation error, Single step methods for first order IVPs and system of IVPs- Family of explicit and implicit Runge–Kutta methods, Taylor series methods, Derivation, Truncation error, Consistency, Stability and convergence analysis.

Unit – 2 **(12 hours)**

IVPs for the system of ODEs, Consistency, Zero stability and convergence of linear multistep methods, Routh–Hurwitz criterion, Order and error constant, First Dahlquist Barrier, Local truncation error and global truncation error, Error bounds, Local error, Linear stability theory, Higher order differential equations.

Unit – 3 **(12 hours)**

Derivation of explicit and implicit multistep methods for IVPs and system of IVPs, Truncation error, Stability and convergence analysis of family of Nystrom method, Adams–Bashforth method,

Adams–Moulton method, Milne–Simpson method, Predictor corrector method, and Modified predictor corrector method, Hybrid method, Multistep methods for second order IVPs.

Unit – 4 **(10 hours)**

Linear BVPs for second order ordinary differential equations, Shooting method, Finite difference method, Collocation method, Derivative boundary conditions, Nonlinear two-point BVPs, Higher order finite difference methods, Stability, Truncation error and convergence analysis.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. K. Jain, S. R. K. Iyenger and R. K. Jain, *Numerical Methods for Scientific and Engineering Computations*, Seventh Edition, New Age International Publisher, 2019.

[2] J. D. Lambert, *Numerical Methods for Ordinary Differential Systems: The Initial Value Problem*, John Wiley & Sons, 1991.

Suggested Readings

(i) K. E. Atkinson, W. Han and D. E. Stewart, *Numerical Solution of Ordinary Differential Equations*, John Wiley & Sons, 2009.

(ii) J. C. Butcher, *The Numerical Analysis of Ordinary Differential Equations*, Second Edition, Wiley, New York, 2008.

(iii) L. Collatz, *The Numerical Treatment of Differential Equations*, Springer-Verlag, 1966.

Syllabi of Courses in Semester-II of One-year M.Sc. Mathematics under Structure-2 (Course work + Research)

Discipline Specific Core (DSC) Courses

DISCIPLINE SPECIFIC CORE – 3: PARTIAL DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSC-3: Partial Differential Equations	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Multivariate Calculus and Differential Equations

Learning Objectives

The main objective of this course is to introduce:

- well-posedness, fundamental solutions, existence and uniqueness of solutions for Laplace equation, Poisson equation and heat equation.
- solution for wave equation by spherical means.
- characteristics, complete integrals, envelopes and conservation laws for first-order nonlinear partial differential equations.
- classical solution techniques such as Green's function, similarity solutions and transform methods.

Learning Outcomes

This course will enable the students to:

- understand Laplace equation, Poisson equation, and Heat equation, their fundamental solutions, uniqueness principles, mean value properties, and Green's function.
- apply the method of spherical means to solve homogeneous and nonhomogeneous wave equations.
- use characteristics to solve nonlinear partial differential equations, construct complete integrals and envelopes, and understand conservation laws.
- implement various techniques such as similarity solutions and transform methods to derive solutions of different types of partial differential equations.

Syllabus

Unit – 1 **(12 hours)**

Well-posed problems, Classical solution, Laplace equation, Poisson equation, Fundamental solution, Strong maximum principle and uniqueness of solution, Mean value formulas, Representation formula, Green's function, Poisson's formula.

Unit – 2 **(10 hours)**

Heat equation, Fundamental solution for homogeneous and nonhomogeneous initial-value problems, Mean value formula, Strong maximum principle and uniqueness of solution, Local estimates for the solution.

Unit – 3 **(13 hours)**

Wave equation: Solution of homogeneous and nonhomogeneous problems by spherical means,

Nonlinear first order partial differential equations: Complete integrals and envelopes, Characteristics, Introduction to conservation laws.

Unit – 4 **(10 hours)**
Other solution methods: Similarity solutions, Fourier transform and Laplace transform, Cole–Hopf transformation, Potential function, Hodograph and Legendre transform.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] L. C. Evans, *Partial Differential Equations*, American Mathematical Society, Providence, RI, 1998.
[2] F. John, *Partial Differential Equations*, Fourth Edition, Springer-Verlag, New York, 1982.

Suggested Readings
(i) P. R. Garabedian, *Partial Differential Equations*, John Wiley & Sons, Inc., New York- London-Sydney, 1964.
(ii) A. K. Nandakumaran and P. S. Datti, *Partial Differential Equations: Classical Theory with a Modern Touch*, Cambridge University Press, 2020.

DISCIPLINE SPECIFIC CORE – 4: ANALYSIS OF SEVERAL VARIABLES

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSC-4: Analysis of Several Variables	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Calculus, Real Analysis including Riemann Integration

Learning Objectives

The primary objective of this course is to:

- introduce differentiation of vector valued functions on $\mathbb{R}^n$ and their properties.
- familiarize students with integration of functions over rectangles and bounded sets in $\mathbb{R}^n$.
- extend integration of functions to unbounded sets in $\mathbb{R}^n$.
- study change of variables and its applications.

Learning Outcomes

This course will enable the students to:

- check differentiability of vector valued functions on $\mathbb{R}^n$, understand the relation between directional derivative and differentiability, apply chain rule, mean value theorems, inverse and implicit function theorems.
- understand higher order derivatives and be able to apply Taylor's formulas with integral remainder, Lagrange's remainder and Peano's remainder.
- master the concepts of integration over rectangles and bounded sets in $\mathbb{R}^n$.
- generalize the integration theory to unbounded sets in $\mathbb{R}^n$.
- grasp the effect of change of variables in integration.

Syllabus

Unit– 1 **(12 hours)**

The differentiability of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$, Partial derivatives and differentiability, Directional derivatives and differentiability, Chain rule, Mean value theorems, Inverse function theorem and Implicit function theorem.

Unit– 2 **(11 hours)**

Derivatives of higher order, Taylor's formulas with integral remainder, Lagrange's remainder and Peano's remainder, Integral over a rectangle, Existence of the integral.

Unit– 3 **(10 hours)**

Evaluation of the integral, Fubini's theorem, Integral over a bounded set.

Unit– 4 **(12 hours)**

Rectifiable sets, Improper integrals, Change of variable theorem, Applications of change of variables.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. Giaquinta and G. Modica, *Mathematical Analysis: An Introduction to Functions of Several Variables*, Birkhäuser, 2009.
[2] J. R. Munkres, *Analysis on Manifolds*, CRC Press, Taylor & Francis, 2018.

Suggested Readings

(i) W. Rudin, *Principles of Mathematical Analysis*, Third Edition, Mc Graw Hill, 1986.
(ii) M. Spivak, *Calculus on Manifolds: A Modern Approach to Classical Theorems of Advanced Calculus*, Taylor & Francis, 2018.

Discipline Specific Elective (DSE) Courses

DSE-3 and DSE-4

Group-1

DISCIPLINE SPECIFIC ELECTIVE: ADVANCED FLUID DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Advanced Fluid Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Calculus and Partial Differential Equations

Learning Objectives

The primary objective of this course is to:

- prepare a foundation for advanced studies in compressible flow, boundary layer theory and magnetohydrodynamics.
- develop concepts, models, and techniques that enable problem-solving in compressible flow, boundary layer theory and magnetohydrodynamics.
- equip students with concepts and techniques to conduct research in the above mentioned domains.

Learning Outcomes

This course will enable the students to:

- learn conservation laws, first and second laws of thermodynamics, internal energy and entropy, different forms of energy equations and dimensional analysis.
- know about compressibility in real fluids, wave motion, sound waves, hyperbolic and dispersive waves, shock waves, their formation, properties and elementary analysis.
- know the concepts of boundary layer, boundary layer equations and their solutions, measurements of boundary layer thickness.
- understand the interaction between hydrodynamic processes and electromagnetic phenomena.
- formulate the basic equations of motion in inviscid and viscous conducting fluid flow and explain Alfven's theorem and magnetohydrodynamic (MHD) waves and MHD shocks.

Syllabus

Unit – 1 **(11 hours)**

Flow characteristics, Conservation laws, Equation of state of a substance, First and second law of thermodynamics, Internal energy and entropy, Energy equation, Nondimensionalizing the basic equations of incompressible viscous fluid flow, Nondimensional numbers.

Unit – 2 **(12 hours)**
Compressibility effects in real fluids, Equations of motion, Sound wave, Hyperbolic and dispersive waves, Isentropic gas flow, Flow through a nozzle, Method of characteristics, Shock jump conditions, Non-linear plane waves, Shock waves and their elementary analysis, Similarity solutions.

Unit – 3 **(11 hours)**
Boundary layer concept, Estimation of boundary layer thickness and friction forces, Prandtl's boundary layer equations, Boundary layer along a flat plate, Boundary layer thickness, General properties of the boundary layer equations, Similar solutions, Momentum and energy integral equations for the boundary layer.

Unit – 4 **(11 hours)**
Maxwell's electromagnetic field equations, Magnetohydrodynamic (MHD) approximations, Magnetic field equation, Magnetic Reynolds number, Magnetic body force, Equations of Motions of conducting fluid, Alfven's theorem, MHD waves, MHD shock waves.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] F. Chorlton, *Text Book of Fluid Dynamics*, CBS Publisher, 2005.
[2] H. Schlichting and K. Gersten, *Boundary Layer Theory*, Ninth Edition, Springer, 2017.
[3] G. B. Witham, *Linear and Nonlinear Waves*, John Wiley & Sons, 1999.

Suggested Readings
(i) K. R. Cramer and S. I. Pai, *Magnetofluid Dynamics for Engineers and Applied Physics*, McGraw Hill Book Co., New York, 1973.
(ii) Y. Shao-Wen, *Foundations of Fluid Mechanics*, PHI, New Delhi, 1960.

DISCIPLINE SPECIFIC ELECTIVE: MODULE THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Module Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Groups and Rings

Learning Objectives

The primary objective of this course is to:

- introduce a new algebraic structure, namely, module which is a generalization of a vector space when the underlying field is replaced by an arbitrary ring. The study of modules over a ring also provides an insight into the structure of the ring.
- study free modules, finitely generated modules, projective and injective modules.
- classify the finitely generated modules over a principal ideal domain (PID).

Learning Outcomes

This course will enable the students to:

- identify and construct examples of modules, and apply homomorphism theorems on the same.
- define and characterize Noetherian, Artinian module, and apply the structure theorem of finitely generated modules over PID.
- distinguish between projective, injective, free, and semi simple modules.
- prove universal property of tensor product of modules, and Hilbert basis theorem.

Syllabus

Unit – 1 **(13 hours)**

Basic concepts of module theory, Fundamental theorems of homomorphism, Direct product and direct sum of modules, Exact sequences, Split exact sequences.

Unit – 2 **(10 hours)**

Free modules, Projective and injective modules, Dual basis lemma, Baer's criterion, Divisible modules.

Unit – 3 **(12 hours)**

Tensor product of modules, Chain conditions, Hilbert basis theorem.

Unit – 4 **(10 hours)**

Modules over PID's, Semi simple modules.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. F. Athiyah and I. G. Macdonald, *Introduction to Commutative Algebra*, Addison Wesley, 1969.
[2] P. M. Cohn, *Basic Algebra*, Springer International Edition, 2003.
[3] P. M. Cohn, *Classic Algebra*, John Wiley & Sons Ltd., 2000.

Suggested Readings

(i) D. S. Dummit and R. M. Foote, *Abstract Algebra*, Wiley India Pvt. Ltd., 2011.
(ii) N. Jacobson, *Basic Algebra*, Volume II, Dover Publications Inc., 2009.
(iii) T. W. Hungerford, *Algebra*, Springer-Verlag, 1981.

DISCIPLINE SPECIFIC ELECTIVE: PROBABILITY THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Probability Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Measure Theory

Learning Objectives
The primary objective of this course is to introduce:
- probability space as a measure space and random variables as measurable functions.
- expectation and moments of random variables.
- notion of convergence in probability.
- conditioning on sub-σ-algebra.

Learning Outcomes
This course will enable the students to learn:
- about probability or uncertainty in abstract setting.
- moments and expectation of random variables which help to understand applications of probability in industry.
- how to apply the idea of convergence in probability.
- weak law and strong law of large numbers and their applications.

Syllabus

Unit – 1 **(11 hours)**
Probability: σ-algebra, Constructing probability triples, The extension theorem, Random variables, Independence of events, Continuity of probabilities, Limit events, The Borel–Cantelli Lemma.

Unit – 2 **(10 hours)**
Expected values: Simple, general non-negative and arbitrary random variables, Moment generating functions, Markov's inequality, Chebyshev's inequality.

Unit – 3 **(12 hours)**
Convergence of random variables: Convergence almost surely, Convergence in probability, Weak law of large numbers, Strong law of large numbers.

Unit – 4 **(12 hours)**
Distributions of random variables: Examples of distributions, Characteristic functions, The central limit theorem, Conditional probability, Conditioning on random variable, Conditioning on a sub-σ-algebra, Conditional variance.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. S. Rosenthal, *A First Look at Rigorous Probability Theory*, Second Edition, World Scientific, Singapore, 2006.

Suggested Readings

(i) W. Feller, *An Introduction to Probability Theory and its Applications*, Volume 1, Third Edition, Wiley, 2008.
(ii) J. E. Michael and J. S. Rosenthal, *Probability and Statistics: The Science of Uncertainty*, Second Edition, W. H. Freeman & Co Ltd., 2009.
(iii) S. Ross, *A First Course in Probability*, Tenth Edition, Pearson Education, 2022.
(iv) D. W. Stroock, *Probability Theory, An Analytic View*, Cambridge University Press, 2024.

DISCIPLINE SPECIFIC ELECTIVE: THEORY OF UNBOUNDED OPERATORS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Theory of Unbounded Operators	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis and Bounded Operators

Learning Objectives

The primary objective of this course is to:

- introduce the notion of unbounded operators.
- develop the theory of operator semigroups and understand their role in applications, particularly for solving differential equations.

Learning Outcomes

This course will enable the students to:

- identify closed and closable linear operators on Banach spaces.
- compute adjoints of unbounded linear operators.
- understand spectral properties of some unbounded operators.
- comprehend the role unbounded operators and semigroups play in applications, particularly in studying solutions of differential equations.

Syllabus

Unit – 1 **(10 hours)**

Unbounded linear operators, Hilbert adjoints, Hellinger–Toeplitz theorem, Hermitian, symmetric and self-adjoint linear operators, Closed linear operators, Closable operators and their closures on Banach spaces.

Unit – 2 **(12 hours)**

Cayley transform, Deficiency indices, Spectral properties of self-adjoint operators, Multiplication and differentiation operators and their spectra.

Unit – 3 **(11 hours)**

Analytic properties of exponential functions, Matrix Semigroups, Uniformly continuous semigroups, Semigroups on Hilbert spaces, Strongly continuous semigroups.

Unit – 4 **(12 hours)**

Generators of semigroups and their resolvents, Hille–Yosida theorem (for contraction semigroup), Dissipative operators and their properties, Lumer–Phillips theorem, Generators of Group, Stone's theorem.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] K. J. Engle and R. Nagel, *One-parameter Semigroups for Linear Evolution Equations*, Springer-Verlag, New York, 2000.
[2] E. Kreyszig*, Introductory Functional Analysis with Applications*, John Wiley & Sons, 2006.

Suggested Readings

(i) S. Goldberg, *Unbounded Linear Operators: Theory and Applications*, Dover Publications, 2006.
(ii) E. Hille and R. S. Phillips, *Functional Analysis and Semi-groups*. American Mathematical Society, Providence, RI, 1957.
(iii) A. Pazy*, Semigroups of Linear Operators and Applications to Partial Differential Equations*, Springer, 1983.
(iv) M. Schechter, *Principles of Functional Analysis*, Second Edition, American Mathematical Society, 2001.
(v) J. Weidmann, *Linear Operators in Hilbert Spaces, Graduate Texts in Mathematics*, Springer, New York, 1980.

Group-2

DISCIPLINE SPECIFIC ELECTIVE: BANACH AND C*-ALGEBRAS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Banach and C*-Algebras	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis and Topology

Learning Objectives

The primary objective of this course is to introduce:

- Banach algebras and C*-algebras.
- various ways to construct new operator algebras using given ones.
- spectrum of elements in Banach algebras and to study its properties.
- Gelfand representations of commutative Banach algebras and of C*-algebras.

Learning Outcomes

This course will enable the students to:

- familiarize with the representations of operator algebras.
- realize commutative Banach algebras and abelian C*-algebras as space of continuous functions on locally compact groups.
- understand the powerful tool of functional calculus.
- identify any C*-algebra as closed *-subalgebra of space of bounded linear operators on a Hilbert space.

Syllabus

Unit – 1 **(11 hours)**

Elementary properties and examples of Banach algebras, Ideals and quotients, Invertible elements, Spectrum and spectral radius, Spectral radius formula, Spectral mapping theorem (for polynomials), Gelfand–Mazur theorem.

Unit – 2 **(11 hours)**

Multiplicative linear functionals, Commutative Banach algebra, w*-topology, Gelfand transform of an element, Structure space, Gelfand representation.

Unit – 3 **(12 hours)**

Elementary properties and examples of C*-algebras, Unitization, Gelfand–Naimark representation of commutative C*-algebras, Continuous functional calculus, Spectral mapping theorem for normal elements, Positive elements of C*-algebras.

Unit – 4 **(11 hours)**

Ideals in C*-algebras, Approximate units, Quotients, Positive linear functionals, Gelfand–Naimark–Segal representation of C*-algebras.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.
[2] G. J. Murphy, *C*-algebras and Operator Theory*, Academic Press Inc., 1990.

Suggested Readings

(i) J. B. Conway, *A Course in Operator Theory, Graduate Texts in Mathematics*, Springer, 2007.
(ii) J. Dixmier, *C*-algebras*, North-Holland Publishing Company, 1977.
(iii) R. G. Douglas, *Banach Algebra Techniques in Operator Theory, Graduate Texts in Mathematics*, Springer, 1998.
(iv) E. Kaniuth, *A Course on Commutative Banach Algebras*, Graduate Texts in Mathematics, Springer, 2009.
(v) S. Kantorovitz, *Introduction to Modern Analysis*, Oxford Graduate Texts in Mathematics, Oxford University Press, 2006.
(vi) M. Takesaki, *Theory of Operator Algebras I*, Springer, 2002.

DISCIPLINE SPECIFIC ELECTIVE: CHAOS THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Chaos Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- introduce some useful and interesting notions like Topological Transitivity and Sensitive dependence on initial conditions.
- study different types of chaos including Devaney's chaos and finding their interrelationships.
- know classical result that period three implies chaos on intervals.
- relate chaos and decomposition theorems.
- study Topological entropy through open covers and also Bowen's definition of entropy, equivalence of these two definitions on compact metric spaces.
- study various interesting results related to topological entropy.

Learning Outcomes

This course will enable the students to:

- construct interesting examples of Topological transitive maps, Topological mixing maps etc.
- know classical examples of Devaney's chaotic maps like tent map, shift maps, logistic maps.
- study and compare different types of chaos.
- find relation between transitivity and chaos on intervals.
- relate chaos theory and classical decomposition theorems.
- study very useful notion of Topological entropy including its properties.
- calculate entropy of any homeomorphism of closed unit interval and of unit circle.

Syllabus

Unit – 1 **(12 hours)**

Topological Transitivity, Locally eventually onto maps, Topological mixing, Sensitive dependence on initial conditions, Devaney's definition of chaos, Transitivity and limit sets for continuous interval maps.

Unit – 2 **(11 hours)**

Characterizing topological mixing in terms of topological transitivity for continuous interval maps, Topological Weakly Mixing, Totally Transitive maps, Relation between transitivity and chaos on intervals, Logistic maps and shift maps as chaotic maps.

Unit – 3 **(12 hours)**

Various other definitions of chaos and their interrelationships. Period three implies chaos, Chaos and decomposition theorems including Bowen's decomposition theorem, Topological Entropy: Definition using open covers, Examples and properties, Bowen's definition of topological entropy,

Equivalence of two definitions, Topological version of Kolmogorov–Sinai theorem.

Unit – 4 **(10 hours)**

Topological Entropy of maps on a compact metric space, Topological Entropy of product maps, of iterations of a map, Topological entropy of an expansive homeomorphism on a compact metric space, of the two-sided shift, of any homeomorphism of the unit interval and of the unit circle.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] N. Aoki and K. Hiraide, *Topological Theory of Dynamical Systems: Recent Advances*, North Holland Publications, 1994.
[2] R. L. Devaney, *A First Course in Chaotic Dynamical Systems*, CRC Press, 2018.
[3] S. Ruette, *Chaos for Continuous Interval Maps: A Survey of Relationship Between Various Kinds of Chaos*, 2018.
[4] Peter Walters, *An Introduction to Ergodic Theory*, Springer, 2000.

Suggested Readings

(i) L. Alsedà, J. Llibre and M. Misiurewicz, *Combinatorial Dynamics and Entropy in Dimension One*, Advanced Series in Nonlinear Dynamics, World Scientific, 2000.
(ii) L. S. Block and W. A. Coppel, *Dynamics in One Dimension*, Springer, 2014.
(iii) M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge University Press, 2015.

DISCIPLINE SPECIFIC ELECTIVE: COMPLEX ANALYSIS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Complex Analysis	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Real Analysis and Metric Spaces

Learning Objectives

The primary objective of this course is to:

- gain insights of well-known classical results in the field of complex analysis.
- investigate various properties of analytic functions, conformal mappings and Möbius transformations.
- derive various forms of Cauchy's theorem, integral formulas and maximum principles.
- represent complex-valued functions as Taylor and Laurent series.

Learning Outcomes

This course will enable the students to:

- construct Möbius transformations using Symmetry and Orientation principles.
- foresee the usage of simply connected regions in the complex plane for the existence of primitives and branch of logarithm.
- understand the behavior of zeros of analytic functions and meromorphic functions through Argument principle and Rouche's theorem.
- evaluate the real integrals involving rational and trigonometric functions by contour integration using Residue theorem.
- apply Schwarz's lemma to characterize the conformal maps of the open unit disk onto itself.

Syllabus

Unit – 1 **(10 hours)**

Extended plane and its spherical representation, Analytic functions, Branch of logarithm, Conformal mappings, Möbius transformations.

Unit – 2 **(10 hours)**

Line integrals, Fundamental theorem of Calculus for line integrals, Power series representation of analytic functions, Zeros of analytic functions, Liouville's theorem.

Unit – 3 **(12 hours)**

Index of a closed curve, Cauchy's theorem and integral formula, Morera's Theorem, Homotopic version of Cauchy's theorem and simple connectivity, Counting Zeros, Open mapping theorem, Goursat's theorem.

Unit – 4 **(13 hours)**

Classification of singularities, Laurent series, Casorati-Weierstrass theorem, Residue theorem with applications, Argument principle, Rouche's theorem, Maximum principles, Schwarz lemma.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] J. B. Conway, *Functions of One Complex Variable*, Second Edition, Narosa Publishing House, New Delhi, 2002.

Suggested Readings
(i) L. V. Ahlfors, *Complex Analysis*, Mc Graw Hill Co., Indian Edition, 2017.
(ii) T. W. Gamelin, *Complex Analysis*, Springer New York, NY, 2001.
(iii) L. Hahn and B. Epstein, *Classical Complex Analysis*, Jones and Bartlett, 1996.
(iv) E. M. Stein and R. Shakarchi, *Complex Analysis*, Princeton University Press, 2003.

DISCIPLINE SPECIFIC ELECTIVE: NONSMOOTH OPTIMIZATION

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Nonsmooth Optimization	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Nonlinear Optimization

Learning Objectives

The primary objective of this course is to:

- understand the tools to deal with nonsmooth convex functions.
- study conjugate duality in terms of conjugate functions for constrained nonlinear optimization problems.
- introduce numerical techniques to solve constrained nonlinear optimization problems.

Learning Outcomes

This course will enable the students to learn:

- the notions of subgradients and subdifferentials for nonsmooth convex functions.
- the use of conjugate functions to develop the theory of conjugate duality.
- about numerical methods like gradient descent method, gradient projection method, Newton's method and conjugate gradient method.
- penalty approach technique to solve constrained nonlinear optimization problems.

Syllabus

Unit – 1 **(11 hours)**

Extended real valued functions, Proper convex functions, Closure of convex functions, Differential derivatives, Subgradients and subdifferentials.

Unit – 2 **(12 hours)**

Conjugate functions, Biconjugate functions, Perturbation functions, Closure of convex functions, Directional derivatives, Subgradients and subdifferentials.

Unit – 3 **(12 hours)**

Gradient descent method, Gradient projection method, Newton's method, Conjugate gradient method.

Unit – 4 **(10 hours)**

Penalty function methods, Exterior penalty function, Interior penalty functions.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. Avriel, *Nonlinear Programming: Analysis and Methods*, Dover Publications, 2003.

[2] O. Güler, *Foundations of Optimization*, Springer, 2010.

Suggested Readings

(i) A. Bagirov, N. Karitsa and M. M. Makela, *Introduction to Nonsmooth Optimization: Theory, Practice and Software*, Springer, 2014.
(ii) M. S. Bazaraa, H. D. Sherali and C. M. Shetty, *Nonlinear Programming: Theory and Algorithms,* Third Edition, John Wiley & Sons, 2013.
(iii) D. G. Luenberger and Y. Ye, *Linear and Nonlinear Programming*, Springer, 2008.

Group-3

DISCIPLINE SPECIFIC ELECTIVE: COMPUTATIONAL FLUID DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Computational Fluid Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Partial Differential Equation (Undergraduate level)

Learning Objectives

The primary objective of this course is to teach:

- various numerical schemes on finite difference and finite volume methods for solving PDEs.
- discretization errors and grid dependence.
- some real-world applications of PDEs and fluid dynamics.
- discretization of governing equations of diffusion, convection-diffusion, fluid flow and thereby computing the numerical solutions using the flow variables using algorithms.

Learning Outcomes

This course will enable the students to learn:

- techniques for solving the PDEs along with some initial and boundary conditions by using the finite difference and finite volume methods.
- the basic conservation principles of mass, momentum, energy, discretization of governing equations.
- discretization techniques.
- some popular algorithms like SIMPLE and SIMPLER used to obtain the solutions of steady and unsteady flow problems by finite volume methods.

Syllabus

Unit – 1 **(12 hours)**

Basics of discretization using finite differences, Various single and multi-step explicit and implicit finite difference schemes for 1-D and 2-D parabolic and hyperbolic initial boundary value problems, Alternating Direction Implicit schemes (ADI) for 2-D parabolic and hyperbolic equations, Order of accuracy, Consistency, Stability and convergence of a finite difference scheme, Courant Friedrich Lewy condition.

Unit – 2 **(12 hours)**

Finite difference schemes for second and fourth order 2-D elliptic boundary value problem and applications, Finite volume method for diffusion and convection-diffusion equations, Discretization of one and two-dimensional steady state diffusion and convection-diffusion equations, Central difference, Upwind, Exponential, Hybrid, Power-law and QUICK schemes and their properties.

Unit – 3 **(11 hours)**

Flow field calculation, Pressure-velocity coupling, Vorticity-stream function approach, Primitive variables, Staggered grid, Pressure and velocity corrections, Pressure correction equation, SIMPLE and SIMPLER algorithms.

Unit – 4 **(10 hours)**

Finite volume methods for unsteady flows, Discretization of one-dimensional transient heat conduction, Explicit, fully implicit and Crank–Nicolson schemes, Implementation of boundary conditions.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. C. Strikweda, *Finite Difference Schemes and Partial Differential Equations*, Second Edition, SIAM, 2004.

[2] H. K. Versteeg and W. Malalasekera, *An Introduction to Computational Fluid Dynamics: The Finite Volume Method*, Second Edition, Pearson, 2008.

Suggested Readings

(i) J. D. Anderson, *Computational Fluid Dynamics*, McGraw-Hill, 1995.

(ii) S. V. Patankar, *Numerical Heat Transfer and Fluid Flow*, CRC Press, Taylor and Francis, Indian Edition, 2017.

(iii) R. H. Pletcher, J. C. Tannehill and D. A. Anderson, *Computational Fluid Mechanics and Heat Transfer*, CRC Press, Taylor and Francis, 2013.

(iv) J. W. Thomas, *Numerical Partial Differential Equations: Finite Difference Methods*, Springer, 2013.

DISCIPLINE SPECIFIC ELECTIVE: DIFFERENTIAL TOPOLOGY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Differential Topology	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- introduce the concepts of topological manifolds, smooth structures, smooth manifolds, and manifolds with boundary.
- develop an understanding of smooth functions, smooth maps, diffeomorphisms, and tangent spaces.
- explain the Inverse function theorem, immersions and submersions.
- develop the fundamental concepts of 2-manifolds and distinguish between orientable and non-orientable surfaces.
- explore the properties of compact and connected surfaces.

Learning Outcomes

This course will enable the students to:

- identify and construct examples of topological manifolds, smooth structures and manifolds with and without boundary.
- demonstrate understanding of diffeomorphisms and tangent spaces.
- apply the Inverse function theorem, immersions and submersions.
- define key concepts such as 2-manifolds, orientability, compactness, connectedness and boundary of a surface.
- differentiate between orientable and non-orientable surfaces using examples such as the sphere, torus, Möbius strip and Klein bottle.

Syllabus

Unit – 1 **(12 hours)**
Topological manifolds, Topological properties of manifolds, Smooth structures, Examples of smooth manifolds, Manifolds with boundary.

Unit – 2 **(11 hours)**
Smooth functions and smooth maps, Lie groups, Diffeomorphisms.

Unit – 3 **(10 hours)**
Derivatives and tangents, Inverse function theorem, Immersions and submersions.

Unit – 4 **(12 hours)**
Complexes, Connected sum of two surfaces, Non-orientable surfaces (2- Manifolds), Compact and connected surfaces, Classification of compact and connected surfaces with and without boundary.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] V. Guillemin and Alan Pollack, *Differential Topology*, Prentice-Hall, 1974.
[2] L. C. Kinsey, *Topology of Surfaces*, Springer Verlag, 1997.
[3] J. M. Lee, *Introduction to Smooth Manifolds*, Second Edition, Springer, 2013.

Suggested Readings

(i) L. Conlon, *Differentiable Manifolds*, Second Edition, Birkhäuser Advanced Texts, 2001.
(ii) M. Spivak, *A Comprehensive Introduction to Differential Geometry*, Volume 1, Third Edition, Publish or Perish, Huston, Texas, 1999.
(iii) L. W. Tu, *Introduction to Manifolds*, Second Edition, Springer, 2011.
(iv) F. W. Warner, *Foundations of Differentiable Manifolds and Lie Group*, Springer-Verlag, 1983.

DISCIPLINE SPECIFIC ELECTIVE: GENERAL MEASURE THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: General Measure Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Measure Theory and Topology

Learning Objectives

The primary objective of this course is to introduce:

- real valued and complex valued measures.
- decomposition of measure spaces and of measures.
- extension of a premeasure to a measure, Lebesgue measure on Euclidean spaces.
- representation of measures and functionals in terms of integrals.
- product measures.

Learning Outcomes

This course will enable the students to:

- appreciate signed measures and complex measures, mutual singularity of measures, Hahn and Jordan decompositions, Lebesgue decomposition, Radon–Nikodym theorem.
- verify conditions under which a set function defined on a collection of subsets of a set has an extension to a measure on a sigma-algebra.
- apply Riesz representation theorem for bounded linear functionals on L^p-spaces.
- understand product measure and the results of Fubini and Tonelli, and express the Lebesgue measure on Euclidean spaces as a product measure.
- apply Riesz–Markov representation theorem for the bounded linear functionals on the space of continuous functions.

Syllabus

Unit – 1 **(13 hours)**

Signed measures, Hahn and Jordan decompositions, Mutually singular measures, Radon–Nikodym theorem, Lebesgue decomposition, Complex measure.

Unit – 2 **(10 hours)**

The Carathéodory extension theorem, Lebesgue measure on $\mathbb{R}^n$, Regularity and translation invariance of Lebesgue measure on $\mathbb{R}^n$.

Unit – 3 **(10 hours)**

Riesz representation theorem for the dual of L^p-spaces, Product measures, Fubini's theorem, Tonelli's theorem.

Unit – 4 **(12 hours)**

Locally compact Hausdorff spaces and construction of Radon measure, Riesz–Markov representation theorem for positive linear functionals on $C_c(X)$, Riesz representation theorem for the dual of $C(X)$.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] H. L. Royden and P. M. Fitzpatrick, *Real Analysis*, Fourth Edition, Pearson, 2015.
[2] M. E. Taylor, *Measure Theory and Integration*, American Mathematical Society, 2006.

Suggested Readings

(i) G. B. Folland, *Real Analysis: Modern Techniques and Their Applications*, Second Edition, Wiley, New York, 1999.
(ii) P. R. Halmos, *Measure Theory*, Springer Science + Business Media, LLC, 2014.
(iii) E. M. Stein and R. Shakarchi, *Real Analysis: Measure Theory, Integration and Hilbert Spaces*, New Age International Publishers, New Delhi, 2010.

DISCIPLINE SPECIFIC ELECTIVE: THEORY OF NON-COMMUTATIVE RINGS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Theory of Non-commutative Rings	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Groups and Rings

Learning Objectives

The primary objective of this course is to:

- give students an understanding of Wedderburn–Artin theory of semisimple rings.
- develop Jacobson's general theory of radicals, prime and semiprime rings, and primitive and semiprimitive rings.
- introduce the structure of primitive rings as a generalisation of the Wedderburn–Artin theorem on Artinian simple rings.

Learning Outcomes

This course will enable the students to:

- know about an extensive variety of rings, including free rings, Weyl algebra, Hilbert twist and triangular ring.
- understand the module theoretic definition of semisimple rings and how it leads to the Wedderburn–Artin structure theorem on their complete classification.
- know Jacobson's general theory of radicals, semiprime rings, prime, primitive and semiprimitive rings and their structures.
- understand the significance of the fundamental result 'Density Theorem' and its consequences on the structure of primitive rings.

Syllabus

Unit – 1 **(11 hours)**

Simple rings, Reduced rings, Dedekind-finite rings, Algebra, Quaternions, Free k-rings, Rings with generators and relations, Weyl algebra, Formal power series ring, Hilbert's twist ring, Differential polynomial rings, Derivation and inner derivation on a ring, Triangular rings, Characterization of one-sided and two-sided ideals in such rings.

Unit – 2 **(11 hours)**

Noetherian and Artinian rings, Examples of one-sided Noetherian and Artinian triangular rings, Twisted polynomial ring and Quotient of free $\mathbb{Z}$-ring, Semisimple rings, Structure of semisimple rings: Wedderburn–Artin's theorem.

Unit – 3 **(10 hours)**

Structure theorem of simple left Artinian rings, Jacobson radical, J-semisimple rings, Nil and nilpotent ideals, Connection between semisimple and J-semisimple rings, Hopkins–Levitzki theorem, Nakayama's lemma.

Unit – 4 **(13 hours)**
Prime radical, Characterisation of prime and semiprime ideals, Prime and semiprime rings, Structure theorem of primitive rings, Density theorem.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] T.-Y. Lam, *A First Course in Noncommutative Rings*, Springer, 2001.

Suggested Readings
(i) I. N. Herstein, *Noncommutative Rings*, The Mathematical Association of America, 2005.
(ii) T. W. Hungerford, *Algebra*, Springer-Verlag, New York, 1981.
(iii) L. H. Rowen, *Ring Theory*, Student Edition, Academic Press, 1991.

Generic Elective (GE) Courses

GENERIC ELECTIVE – 2 (i): NONSMOOTH OPTIMIZATION

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
GE-2 (i): Nonsmooth Optimization	4	3	1	0	Class XII pass with Mathematics	Basics of Nonlinear Optimization

Learning Objectives

The primary objective of this course is to:

- understand the tools to deal with nonsmooth convex functions.
- study conjugate duality in terms of conjugate functions for constrained nonlinear optimization problems.
- introduce numerical techniques to solve constrained nonlinear optimization problems.

Learning Outcomes

This course will enable the students to learn:

- the notions of subgradients and subdifferentials for nonsmooth convex functions.
- the use of conjugate functions to develop the theory of conjugate duality.
- about numerical methods like gradient descent method, gradient projection method, Newton's method and conjugate gradient method.
- penalty approach technique to solve constrained nonlinear optimization problems.

Syllabus

Unit – 1 **(11 hours)**

Extended real valued functions, Proper convex functions, Closure of convex functions, Differential derivatives, Subgradients and subdifferentials.

Unit – 2 **(12 hours)**

Conjugate functions, Biconjugate functions, Perturbation functions, Closure of convex functions, Directional derivatives, Subgradients and subdifferentials.

Unit – 3 **(12 hours)**

Gradient descent method, Gradient projection method, Newton's method, Conjugate gradient method.

Unit – 4 **(10 hours)**

Penalty function methods, Exterior penalty function, Interior penalty functions.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. Avriel, *Nonlinear Programming: Analysis and Methods*, Dover Publications, 2003.

[2] O. Güler, *Foundations of Optimization*, Springer, 2010.

Suggested Readings

(i) A. Bagirov, N. Karitsa and M. M. Makela, *Introduction to Nonsmooth Optimization: Theory, Practice and Software*, Springer, 2014.

(ii) M. S. Bazaraa, H. D. Sherali and C. M. Shetty, *Nonlinear Programming: Theory and Algorithms,* Third Edition, John Wiley & Sons, 2013.

(iii) D. G. Luenberger and Y. Ye, *Linear and Nonlinear Programming*, Springer, 2008.

GENERIC ELECTIVE – 2 (ii): PROBABILITY THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
GE-2 (ii): Probability Theory	4	3	1	0	Class XII pass with Mathematics	Basics of Integration

Learning Objectives
The primary objective of this course is to introduce:
- probability space as a measure space and random variables as measurable functions.
- expectation and moments of random variables.
- notion of convergence in probability.
- conditioning on sub-σ-algebra.

Learning Outcomes
This course will enable the students to learn:
- about probability or uncertainty in abstract setting.
- moments and expectation of random variables which help to understand applications of probability in industry.
- how to apply the idea of convergence in probability.
- weak law and strong law of large numbers and their applications.

Syllabus

Unit – 1 **(11 hours)**
Probability: σ-algebra, Constructing probability triples, The extension theorem, Random variables, Independence of events, Continuity of probabilities, Limit events, The Borel–Cantelli Lemma.

Unit – 2 **(10 hours)**
Expected values: Simple, general non-negative and arbitrary random variables, Moment generating functions, Markov's inequality, Chebyshev's inequality.

Unit – 3 **(12 hours)**
Convergence of random variables: Convergence almost surely, Convergence in probability, Weak law of large numbers, Strong law of large numbers.

Unit – 4 **(12 hours)**
Distributions of random variables: Examples of distributions, Characteristic functions, The central limit theorem, Conditional probability, Conditioning on random variable, Conditioning on a sub-σ-algebra, Conditional variance.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. S. Rosenthal, *A First Look at Rigorous Probability Theory*, Second Edition, World Scientific, Singapore, 2006.

Suggested Readings

(i) W. Feller, *An Introduction to Probability Theory and its Applications*, Volume 1, Third Edition, Wiley, 2008.
(ii) J. E. Michael and J. S. Rosenthal, *Probability and Statistics: The Science of Uncertainty*, Second Edition, W. H. Freeman & Co Ltd., 2009.
(iii) S. Ross, *A First Course in Probability*, Tenth Edition, Pearson Education, 2022.
(iv) D. W. Stroock, *Probability Theory, An Analytic View*, Cambridge University Press, 2024.

Syllabi of Courses
in
Semester-I
of
One-year M.Sc. Mathematics
under Structure-3
(Research)

Discipline Specific Core (DSC) Course

DISCIPLINE SPECIFIC CORE: MATRIX GROUPS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSC: Matrix Groups	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Calculus and Linear Algebra

Learning Objectives
The primary objective of this course is to:
- introduce the notion of matrix groups which serves as a bridge between algebra, geometry and analysis.
- use exponential and logarithm maps to connect one-parameter subgroups with Lie algebras.
- connect Clifford algebra constructions to orthogonal groups, spin groups and isomorphism questions.

Learning Outcomes
This course will enable the students to:
- distinguish between general linear, orthogonal and special orthogonal groups with explicit examples.
- construct and compute reflection and rotation matrices in $\mathbb{R}^n$and analyze their properties.
- comprehend maximal tori, their coverings and conjugacy results in compact matrix groups.
- appreciate the applications of matrix groups in algebraic geometry, complex analysis, group and ring theory, number theory, quantum physics, Einstein's special relativity, Heisenberg's uncertainty principle, quark theory, Fourier series, combinatorics and many more areas.

Syllabus

Unit – 1 **(10 hours)**
The general linear groups, The orthogonal groups, The isomorphism question, Reflection in $\mathbb{R}^n$, Curves in a vector space, Smooth homeomorphisms, The special orthogonal groups.

Unit – 2 **(11 hours)**
Orthogonal matrices and isometries, Exponential and Logarithm of a matrix, One-parameter subgroups, Lie Algebras, $SO(3)$ and $Sp(1)$.

Unit – 3 **(12 hours)**
Maximal tori, Covering by maximal tori, Reflections in $\mathbb{R}^n$, Monogenic groups.

Unit – 4 **(12 hours)**
Conjugacy of maximal tori, Clifford algebras, $Pin(k)$, $Spin(k)$ and isomorphisms.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] M. L. Curtis, *Matrix Groups*, Springer, 1984.

Suggested Readings
(i) A. Baker, *Matrix Groups: An Introduction to Lie Group Theory*, Springer Undergraduate Mathematics Series, 2001.
(ii) K. Tapp, *Matrix Groups for Undergraduates*, American Mathematical Society, 2016.

Discipline Specific Elective (DSE) Courses

DISCIPLINE SPECIFIC ELECTIVE: ADVANCED FUNCTIONAL ANALYSIS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Advanced Functional Analysis	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis and Topology

Learning Objectives

The primary objective of this course is to:

- define and explain the structure of a topological vector space and its fundamental properties.
- differentiate between normed, metrizable, locally convex and Hausdorff topological vector spaces.
- introduce the foundational theorems of functional analysis, including the Hahn–Banach, Banach–Steinhaus, Open mapping, and Closed graph theorems in the context of locally convex spaces.
- explain some applications of Banach–Alaoglu theorem and Krein–Milman theorem.

Learning Outcomes

This course will enable the students to:

- appreciate types of topological vector spaces and their separation properties.
- understand quotient spaces, weak topology and weak*-topology.
- analyze concepts of continuity, boundedness, and convergence for linear operators and functionals on topological vector spaces.
- understand the notion of local convexity and the role of seminorms in defining locally convex topologies.

Syllabus

Unit – 1 **(12 hours)**

Topological vector spaces, Types of Topological vector spaces, Separation properties, Linear mappings, Finite dimensional spaces, Metrization, Boundedness and continuity, Seminorms and local convexity, Normability.

Unit – 2 **(11 hours)**

Quotient spaces, Seminorms and quotient spaces, Examples, Baire category theorem, Banach–Steinhaus theorem, The open mapping theorem and the closed graph theorem on topological vector spaces.

Unit – 3 **(11 hours)**

Hahn–Banach separation theorem on topological vector spaces, Continuous extension theorem, Weak topologies, Weak topology and convexity, Weak topology and metrizability, Weak*-

topology of a dual space, Compact convex sets, Banach–Alaoglu theorem and applications, Goldstine theorem.

Unit – 4 **(11 hours)**

Extreme points, Krein–Milman theorem, Convex hull of compact sets, Applications of Krein–Milman theorem: Stone–Weierstrass theorem, Markov–Kakutani fixed point theorem.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.
[2] W. Rudin, *Functional Analysis*, Second Edition, Tata Mc Graw-Hill, 2011.

Suggested Readings

(i) V. I. Bogachev and O. G. Smolyanov, *Topological Vector Spaces and Their Applications*, Springer, 2017.
(ii) S. Kantorovitz, *Introduction to Modern Analysis*, Oxford Graduate Texts in Mathematics, Oxford University Press, 2006.
(iii) J. Voigt, *A Course on Topological Vector Spaces*, Birkhäuser, 2020.

DISCIPLINE SPECIFIC ELECTIVE: ALGEBRAIC CODING THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Algebraic Coding Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra, Groups and Rings

Learning Objectives
The primary objective of this course is to:
- provide an introduction to algebraic coding theory, particularly linear codes.
- discuss bounds on the parameters along with cyclic codes.
- describe some well-known codes, such as Reed–Muller and Golay codes.
- explore the algebraic structure of Cyclic and Quadratic residue codes over fields and rings.

Learning Outcomes
This course will enable the students to:
- get an insight into the matrix representation of a code, as well as encoding and decoding.
- understand Hamming, MDS and Reed–Muller codes.
- describe cyclic codes and their generator polynomial.
- learn about special cyclic codes, such as Quadratic residue codes, and their properties over the ring $\mathbb{Z}_4$.

Syllabus

Unit – 1 **(10 hours)**
Error detecting and error correcting codes, Maximum likelihood decoding, Hamming distance, Linear codes, Hamming weight, Generator matrix, Parity check matrix, Equivalence of linear codes, Encoding and decoding of linear codes, Syndrome decoding.

Unit – 2 **(11 hours)**
Bounds on codes, Sphere covering bound, Hamming bound, Perfect codes, Binary Hamming codes, Binary Golay codes, Singleton bound and MDS codes. Propagation rules, Reed–Muller codes.

Unit – 3 **(12 hours)**
Cyclic codes, Cyclic codes as ideals, Generator polynomial of cyclic codes, Generator and parity-check matrices of cyclic codes**,** Decoding of cyclic codes, Burst error correcting codes.

Unit – 4 **(12 hours)**
Quadratic residue codes: QR codes over fields of characteristic 2 and 3, Cyclic codes and their generating polynomial over $\mathbb{Z}_4$, QR codes over $\mathbb{Z}_4$.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] S. Ling and C. Xing, *Coding Theory: A First Course*, Cambridge University Press, 2004.
[2] W. C. Huffman and V. Pless, *Fundamentals of Error Correcting Codes*, Cambridge University Press, 2010.

Suggested Readings

(i) R. Hill, *A First Course in Coding Theory*, Oxford University Press, 1986.
(ii) F. J. Mac William and N. J. A. Sloane, *Theory of Error Correcting Codes, Part I & II,* Elsevier/North-Holland, Amsterdam, 1977.

DISCIPLINE SPECIFIC ELECTIVE: COMMUTATIVE ALGEBRA

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Commutative Algebra	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra and Field Theory

Learning Objectives

The objective of this course is to:

- develop a solid understanding of the structure of commutative rings, ideals, their radicals, extension, contraction etc.
- study important constructions such as total quotient rings, localizations.
- develop basic foundation in other areas of mathematics such as algebraic geometry, algebraic number theory.

Learning Outcomes

This course will enable the students to:

- know the localization of rings at a prime ideal that is an algebraic analogue of the geometric notion concentrating attention near a point.
- know more closely the polynomial rings, power series rings in one or more variables over a commutative ring and their prime spectrum.
- define, identify, and elaborate integral closure of rings, valuations rings, discrete valuation rings, structure theorem of Artin rings.

Syllabus

Unit – 1 **(12 hours)**

Radical of an ideal, Prime avoidance lemma, Chinese remainder theorem, Extension and contraction of ideals, Jacobson radical of a ring, Nakayama lemma, Tensor product of modules.

Unit – 2 **(13 hours)**

Rings and modules of fractions, Localization, Local properties, Primary decomposition, First and second uniqueness theorem of primary decomposition, Associated prime ideals of decomposable ideals.

Unit – 3 **(10 hours)**

Integral ring extensions, Going up theorem, Going down theorem, Integrally closed domains, Valuation rings, Hilbert’s Nullstellensatz theorem.

Unit – 4 **(10 hours)**

Noetherian rings, Primary decomposition in Noetherian rings, Artin rings, Structure theorem for Artin rings, Discrete valuation rings.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. F. Atiyah and I. G. MacDonald, *Introduction to Commutative Algebra*, CRC Press, Taylor & Francis, 2018.

Suggested Readings

(i) D. Eisenbud, *Commutative Algebra with a View Towards Algebraic Geometry*, Springer, 2004.
(ii) R. Y. Sharp, *Steps in Commutative Algebra*, Cambridge University Press, 2000.
(iii) B. Singh, *Basic Commutative Algebra*, World Scientific, 2011.
(iv) O. Zariski and P. Samuel, *Commutative Algebra*, Volume I & II, Springer, 1975.

DISCIPLINE SPECIFIC ELECTIVE: DIFFERENTIAL GEOMETRY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Differential Geometry	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra, Multivariate Calculus and Differential Equations

Learning Objectives

The primary objective of this course is to introduce:

- surfaces and parametrized surfaces.
- orientation on connected surfaces.
- geodesics on surfaces.
- Weingarten maps on oriented surfaces.
- arc length and curvature of oriented plane curves.
- curvatures of oriented surfaces.

Learning Outcomes

This course will enable the students to:

- understand the concepts of level sets and graphs of functions, smooth vector fields, tangent spaces of level sets.
- appreciate surfaces and parametrized surfaces, Gauss map, geodesics and parallel transport on oriented surfaces.
- know what the Weingarten map of an oriented surface is, realize it as shape operator and use it to compute curvature of oriented plane curves.
- find global parametrization and hence arc length of an oriented plane curve.
- compute various types of curvatures of surfaces.

Syllabus

Unit – 1 **(10 hours)**

Level sets in $\mathbb{R}^{n+1}$ and graphs of functions, Smooth vector fields and existence and uniqueness of their integral curves, Tangent spaces of level sets at regular points, Surfaces in $\mathbb{R}^{n+1}$ as inverse images of regular values of smooth functions, Necessary condition for extrema of functions on surfaces-Lagrange multipliers, Existence of a normal vector field on a connected surface, Orientation, Gauss map.

Unit – 2 **(13 hours)**

The notion of a geodesic on a surface, Existence and uniqueness of a geodesic on a surface through a given point with a given velocity vector thereof, Covariant derivative of a smooth vector field, Parallel vector field along a curve, Existence and uniqueness of a parallel vector field along a curve with a given initial vector, Weingarten map of a surface at a point, Local parametrization and curvature of a plane curve.

Unit – 3 **(10 hours)**
Global parametrization and arc length of an oriented plane curve, Differential 1-forms, Line integral of 1-forms over parametrized curves.

Unit – 4 **(12 hours)**
Parametrized surfaces with examples, Curvature of surfaces, Normal curvature of a surface at a point in a given direction, Principal curvatures, First and second fundamental forms, Gauss-Kronecker curvature and mean curvature.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] A. Pressley, *Elementary Differential Geometry*, Springer-Verlag London Limited, 2012.
[2] J. A. Thorpe, *Elementary Topics in Differential Geometry*, Springer (India) Pvt. Limited, 2004.

Suggested Readings
(i) W. Kuhnel, *Differential Geometry: Curves-Surfaces-Manifolds*, Third Edition, American Mathematical Society, 2015.
(ii) B. O' Neill, *Elementary Differential Geometry*, Second Edition, Academic Press INC., Academic Press, New York, 2006.

DISCIPLINE SPECIFIC ELECTIVE: DYNAMICAL SYSTEMS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Dynamical Systems	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology and Ordinary Differential Equations

Learning Objectives
The primary objective of this course is to:
- understand discrete and continuous systems with case studies to study nonlinear systems of ordinary differential equations and dynamical systems.
- understand the concepts, models and techniques to realize the real-world problems and stability of the systems along with the chaotic dynamic behaviour of models by understanding bifurcations.

Learning Outcomes
This course will enable the students to learn:
- formulation of mathematical models with the stability analysis near the equilibrium points.
- how the concept of phase portraits helps to analyse mathematical model graphically.
- the qualitative behaviour of the solution set of a given system of differential equations including the invariant sets and limiting behaviour of the dynamical system or flow defined by the system of differential equations.
- how different bifurcations explain the chaotic behaviour of the system.

Syllabus

Unit – 1 **(13 hours)**
Linear systems: Jordan forms, Stability theory; Nonlinear systems: Fundamental existence-uniqueness theorem, Dependence on initial conditions and parameters, Flow of a differential equation, Linearization, Stable manifold theorem, Hartman–Grobman theorem.

Unit – 2 **(10 hours)**
Stability and Lyapunov functions, Saddle points, Nodes, Foci, Centers and nonhyperbolic critical points, Center manifold theorem.

Unit – 3 **(12 hours)**
Limit sets and attractors, Periodic orbits and limit cycles, Poincaré map, Stable manifold theorem for periodic orbits, Poincare-Bendixson theorem.

Unit – 4 **(10 hours)**
Bifurcations at nonhyperbolic equilibrium points, Saddle node, Transcritical and Pitchfork bifurcations, Hopf bifurcation.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. W. Hirsch, S. Smale and R. L. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, Third Edition, Academic Press, 2013.
[2] L. Perko, *Differential Equations and Dynamical Systems*, Third Edition, Springer Verlag, 2001.

Suggested Readings

(i) R. L. Devaney, *A First Course in Chaotic Dynamical Systems: Theory and Experiment*, CRC Press, Taylor & Francis, 2018.
(ii) S. H. Strogatz, *Nonlinear Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 2018.
(iii) S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, TAM Volume 2, Springer-Verlag, NY, 1990.

DISCIPLINE SPECIFIC ELECTIVE: FINITE ELEMENT METHODS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Finite Element Methods	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Differential Equations

Learning Objectives

The primary objective of this course is to:

- introduce basic aspects of finite element methods (FEM) including domain discretization, polynomial interpolation, application of boundary conditions, assembly of global arrays, and solution of the resulting algebraic systems.
- discuss the use of finite element methods in solving engineering problems in the domain of solid mechanics, fluid mechanics, heat transfer and electromagnetism.

Learning Outcomes

This course will enable the students to:

- use integral statement to deduce finite element approximations for the underlying linear partial differential equations.
- write special-purpose finite element programs within a procedural programming environment.
- use finite element methods to solve engineering problems in solids mechanics, fluid mechanics, heat transfer, and electromagnetism.
- assess the accuracy and reliability of finite element solutions and troubleshoot problems arising from errors in a given finite element analysis.

Syllabus

Unit – 1 **(12 hours)**

Basic concepts of weak formulation, Variational formulation of a one dimensional model equation, Basis function and finite element solutions, Collocation method, Ritz method, Least square method, Standard Galerkin method, FEM for model problem, Error estimate for FEM for model equation, Convergence analysis.

Unit – 2 **(11 hours)**

Various shapes of finite element, Higher order basis functions, Finite element methods for elliptic problems: Variational methods, Standard Galerkin method, Error estimate for FEM for elliptic problem, FEM for Poisson equation.

Unit – 3 **(12 hours)**

Finite element methods for parabolic problems: One dimensional model problems, Semi-discretization in space, Error estimates, Discretization in space and time, Galerkin method, Finite element methods for hyperbolic problems: Standard Galerkin method, Standard Galerkin method with strongly and weakly imposed boundary conditions.

Unit – 4 **(10 hours)**

Applications of the FEM to second order BVPs in one dimension, Applications of the FEM to linear elliptic, parabolic and hyperbolic equations.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] G. Evans, J. Blackledge and P. Yardley, *Numerical Methods for Partial Differential Equations*, Springer-Verlag, London, 2000.
[2] C. Johnson, *Numerical Solutions of Partial Differential Equations by Finite Element Methods*, *Cambridge University Press*, Cambridge, 1987.
[3] J. Whiteley, *Finite Element Methods - A Practical Guide*, Springer, 2016.

DISCIPLINE SPECIFIC ELECTIVE: NUMERICAL METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Numerical Methods for Ordinary Differential Equations	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Ordinary Differential Equations

Learning Objectives

The primary objective of this course is to:

- develop the basic theory underlying the numerical solution of differential equations.
- introduce the concepts of consistency, stability and convergence of finite difference methods.
- execute the numerical schemes for the solution of differential equations.

Learning Outcomes

This course will enable the students to:

- gain a thorough understanding of the fundamental concepts involved in the construction and analysis of finite difference schemes for solving ordinary differential equations (ODEs).
- apply various numerical methods based on finite difference approaches to obtain approximate solutions for both initial value problems (IVPs) and boundary value problems (BVPs).
- develop the ability to select appropriate finite difference methods for specific types of problems and effectively apply them to real world applications.

Syllabus

Unit – 1 **(11 hours)**

Initial value problems: Existence and uniqueness of solution, Finite difference equation, Truncation error, Single step methods for first order IVPs and system of IVPs- Family of explicit and implicit Runge–Kutta methods, Taylor series methods, Derivation, Truncation error, Consistency, Stability and convergence analysis.

Unit – 2 **(12 hours)**

IVPs for the system of ODEs, Consistency, Zero stability and convergence of linear multistep methods, Routh–Hurwitz criterion, Order and error constant, First Dahlquist Barrier, Local truncation error and global truncation error, Error bounds, Local error, Linear stability theory, Higher order differential equations.

Unit – 3 **(12 hours)**

Derivation of explicit and implicit multistep methods for IVPs and system of IVPs, Truncation error, Stability and convergence analysis of family of Nystrom method, Adams–Bashforth method,

Adams–Moulton method, Milne–Simpson method, Predictor corrector method, and Modified predictor corrector method, Hybrid method, Multistep methods for second order IVPs.

Unit – 4 **(10 hours)**

Linear BVPs for second order ordinary differential equations, Shooting method, Finite difference method, Collocation method, Derivative boundary conditions, Nonlinear two-point BVPs, Higher order finite difference methods, Stability, Truncation error and convergence analysis.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. K. Jain, S. R. K. Iyenger and R. K. Jain, *Numerical Methods for Scientific and Engineering Computations*, Seventh Edition, New Age International Publisher, 2019.

[2] J. D. Lambert, *Numerical Methods for Ordinary Differential Systems: The Initial Value Problem*, John Wiley & Sons, 1991.

Suggested Readings

(i) K. E. Atkinson, W. Han and D. E. Stewart, *Numerical Solution of Ordinary Differential Equations*, John Wiley & Sons, 2009.

(ii) J. C. Butcher, *The Numerical Analysis of Ordinary Differential Equations*, Second Edition, Wiley, New York, 2008.

(iii) L. Collatz, *The Numerical Treatment of Differential Equations*, Springer-Verlag, 1966.

DISCIPLINE SPECIFIC ELECTIVE: REPRESENTATION OF FINITE GROUPS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Representation of Finite Groups	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Linear Algebra and Group Theory

Learning Objectives

The primary objective of this course is to:

- represent finite groups as groups of matrices (via homomorphisms) and apply the tools of linear algebra to study the group structure.
- introduce the notion of Group algebra, which plays an essential role in classifying representations of groups.
- to discuss some applications of representations of finite groups, such as the Burnside's theorem.

Learning Outcomes

This course will enable the students to:

- define and construct examples of group representations, *FG*-modules, group algebras.
- grasp key concepts and tools of representation theory and establish a link between *FG*-modules and group representations.
- prove and apply Maschke's theorem and Schur's lemma to describe all irreducible representations of finite groups over the field of complex numbers.
- apply the theory of characters and group representations to gain insight into group structure, such as normal subgroups, and the solubility of groups.

Syllabus

Unit – 1 **(11 hours)**

Representation of groups, *FG*-modules and *FG*-submodules, and reducibility, Permutation modules, *FG*-modules and equivalent representations, Reducible and irreducible *FG*-modules, Group algebra of *G*, Regular *FG*-module and regular representations, *FG*-homomorphisms, Direct sum of *FG*-modules.

Unit – 2 **(11 hours)**

Maschke's theorem for *FG*-modules and consequences. Schur's lemma and its converse, Application of Schur's lemma, Irreducible modules and group algebra, Structure of group algebra and space of *CG*-homomorphisms.

Unit – 3 **(10 hours)**

Characters and their properties, Permutation and regular characters, Inner product, Number of irreducible characters, Orthogonality relations and finding normal subgroups.

Unit – 4 **(13 hours)**

Algebraic numbers, Algebraic integers and their properties, Character values, The Burnside's *(p,q)*-theorem and solubility of some particular groups.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] G. James and M. Liebeck, *Representations and Characters of Groups*, Second Edition, Cambridge University Press, 2005.

Suggested Readings

(i) C. W. Curtis and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras*, American Mathematical Society, 2006.
(ii) W. Fulton and J. Harris, *Representation Theory - A First Course*, Springer-Verlag, 2004.
(iii) I. M. Issacs, *Character Theory of Finite Groups*, American Mathematical Society reprint, 2006.

DISCIPLINE SPECIFIC ELECTIVE: THEORY OF BOUNDED OPERATORS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Theory of Bounded Operators	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis

Learning Objectives
The primary objective of this course is to:
- introduce some classes of bounded linear operators which play a central role in both pure and applied mathematics.
- study the properties and spectral theory of these operators.

Learning Outcomes
This course will enable the students to understand:
- the spectrum and sub-divisions of spectrum of standard operators like shifts and multiplication.
- the spectral theorem for some classes of bounded linear operators.
- the concepts of compactness, self-adjointness and positivity of bounded linear operators.
- trace class and Hilbert–Schmidt operators.

Syllabus

Unit – 1 **(11 hours)**
Properties of spectrum and resolvent of bounded operators, Subdivision of the spectrum including point, approximate and compression spectrum.

Unit – 2 **(10 hours)**
Operators on Hilbert spaces, Adjoint operator, Projections and idempotents, Operations with projections, Invariant and reducing subspaces.

Unit – 3 **(14 hours)**
Compact operators on Hilbert spaces, Diagonalisation of compact self-adjoint operators, Spectral theorem and functional calculus for Compact normal operators, Positive operators, Compact operators on Banach spaces, Spectral theory of compact operators.

Unit – 4 **(10 hours)**
Polar decomposition, Singular values, Trace class operators, Trace norm and Hilbert Schmidt operators.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] R. Bhatia, *Notes on Functional Analysis*, Hindustan Book Agency, 2009.
[2] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.

Suggested Readings

(i) E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, 2006.
(ii) B. Simon, *Operator Theory: A Comprehensive Course in Analysis*, Part 4, American Mathematical Society, 2015.
(iii) S. R. Garcia, J. Mashregi and W. T. Ross, *Operator Theory by Example*, Oxford University Press, 2023.

DISCIPLINE SPECIFIC ELECTIVE: TOPOLOGICAL DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Topological Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- provide a strong background of topological dynamical systems including their applications.
- develop some useful and interesting dynamical properties like expansivity, shadowing and topological stability with supporting examples and results from symbolic and topological dynamics.
- introduce the celebrated Sarkovskii's theorem.

Learning Outcomes

This course will enable the students to:

- construct interesting examples of dynamical systems and topological conjugacy.
- visualize stable sets, omega sets and alpha limit sets.
- understand the applications of Sarkovskii's theorem.
- use subshifts of finite type to characterize irreducible matrices.
- prove key results on expansivity and shadowing regarding existence/non-existence, product, subspace and their different characterizations etc.
- find the class of topologically stable homeomorphisms.

Syllabus

Unit – 1 **(10 hours)**

Definition and examples (including real life examples) of dynamical systems, Orbits, Types of orbits, Topological conjugacy and orbits, Phase portrait-graphical analysis of orbit, Periodic points and stable sets, Omega and alpha limit sets and their properties.

Unit – 2 **(10 hours)**

Sarkovskii's theorem, Shift spaces and subshift, Subshift of finite type, Subshift represented by a matrix, Characterizations of irreducible matrices.

Unit – 3 **(13 hours)**

Definition and examples of expansive homeomorphisms, Properties of expansive homeomorphisms, Non-existence of expansive homeomorphism on the unit interval and unit circle, Generators and weak generators, Generators and expansive homeomorphisms.

Unit – 4 **(12 hours)**

Converging semi-orbits for expansive homeomorphisms, Definition, examples and properties of maps having shadowing property, Topological Anosov homeomorphisms and topological stability.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] N. Aoki and K. Hiraide, *Topological Theory of Dynamical Systems: Recent Advances*, North Holland Publications, 1994.
[2] M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge University Press, 2004.

Suggested Readings

(i) D. C. Hanselman and B. Little field, *Mastering MATLAB*, Pearson, 2012.
(ii) D. Lind and B. Marcus, *An Introduction to Symbolic Dynamics and Coding*, Cambridge University Press, 1996.
(iii) C. Robinson, *Dynamical Systems*, *Stability, Symbolic Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 1998.
(iv) J. de. Vries, *Elements of Topological Dynamics*, Springer, 1993.

Research Methods/ Tools/ Writing Courses

ADVANCED RESEARCH METHODOLOGY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Advanced Research Methodology	2	1	0	1	Same as for entry to M.Sc. Mathematics	NIL

Learning Objectives

The primary objective of this course is to introduce:

- potential ethical problems in research design, data collection, analysis, and reporting.
- important issues like plagiarism, fabrication, falsification, informed consent and confidentiality.
- ways in which conflicts of interest and research misconduct arise.
- the rich contribution of India in the field of Mathematics.

Learning Outcomes

This course will enable the students to:

- appreciate the importance of ethical conduct for the credibility and societal trust in science.
- foster a culture of responsibility, respect, and fairness in research environments.
- use national and international ethical standards in planning and conducting research.
- inculcate the habit of self-reading and acquire the in-depth knowledge of history of the core discipline.

Syllabus

Unit – 1 **(8 hours)**

Research Ethics: Ethics with respect to science and research, Intellectual honesty and research integrity; Scientific misconducts: Falsification, Fabrication and Plagiarism (FFP).

Unit – 2 **(7 hours)**

History of Mathematics/ Indian Mathematics, Exploring web, Exploring web resources: MAA, AMS, SIAM, arXiv, ResearchGate; Journal metrics: Impact factor of journal as per JCR, MCQ, SNIP, SJR, Google Scholar metric; Reviews/Databases: MathSciNet, zbMath, Web of Science, Scopus.

Practical (30 hours)

- Self-reading
- Seminar

on broad research area of the core discipline.

Essential Readings

[1] G. G. Joseph, *Indian Mathematics: Engaging with the World from Ancient to Modern Times*, World Scientific Publishing Europe Ltd., 2016.
[2] Committee on Publication Ethics- COPE (https://publicationethics.org/)
[3] University Grants Commission (Promotion of Academic Integrity and Prevention of Plagiarism in Higher Educational Institutions) Regulations 2018 (The Gazette of India: Extraordinary, Part-iii-Sec.4)

TOOLS FOR RESEARCH

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Tools for Research	2	0	0	2	Same as for entry to M.Sc. Mathematics	NIL

Learning Objectives

The primary objective of this course is to introduce:

- some advanced tools of LaTeX.
- research article/dissertation writing.
- beamer as tool for mathematical talk.
- the art of making mathematical poster.
- modern mathematical software to enhance research skills.

Learning Outcomes

This course will enable the students to:

- write research article/dissertation.
- prepare a mathematical talk/poster.
- use software to perform research activities.

Syllabus

Unit – 1 **(30 hours)**

Practical: Preparing a research article/dissertation, Preparing a mathematical talk and poster using beamer.

Unit – 2 **(30 hours)**

Practical: Learning and using Mathematical Software like MATLAB, Mathematica and Scilab.

Essential Readings

[1] M. Goossens, F. Mittelbach, S. Rahtz, D. Roegel and H. Voss, *The LaTeX Graphics Companion*, Addison-Wesley, 2008.
[2] N. J. Higham, *Handbook of Writing for the Mathematical Sciences*, SIAM, 1998
[3] L. Lamport, *LaTeX, a Document Preparation System*, Pearson, 2008.
[4] P. Wellin, S. Kemin and R. Gaylord, *An Introduction to Programming with Mathematica*, Third Edition, Cambridge University Press, UK, 2005.

Syllabi of Courses in Semester-II of One-year M.Sc. Mathematics under Structure-3 (Research)

Discipline Specific Elective (DSE) Courses

DISCIPLINE SPECIFIC ELECTIVE: ADVANCED FLUID DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Advanced Fluid Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Calculus and Partial Differential Equations

Learning Objectives

The primary objective of this course is to:

- prepare a foundation for advanced studies in compressible flow, boundary layer theory and magnetohydrodynamics.
- develop concepts, models, and techniques that enable problem-solving in compressible flow, boundary layer theory and magnetohydrodynamics.
- equip students with concepts and techniques to conduct research in the above mentioned domains.

Learning Outcomes

This course will enable the students to:

- learn conservation laws, first and second laws of thermodynamics, internal energy and entropy, different forms of energy equations and dimensional analysis.
- know about compressibility in real fluids, wave motion, sound waves, hyperbolic and dispersive waves, shock waves, their formation, properties and elementary analysis.
- know the concepts of boundary layer, boundary layer equations and their solutions, measurements of boundary layer thickness.
- understand the interaction between hydrodynamic processes and electromagnetic phenomena.
- formulate the basic equations of motion in inviscid and viscous conducting fluid flow and explain Alfven's theorem and magnetohydrodynamic (MHD) waves and MHD shocks.

Syllabus

Unit – 1 **(11 hours)**

Flow characteristics, Conservation laws, Equation of state of a substance, First and second law of thermodynamics, Internal energy and entropy, Energy equation, Nondimensionalizing the basic equations of incompressible viscous fluid flow, Nondimensional numbers.

Unit – 2 **(12 hours)**

Compressibility effects in real fluids, Equations of motion, Sound wave, Hyperbolic and dispersive waves, Isentropic gas flow, Flow through a nozzle, Method of characteristics, Shock jump conditions, Non-linear plane waves, Shock waves and their elementary analysis, Similarity solutions.

Unit – 3 **(11 hours)**

Boundary layer concept, Estimation of boundary layer thickness and friction forces, Prandtl's boundary layer equations, Boundary layer along a flat plate, Boundary layer thickness, General properties of the boundary layer equations, Similar solutions, Momentum and energy integral equations for the boundary layer.

Unit – 4 **(11 hours)**

Maxwell's electromagnetic field equations, Magnetohydrodynamic (MHD) approximations, Magnetic field equation, Magnetic Reynolds number, Magnetic body force, Equations of Motions of conducting fluid, Alfven's theorem, MHD waves, MHD shock waves.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] F. Chorlton, *Text Book of Fluid Dynamics*, CBS Publisher, 2005.
[2] H. Schlichting and K. Gersten, *Boundary Layer Theory*, Ninth Edition, Springer, 2017.
[3] G. B. Witham, *Linear and Nonlinear Waves*, John Wiley & Sons, 1999.

Suggested Readings

(i) K. R. Cramer and S. I. Pai, *Magnetofluid Dynamics for Engineers and Applied Physics*, McGraw Hill Book Co., New York, 1973.
(ii) Y. Shao-Wen, *Foundations of Fluid Mechanics*, PHI, New Delhi, 1960.

DISCIPLINE SPECIFIC ELECTIVE: BANACH AND C*-ALGEBRAS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Banach and C*-Algebras	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis and Topology

Learning Objectives

The primary objective of this course is to introduce:

- Banach algebras and C*-algebras.
- various ways to construct new operator algebras using given ones.
- spectrum of elements in Banach algebras and to study its properties.
- Gelfand representations of commutative Banach algebras and of C*-algebras.

Learning Outcomes

This course will enable the students to:

- familiarize with the representations of operator algebras.
- realize commutative Banach algebras and abelian C*-algebras as space of continuous functions on locally compact groups.
- understand the powerful tool of functional calculus.
- identify any C*-algebra as closed *-subalgebra of space of bounded linear operators on a Hilbert space.

Syllabus

Unit – 1 **(11 hours)**

Elementary properties and examples of Banach algebras, Ideals and quotients, Invertible elements, Spectrum and spectral radius, Spectral radius formula, Spectral mapping theorem (for polynomials), Gelfand–Mazur theorem.

Unit – 2 **(11 hours)**

Multiplicative linear functionals, Commutative Banach algebra, w*-topology, Gelfand transform of an element, Structure space, Gelfand representation.

Unit – 3 **(12 hours)**

Elementary properties and examples of C*-algebras, Unitization, Gelfand–Naimark representation of commutative C*-algebras, Continuous functional calculus, Spectral mapping theorem for normal elements, Positive elements of C*-algebras.

Unit – 4 **(11 hours)**

Ideals in C*-algebras, Approximate units, Quotients, Positive linear functionals, Gelfand–Naimark–Segal representation of C*-algebras.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.
[2] G. J. Murphy, *C*-algebras and Operator Theory*, Academic Press Inc., 1990.

Suggested Readings

(i) J. B. Conway, *A Course in Operator Theory, Graduate Texts in Mathematics*, Springer, 2007.
(ii) J. Dixmier, *C*-algebras*, North-Holland Publishing Company, 1977.
(iii) R. G. Douglas, *Banach Algebra Techniques in Operator Theory, Graduate Texts in Mathematics*, Springer, 1998.
(iv) E. Kaniuth, *A Course on Commutative Banach Algebras*, Graduate Texts in Mathematics, Springer, 2009.
(v) S. Kantorovitz, *Introduction to Modern Analysis*, Oxford Graduate Texts in Mathematics, Oxford University Press, 2006.
(vi) M. Takesaki, *Theory of Operator Algebras I*, Springer, 2002.

DISCIPLINE SPECIFIC ELECTIVE: CHAOS THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Chaos Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- introduce some useful and interesting notions like Topological Transitivity and Sensitive dependence on initial conditions.
- study different types of chaos including Devaney's chaos and finding their interrelationships.
- know classical result that period three implies chaos on intervals.
- relate chaos and decomposition theorems.
- study Topological entropy through open covers and also Bowen's definition of entropy, equivalence of these two definitions on compact metric spaces.
- study various interesting results related to topological entropy.

Learning Outcomes

This course will enable the students to:

- construct interesting examples of Topological transitive maps, Topological mixing maps etc.
- know classical examples of Devaney's chaotic maps like tent map, shift maps, logistic maps.
- study and compare different types of chaos.
- find relation between transitivity and chaos on intervals.
- relate chaos theory and classical decomposition theorems.
- study very useful notion of Topological entropy including its properties.
- calculate entropy of any homeomorphism of closed unit interval and of unit circle.

Syllabus

Unit – 1 **(12 hours)**

Topological Transitivity, Locally eventually onto maps, Topological mixing, Sensitive dependence on initial conditions, Devaney's definition of chaos, Transitivity and limit sets for continuous interval maps.

Unit – 2 **(11 hours)**

Characterizing topological mixing in terms of topological transitivity for continuous interval maps, Topological Weakly Mixing, Totally Transitive maps, Relation between transitivity and chaos on intervals, Logistic maps and shift maps as chaotic maps.

Unit – 3 **(12 hours)**

Various other definitions of chaos and their interrelationships. Period three implies chaos, Chaos and decomposition theorems including Bowen's decomposition theorem, Topological Entropy: Definition using open covers, Examples and properties, Bowen's definition of topological entropy,

Equivalence of two definitions, Topological version of Kolmogorov–Sinai theorem.

Unit – 4 **(10 hours)**
Topological Entropy of maps on a compact metric space, Topological Entropy of product maps, of iterations of a map, Topological entropy of an expansive homeomorphism on a compact metric space, of the two-sided shift, of any homeomorphism of the unit interval and of the unit circle.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] N. Aoki and K. Hiraide, *Topological Theory of Dynamical Systems: Recent Advances*, North Holland Publications, 1994.
[2] R. L. Devaney, *A First Course in Chaotic Dynamical Systems*, CRC Press, 2018.
[3] S. Ruette, *Chaos for Continuous Interval Maps: A Survey of Relationship Between Various Kinds of Chaos*, 2018.
[4] Peter Walters, *An Introduction to Ergodic Theory*, Springer, 2000.

Suggested Readings
(i) L. Alsedà, J. Llibre and M. Misiurewicz, *Combinatorial Dynamics and Entropy in Dimension One*, Advanced Series in Nonlinear Dynamics, World Scientific, 2000.
(ii) L. S. Block and W. A. Coppel, *Dynamics in One Dimension*, Springer, 2014.
(iii) M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge University Press, 2015.

DISCIPLINE SPECIFIC ELECTIVE: COMPUTATIONAL FLUID DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Computational Fluid Dynamics	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Partial Differential Equation (Undergraduate level)

Learning Objectives
The primary objective of this course is to teach:
- various numerical schemes on finite difference and finite volume methods for solving PDEs.
- discretization errors and grid dependence.
- some real-world applications of PDEs and fluid dynamics.
- discretization of governing equations of diffusion, convection-diffusion, fluid flow and thereby computing the numerical solutions using the flow variables using algorithms.

Learning Outcomes
This course will enable the students to learn:
- techniques for solving the PDEs along with some initial and boundary conditions by using the finite difference and finite volume methods.
- the basic conservation principles of mass, momentum, energy, discretization of governing equations.
- discretization techniques.
- some popular algorithms like SIMPLE and SIMPLER used to obtain the solutions of steady and unsteady flow problems by finite volume methods.

Syllabus

Unit – 1 **(12 hours)**
Basics of discretization using finite differences, Various single and multi-step explicit and implicit finite difference schemes for 1-D and 2-D parabolic and hyperbolic initial boundary value problems, Alternating Direction Implicit schemes (ADI) for 2-D parabolic and hyperbolic equations, Order of accuracy, Consistency, Stability and convergence of a finite difference scheme, Courant Friedrich Lewy condition.

Unit – 2 **(12 hours)**
Finite difference schemes for second and fourth order 2-D elliptic boundary value problem and applications, Finite volume method for diffusion and convection-diffusion equations, Discretization of one and two-dimensional steady state diffusion and convection-diffusion equations, Central difference, Upwind, Exponential, Hybrid, Power-law and QUICK schemes and their properties.

Unit – 3 **(11 hours)**
Flow field calculation, Pressure-velocity coupling, Vorticity-stream function approach, Primitive

variables, Staggered grid, Pressure and velocity corrections, Pressure correction equation, SIMPLE and SIMPLER algorithms.

Unit – 4 **(10 hours)**
Finite volume methods for unsteady flows, Discretization of one-dimensional transient heat conduction, Explicit, fully implicit and Crank–Nicolson schemes, Implementation of boundary conditions.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] J. C. Strikweda, *Finite Difference Schemes and Partial Differential Equations*, Second Edition, SIAM, 2004.
[2] H. K. Versteeg and W. Malalasekera, *An Introduction to Computational Fluid Dynamics: The Finite Volume Method*, Second Edition, Pearson, 2008.

Suggested Readings
(i) J. D. Anderson, *Computational Fluid Dynamics*, McGraw-Hill, 1995.
(ii) S. V. Patankar, *Numerical Heat Transfer and Fluid Flow*, CRC Press, Taylor and Francis, Indian Edition, 2017.
(iii) R. H. Pletcher, J. C. Tannehill and D. A. Anderson, *Computational Fluid Mechanics and Heat Transfer*, CRC Press, Taylor and Francis, 2013.
(iv) J. W. Thomas, *Numerical Partial Differential Equations: Finite Difference Methods*, Springer, 2013.

DISCIPLINE SPECIFIC ELECTIVE: DIFFERENTIAL TOPOLOGY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Differential Topology	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Topology

Learning Objectives

The primary objective of this course is to:

- introduce the concepts of topological manifolds, smooth structures, smooth manifolds, and manifolds with boundary.
- develop an understanding of smooth functions, smooth maps, diffeomorphisms, and tangent spaces.
- explain the Inverse function theorem, immersions and submersions.
- develop the fundamental concepts of 2-manifolds and distinguish between orientable and non-orientable surfaces.
- explore the properties of compact and connected surfaces.

Learning Outcomes

This course will enable the students to:

- identify and construct examples of topological manifolds, smooth structures and manifolds with and without boundary.
- demonstrate understanding of diffeomorphisms and tangent spaces.
- apply the Inverse function theorem, immersions and submersions.
- define key concepts such as 2-manifolds, orientability, compactness, connectedness and boundary of a surface.
- differentiate between orientable and non-orientable surfaces using examples such as the sphere, torus, Möbius strip and Klein bottle.

Syllabus

Unit – 1 **(12 hours)**

Topological manifolds, Topological properties of manifolds, Smooth structures, Examples of smooth manifolds, Manifolds with boundary.

Unit – 2 **(11 hours)**

Smooth functions and smooth maps, Lie groups, Diffeomorphisms.

Unit – 3 **(10 hours)**

Derivatives and tangents, Inverse function theorem, Immersions and submersions.

Unit – 4 **(12 hours)**

Complexes, Connected sum of two surfaces, Non-orientable surfaces (2- Manifolds), Compact and connected surfaces, Classification of compact and connected surfaces with and without boundary.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] V. Guillemin and Alan Pollack, *Differential Topology*, Prentice-Hall, 1974.
[2] L. C. Kinsey, *Topology of Surfaces*, Springer Verlag, 1997.
[3] J. M. Lee, *Introduction to Smooth Manifolds*, Second Edition, Springer, 2013.

Suggested Readings

(i) L. Conlon, *Differentiable Manifolds*, Second Edition, Birkhäuser Advanced Texts, 2001.
(ii) M. Spivak, *A Comprehensive Introduction to Differential Geometry*, Volume 1, Third Edition, Publish or Perish, Huston, Texas, 1999.
(iii) L. W. Tu, *Introduction to Manifolds*, Second Edition, Springer, 2011.
(iv) F. W. Warner, *Foundations of Differentiable Manifolds and Lie Group*, Springer-Verlag, 1983.

DISCIPLINE SPECIFIC ELECTIVE: GENERAL MEASURE THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: General Measure Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Measure Theory and Topology

Learning Objectives
The primary objective of this course is to introduce:
- real valued and complex valued measures.
- decomposition of measure spaces and of measures.
- extension of a premeasure to a measure, Lebesgue measure on Euclidean spaces.
- representation of measures and functionals in terms of integrals.
- product measures.

Learning Outcomes
This course will enable the students to:
- appreciate signed measures and complex measures, mutual singularity of measures, Hahn and Jordan decompositions, Lebesgue decomposition, Radon–Nikodym theorem.
- verify conditions under which a set function defined on a collection of subsets of a set has an extension to a measure on a sigma-algebra.
- apply Riesz representation theorem for bounded linear functionals on L^p-spaces.
- understand product measure and the results of Fubini and Tonelli, and express the Lebesgue measure on Euclidean spaces as a product measure.
- apply Riesz–Markov representation theorem for the bounded linear functionals on the space of continuous functions.

Syllabus

Unit – 1 **(13 hours)**
Signed measures, Hahn and Jordan decompositions, Mutually singular measures, Radon–Nikodym theorem, Lebesgue decomposition, Complex measure.

Unit – 2 **(10 hours)**
The Carathéodory extension theorem, Lebesgue measure on $\mathbb{R}^n$, Regularity and translation invariance of Lebesgue measure on $\mathbb{R}^n$.

Unit – 3 **(10 hours)**
Riesz representation theorem for the dual of L^p-spaces, Product measures, Fubini's theorem, Tonelli's theorem.

Unit – 4 **(12 hours)**
Locally compact Hausdorff spaces and construction of Radon measure, Riesz–Markov representation theorem for positive linear functionals on $C_c(X)$, Riesz representation theorem for the dual of $C(X)$.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] H. L. Royden and P. M. Fitzpatrick, *Real Analysis*, Fourth Edition, Pearson, 2015.
[2] M. E. Taylor, *Measure Theory and Integration*, American Mathematical Society, 2006.

Suggested Readings

(i) G. B. Folland, *Real Analysis: Modern Techniques and Their Applications*, Second Edition, Wiley, New York, 1999.
(ii) P. R. Halmos, *Measure Theory*, Springer Science + Business Media, LLC, 2014.
(iii) E. M. Stein and R. Shakarchi, *Real Analysis: Measure Theory, Integration and Hilbert Spaces*, New Age International Publishers, New Delhi, 2010.

DISCIPLINE SPECIFIC ELECTIVE: NONSMOOTH OPTIMIZATION

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Nonsmooth Optimization	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Nonlinear Optimization

Learning Objectives

The primary objective of this course is to:

- understand the tools to deal with nonsmooth convex functions.
- study conjugate duality in terms of conjugate functions for constrained nonlinear optimization problems.
- introduce numerical techniques to solve constrained nonlinear optimization problems.

Learning Outcomes

This course will enable the students to learn:

- the notions of subgradients and subdifferentials for nonsmooth convex functions.
- the use of conjugate functions to develop the theory of conjugate duality.
- about numerical methods like gradient descent method, gradient projection method, Newton's method and conjugate gradient method.
- penalty approach technique to solve constrained nonlinear optimization problems.

Syllabus

Unit – 1 **(11 hours)**

Extended real valued functions, Proper convex functions, Closure of convex functions, Differential derivatives, Subgradients and subdifferentials.

Unit – 2 **(12 hours)**

Conjugate functions, Biconjugate functions, Perturbation functions, Closure of convex functions, Directional derivatives, Subgradients and subdifferentials.

Unit – 3 **(12 hours)**

Gradient descent method, Gradient projection method, Newton's method, Conjugate gradient method.

Unit – 4 **(10 hours)**

Penalty function methods, Exterior penalty function, Interior penalty functions.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] M. Avriel, *Nonlinear Programming: Analysis and Methods*, Dover Publications, 2003.

[2] O. Güler, *Foundations of Optimization*, Springer, 2010.

Suggested Readings

(i) A. Bagirov, N. Karitsa and M. M. Makela, *Introduction to Nonsmooth Optimization: Theory, Practice and Software*, Springer, 2014.
(ii) M. S. Bazaraa, H. D. Sherali and C. M. Shetty, *Nonlinear Programming: Theory and Algorithms,* Third Edition, John Wiley & Sons, 2013.
(iii) D. G. Luenberger and Y. Ye, *Linear and Nonlinear Programming*, Springer, 2008.

DISCIPLINE SPECIFIC ELECTIVE: PROBABILITY THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Probability Theory	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Measure Theory

Learning Objectives

The primary objective of this course is to introduce:

- probability space as a measure space and random variables as measurable functions.
- expectation and moments of random variables.
- notion of convergence in probability.
- conditioning on sub-σ-algebra.

Learning Outcomes

This course will enable the students to learn:

- about probability or uncertainty in abstract setting.
- moments and expectation of random variables which help to understand applications of probability in industry.
- how to apply the idea of convergence in probability.
- weak law and strong law of large numbers and their applications.

Syllabus

Unit – 1 **(11 hours)**

Probability: σ-algebra, Constructing probability triples, The extension theorem, Random variables, Independence of events, Continuity of probabilities, Limit events, The Borel–Cantelli Lemma.

Unit – 2 **(10 hours)**

Expected values: Simple, general non-negative and arbitrary random variables, Moment generating functions, Markov's inequality, Chebyshev's inequality.

Unit – 3 **(12 hours)**

Convergence of random variables: Convergence almost surely, Convergence in probability, Weak law of large numbers, Strong law of large numbers.

Unit – 4 **(12 hours)**

Distributions of random variables: Examples of distributions, Characteristic functions, The central limit theorem, Conditional probability, Conditioning on random variable, Conditioning on a sub-σ-algebra, Conditional variance.

Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] J. S. Rosenthal, *A First Look at Rigorous Probability Theory*, Second Edition, World Scientific, Singapore, 2006.

Suggested Readings

(i) W. Feller, *An Introduction to Probability Theory and its Applications*, Volume 1, Third Edition, Wiley, 2008.
(ii) J. E. Michael and J. S. Rosenthal, *Probability and Statistics: The Science of Uncertainty*, Second Edition, W. H. Freeman & Co Ltd., 2009.
(iii) S. Ross, *A First Course in Probability*, Tenth Edition, Pearson Education, 2022.
(iv) D. W. Stroock, *Probability Theory, An Analytic View*, Cambridge University Press, 2024.

DISCIPLINE SPECIFIC ELECTIVE: THEORY OF NON-COMMUTATIVE RINGS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Theory of Non-commutative Rings	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Groups and Rings

Learning Objectives

The primary objective of this course is to:

- give students an understanding of Wedderburn–Artin theory of semisimple rings.
- develop Jacobson's general theory of radicals, prime and semiprime rings, and primitive and semiprimitive rings.
- introduce the structure of primitive rings as a generalisation of the Wedderburn–Artin theorem on Artinian simple rings.

Learning Outcomes

This course will enable the students to:

- know about an extensive variety of rings, including free rings, Weyl algebra, Hilbert twist and triangular ring.
- understand the module theoretic definition of semisimple rings and how it leads to the Wedderburn–Artin structure theorem on their complete classification.
- know Jacobson's general theory of radicals, semiprime rings, prime, primitive and semiprimitive rings and their structures.
- understand the significance of the fundamental result 'Density Theorem' and its consequences on the structure of primitive rings.

Syllabus

Unit – 1 **(11 hours)**

Simple rings, Reduced rings, Dedekind-finite rings, Algebra, Quaternions, Free k-rings, Rings with generators and relations, Weyl algebra, Formal power series ring, Hilbert's twist ring, Differential polynomial rings, Derivation and inner derivation on a ring, Triangular rings, Characterization of one-sided and two-sided ideals in such rings.

Unit – 2 **(11 hours)**

Noetherian and Artinian rings, Examples of one-sided Noetherian and Artinian triangular rings, Twisted polynomial ring and Quotient of free $\mathbb{Z}$-ring, Semisimple rings, Structure of semisimple rings: Wedderburn–Artin's theorem.

Unit – 3 **(10 hours)**

Structure theorem of simple left Artinian rings, Jacobson radical, J-semisimple rings, Nil and nilpotent ideals, Connection between semisimple and J-semisimple rings, Hopkins–Levitzki theorem, Nakayama's lemma.

Unit – 4 **(13 hours)**
Prime radical, Characterisation of prime and semiprime ideals, Prime and semiprime rings, Structure theorem of primitive rings, Density theorem.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings
[1] T.-Y. Lam, *A First Course in Noncommutative Rings*, Springer, 2001.

Suggested Readings
(i) I. N. Herstein, *Noncommutative Rings*, The Mathematical Association of America, 2005.
(ii) T. W. Hungerford, *Algebra*, Springer-Verlag, New York, 1981.
(iii) L. H. Rowen, *Ring Theory*, Student Edition, Academic Press, 1991.

DISCIPLINE SPECIFIC ELECTIVE: THEORY OF UNBOUNDED OPERATORS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
DSE: Theory of Unbounded Operators	4	3	1	0	Same as for entry to M.Sc. Mathematics	Basics of Functional Analysis and Bounded Operators

Learning Objectives
The primary objective of this course is to:
- introduce the notion of unbounded operators.
- develop the theory of operator semigroups and understand their role in applications, particularly for solving differential equations.

Learning Outcomes
This course will enable the students to:
- identify closed and closable linear operators on Banach spaces.
- compute adjoints of unbounded linear operators.
- understand spectral properties of some unbounded operators.
- comprehend the role unbounded operators and semigroups play in applications, particularly in studying solutions of differential equations.

Syllabus

Unit – 1 **(10 hours)**
Unbounded linear operators, Hilbert adjoints, Hellinger–Toeplitz theorem, Hermitian, symmetric and self-adjoint linear operators, Closed linear operators, Closable operators and their closures on Banach spaces.

Unit – 2 **(12 hours)**
Cayley transform, Deficiency indices, Spectral properties of self-adjoint operators, Multiplication and differentiation operators and their spectra.

Unit – 3 **(11 hours)**
Analytic properties of exponential functions, Matrix Semigroups, Uniformly continuous semigroups, Semigroups on Hilbert spaces, Strongly continuous semigroups.

Unit – 4 **(12 hours)**
Generators of semigroups and their resolvents, Hille–Yosida theorem (for contraction semigroup), Dissipative operators and their properties, Lumer–Phillips theorem, Generators of Group, Stone's theorem.

Tutorial
Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

Essential Readings

[1] K. J. Engle and R. Nagel, *One-parameter Semigroups for Linear Evolution Equations*, Springer-Verlag, New York, 2000.

[2] E. Kreyszig*, Introductory Functional Analysis with Applications*, John Wiley & Sons, 2006.

Suggested Readings

(i) S. Goldberg, *Unbounded Linear Operators: Theory and Applications*, Dover Publications, 2006.

(ii) E. Hille and R. S. Phillips, *Functional Analysis and Semi-groups*. American Mathematical Society, Providence, RI, 1957.

(iii) A. Pazy*, Semigroups of Linear Operators and Applications to Partial Differential Equations*, Springer, 1983.

(iv) M. Schechter, *Principles of Functional Analysis*, Second Edition, American Mathematical Society, 2001.

(v) J. Weidmann, *Linear Operators in Hilbert Spaces, Graduate Texts in Mathematics*, Springer, New York, 1980.

Research Methods/ Tools/ Writing Courses

TECHNIQUES OF RESEARCH WRITING

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Techniques of Research Writing	2	0	0	2	Same as for entry to M.Sc. Mathematics	NIL

Learning Objectives

The primary objective of this course is to:

- equip the students with the techniques for writing mathematics properly.
- provide knowledge about differentiating between good and bad mathematics writing.
- prepare the students with basic skills required for writing a research article in mathematics.

Learning Outcomes

This course will enable the students to:

- understand the difference between good and bad mathematical writing.
- know the techniques of writing a good research article in mathematics.

Syllabus

Unit – 1 **(30 hours)**
Practical: Learning basic skills of writing mathematics.

Unit – 2 **(30 hours)**
Practical: Writing a report on any topic of core discipline.

Essential Readings

[1] N. J. Higham, *Handbook for writing for the Mathematical Sciences*, SIAM, 1998.
[2] N. E. Steenrod, P. R. Halmos, M. M. Schiffer and J. A. Dieudonné, *How to Write Mathematics*, American Mathematical Society, 1973.