Two-year M.Sc. Mathematics

(based on Post Graduate Curriculum Framework (PGCF) -2024)

Effective from Academic Session 2025-26

Department of Mathematics
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Programme Objectives and Outcomes

Programme Objectives

The M.Sc. Mathematics programme's main objectives are to:

- inculcate and develop in the students mathematical aptitude and the ability to think abstractly.
- develop computational abilities and programming skills of the students.
- develop the ability to read, follow and appreciate mathematical text among the students.
- train students to communicate mathematical ideas in a lucid and effective manner.
- train students to apply their theoretical knowledge to solve real-life problems.
- prepare the students for higher education and research in mathematics.

Programme Outcomes

On successful completion of the M.Sc. Mathematics programme, a student will:

- have a strong foundation in core areas of Mathematics, both pure and applied.
- be able to apply mathematical skills and logical reasoning for problem solving.
- communicate mathematical ideas effectively, in writing as well as orally.
- have sound knowledge of mathematical modelling, programming and computational techniques as required for employment in industry.

Curricular Structure for First Year

Semester	DSC	DSE	2 Credit Course	Total Credits
Semester-I	**DSC-1** **DSC-2** **DSC-3** (12 Credits)	**DSE-1** **DSE-2** OR **DSE-1 & GE-1** (8 Credits)	**Skill-based course/ Workshop/ Specialized laboratory/ Hands on learning** (2 Credits)	**22**
Semester-II	**DSC-4** **DSC-5** **DSC-6** (12 Credits)	**DSE-3** **DSE-4** OR **DSE-2 & GE-2** (8 Credits)	**Skill-based course/ Workshop/ Specialized laboratory/ Hands on learning** (2 Credits)	**22**

Details of Courses in 1st Year of Two-year M.Sc. Mathematics

Semester	DSC	DSE	2 Credit Course
Semester-I	**DSC-1:** Field Theory **DSC-2**: Introduction to Topology **DSC-3**: Ordinary Differential Equations	**DSE-1:** (i) Matrix Analysis (ii) Numerical Analysis **DSE-2*:** (i) Advanced Group Theory (ii) Nonlinear Optimization *Student will opt for DSE-2 or GE-1	Communicating Mathematics
Semester-II	**DSC-4**: Module Theory **DSC-5**: Functional Analysis **DSC-6**: Complex Analysis	**DSE-3:** (i) Algebraic Number Theory (ii) General Topology **DSE-4#:** (i) Fourier Analysis (ii) Integral Equations #Student will opt for DSE-4 or GE-2	Appreciating Mathematics via Workshops and Seminars

GE Courses (offered by the Department of Mathematics)

Semester-I	**GE-1:** (i) Matrix Analysis (ii) Nonlinear Optimization
Semester-II	**GE-2**: (i) Fourier Analysis (ii) Integral Equations

Semester-I

Discipline Specific Core (DSC) Courses

DISCIPLINE SPECIFIC CORE – 1: FIELD THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course		
		Lecture	Tutorial	Practical
DSC-1: Field Theory	4	3	1	0

Learning Objectives
The primary objective of this course is to:
- study the ancient problem of solvability of polynomials over the rational field.
- understand a necessary and sufficient condition for the solvability of polynomials in terms of radical expression.
- apply Galois theory to classical problems, such as the insolvability of the general quintic.

Learning Outcomes
This course will enable the students to:
- identify and construct examples of fields, distinguish between algebraic and transcendental extensions, characterize normal extensions in terms of splitting fields and prove the existence of algebraic closure of a field.
- characterize perfect fields using separable extensions, construct examples of automorphism group of a field and Galois extensions as well as prove Artin's theorem and the fundamental theorem of Galois theory.
- classify finite fields using roots of unity and Galois theory and prove that every finite separable extension is simple.
- use Galois theory of equations to prove that a polynomial equation over a field of characteristic is solvable by radicals if and only if its group (Galois) is a solvable group and hence deduce that a general quintic equation is not solvable by radicals.

SYLLABUS OF DSC-1

Unit – 1 **(16 hours)**
Fields and their extensions, Splitting fields, Normal extensions, Algebraic closure of a field, Separability, Perfect fields.

Unit – 2 **(18 hours)**
Automorphisms of field extensions, Artin's theorem, Galois extensions, Fundamental theorem of Galois theory, Roots of unity, Cyclotomic polynomials and extensions, Finite fields.

Unit – 3 **(11 hours)**
Theorem of primitive element and Steinitz's theorem, Galois theory of equations, Theorem on natural irrationalities, Radical extension and solvability by radicals.

Essential Readings

[1] P. M. Cohn, *Basic Algebra*, Springer International Edition, 2003.

Suggested Readings

(i) D. S. Dummit & R. S. Foote, *Abstract Algebra*, Wiley Student Edition, 2011.
(ii) T. W. Hungerford, *Algebra*, Springer-Verlag, 1981.
(iii) N. Jacobson, *Basic Algebra*, Volume I, Dover Publications Inc., 2009.
(iv) I. Stwart, *Galois Theory*, CRC Press, 2015.

DISCIPLINE SPECIFIC CORE – 2: INTRODUCTION TO TOPOLOGY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course		
		Lecture	Tutorial	Practical
DSC-2: Introduction to Topology	4	3	1	0

Learning Objectives

The primary objective of this course is to introduce:

- basic principles of point-set topology, including bases and subbases for a topology.
- continuity, homeomorphisms, and different types of topologies, such as product and box topologies.
- key notions of connectedness and local connectedness.
- compactness and its significance in topological spaces.

Learning Outcomes

This course will enable the students to:

- analyze subsets of topological spaces by determining their interior, closure, boundary, and limit points, as well as identifying bases and subbases.
- identify continuous functions between topological spaces, analyze mappings into product spaces, and compare topological properties of different spaces.
- evaluate the connectedness and path connectedness of the product of an arbitrary family of spaces.
- understand key classifications of topological spaces, including Hausdorff spaces, first and second countable spaces, and separable spaces.
- explore advanced concepts such as limit point compactness and Tychonoff's theorem.

SYLLABUS OF DSC-2

Unit – 1 **(14 hours)**

Topological spaces, Basis, Order topology, Subspace topology, Metric topology, Closed set and limit points, Hausdorff spaces, Continuous functions, Homeomorphism.

Unit – 2 **(16 hours)**

The box and product topologies, Metrizability of products of metric spaces, Connected and path connected spaces, Locally connected and locally path connected spaces, Connectedness of product of spaces.

Unit – 3 **(15 hours)**

First and second countable spaces, Separable spaces, Compact spaces, The Tychonoff theorem, Limit point compactness.

Essential Readings

[1] J. R. Munkres, *Topology*, Updated Second Edition, Pearson, 2021.
[2] T. B. Singh, *Introduction to Topology*, Springer Nature, 2019.

Suggested Readings

(i) G. E. Bredon, *Topology and Geometry*, Springer, 2014.
(ii) J. Dugundji, *Topology*, Allyn and Bacon Inc., Boston, 1978.
(iii) J. L. Kelley, *General Topology*, Dover Publications, 2017.
(iv) G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill Education, 2017.
(v) L. A. Steen & J. A. Seebach, *Counterexamples in Topology*, Dover Publications, 2013.
(vi) S. Willard, *General Topology*, Dover Publications, 2004.

DISCIPLINE SPECIFIC CORE – 3: ORDINARY DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course		
		Lecture	Tutorial	Practical
DSC-3: Ordinary Differential Equations	4	3	1	0

Learning Objectives

The objective of this course is to study:

- existence, uniqueness, and continuity of solutions of initial value problems (IVPs)
- homogeneous and non-homogeneous linear systems
- stability of solutions for systems of ordinary differential equations.
- eigenvalues and eigenfunctions of Sturm-Liouville systems and Green's functions
- applications of theory of ordinary differential equations in real world problems.

Learning Outcomes

After studying this course, the student will be able to:

- know about the existence, uniqueness, and continuity of solutions of IVPs.
- apply the matrix method of solution for linear systems of differential equations.
- analyze the stability of solutions for systems of ordinary differential equations.
- understand Green's functions and their applications in the solution of boundary value problems (BVPs).
- comprehend the properties of eigenvalues and eigenfunctions of Sturm-Liouville systems.

SYLLABUS OF DSC-3

Unit – 1 **(18 hours)**

Well-posed problems, Existence, uniqueness, and continuity theorems for the solution of IVPs of the first order, Picard's method, Existence and uniqueness of solution for systems and higher order IVPs, Global existence theorem, Homogeneous and non-homogeneous linear systems, Linear systems with constant coefficients and their solution by matrix method, Linear equations with periodic coefficients.

Unit – 2 **(15 hours)**

Stability of autonomous system of differential equations, Critical point of an autonomous system and their classification. Stability of linear systems with constant coefficients, Linear plane autonomous system and phase portrait analysis, Perturbed systems, Method of Lyapunov for nonlinear systems, Limit cycles, Poincare-Bendixson's theorem and its applications.

Unit – 3 **(12 hours)**

Sturm separation and comparison theorems, Adjoint forms and Lagrange's identity, Two-point boundary value problems, Green's functions, Construction of Green's functions, Sturm-Liouville systems, eigenvalues and eigenfunctions.

Essential Readings

[1] E. A. Coddington, *An Introduction to Ordinary Differential Equations*, Dover

Publications, 2012.
[2] T. Myint-U, *Ordinary Differential Equations*, Elsevier, North-Holland, 1978.
[3] S. L. Ross, *Differential Equation*s, Second Edition, John Wiley & Sons, India, 2007.

Suggested Readings

(i) L. Perko, *Differential Equations and Dynamical Systems*, Springer, 2001.
(ii) G. F. Simmons, *Differential Equations with Applications and Historical Notes*, Third Edition, CRC Press, 2017.

Discipline Specific Elective (DSE) Courses

DISCIPLINE SPECIFIC ELECTIVE – 1 (i): MATRIX ANALYSIS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course		
		Lecture	Tutorial	Practical
DSE-1 (i): Matrix Analysis	4	3	1	0

Learning Objectives
The primary objective of this course is to:
- create a bridge between undergraduate-level matrix theory and matrix theory used in applications and research.
- give exposure to some topics of linear algebra that find application in analysis.

Learning Outcomes
This course will enable the students to learn:
- various useful matrix decompositions.
- variety of matrix norms and useful facts about the spectral radius.
- positive definite matrices and matrix ordering.
- some useful products of matrices.
- the powerful tool of non-negative matrices and the celebrated Perron's theorem which arises in many applications.

SYLLABUS OF DSE-1 (i)

Unit – 1 **(18 hours)**
Schur decomposition, Spectral decomposition, Jordan decomposition, Singular value decomposition, QR factorization, LU factorization, Matrix norms, Gelfand formula for spectral radius, Matrix functions including matrix exponential.

Unit – 2 **(11 hours)**
Positive definite and semidefinite matrices and their properties, A pair of positive semidefinite matrices (matrix-ordering), Polar value decomposition, Fischer's inequality, Hadamard's inequality, Minkowski's inequality.

Unit – 3 **(16 hours)**
Schur and Kroneckar products and their properties, Schur and Kroneckar products of positive definite matrices, Gershgorin regions, Condition number of matrix, Eigenvalue perturbation theorem, Bauer-Fike's theorem, Permutation and Doubly Stochastic matrices, Birkhoff's theorem, Non-negative matrices and their properties, Perron's theorem.

Essential Readings
[1] R. A. Horn & C. R. Johnson, *Matrix Analysis*, Second Edition, Cambridge University Press, 2013.
[2] F. Zhang, *Matrix Theory: Basic Results and Techniques*, Second Edition, Springer, 2011.

Suggested Readings

(i) R. Bhatia, *Matrix Analysis*, Springer, 1997.
(ii) A. J. Laub, *Matrix Analysis for Scientists and Engineers*, SIAM, 2005.
(iii) C. D. Meyer, *Matrix Analysis and Applied Linear Algebra*, SIAM, 2000.

DISCIPLINE SPECIFIC ELECTIVE – 1 (ii): NUMERICAL ANALYSIS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course		
		Lecture	Tutorial	Practical
DSE-1 (ii): Numerical Analysis	4	3	1	0

Learning Objectives
The main objective of this course is to:
- get robust understanding of the basic theory behind numerical methods.
- analyze the accuracy, efficiency, and stability of numerical algorithms.
- develop an ability to evaluate the suitability of various numerical techniques based on problem context and computational constraints.

Learning Outcomes
This course will enable the students to:
- apply algorithms for solving linear and nonlinear equations, analyze the convergence properties and potential pitfalls of these methods.
- develop a comprehensive understanding of the core concepts and principles behind both direct and indirect solvers for system of linear equations.
- use various interpolation techniques and understand their error estimates.
- implement various techniques to approximate derivatives and integrals and analyze them.
- construct and analyze numerical methods for solving initial value problems, covering key aspects like convergence, stability, and error propagation.

SYLLABUS OF DSE-1 (ii)

Unit – 1 **(18 hours)**
Floating-point approximation of a number, Source and propagation of error, Stability in numerical computation, Polynomial deflation, Laguerre's method, Horner's method, Muller's method for real and complex roots, Their order of convergence and convergence analysis, Existence and uniqueness of interpolating polynomial, Interpolation error, Neville's method of interpolation, Cubic spline interpolation, Trigonometric interpolation.

Unit – 2 **(12 hours)**
Special Matrices: Diagonal dominant matrices, Positive definite matrices, Band matrices and Tridiagonal matrices and their important properties, Eigen value problem by using Power method, Data fitting, Least square approximation and Chebyshev approximation.

Unit – 3 **(15 hours)**
Adaptive quadrature, Gaussian integration methods, Initial value problems, Difference equations, Local and global truncation error, Convergence and stability, Single step and multi-step methods, First and fourth-order Taylor series method, Extrapolation method, Adams-Moulton and Adams-Bashforth methods, Nystrom method.

Essential Readings

[1] K. E. Atkinson, *An Introduction to Numerical Analysis*, Second Edition, Wiley-India, 1989.

[2] R. L. Burden & J. D. Faires, *Numerical Analysis*, Ninth Edition, Cengage Learning India Private Limited, 2012.

[3] S. D. Conte & C. de Boor, *Elementary Numerical Analysis - An Algorithmic Approach*, Third Edition, McGraw-Hill, 1981.

Suggested Readings

(i) B. Bradie, *A Friendly Introduction to Numerical Analysis*, Pearson Education, India, 2006.

(ii) C. F. Gerald & P. O. Wheatley, *Applied Numerical Analysis*, Seventh Edition, Pearson Education, 2004.

(iii) F. B. Hildebrand, *Introduction to Numerical Analysis*, Second Edition, Dover Publications, 2008.

DISCIPLINE SPECIFIC ELECTIVE – 2 (i): ADVANCED GROUP THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course		
		Lecture	Tutorial	Practical
DSE-2 (i): Advanced Group Theory	4	3	1	0

Learning Objectives
The objective of this course is to introduce:
- the concepts of normal series, composition series and Zessenhaus lemma.
- solvable groups, nilpotent group and fitting and Frattini subgroup.
- free group, presentation of a group and properties of a free group.

Learning Outcomes
This course will enable the students to:
- understand Schreier's refinement theorem, Jordan-Hölder theorem, and fundamental theorem of arithmetic using Jordan-Hölder theorem.
- learn the significance and proof of Hall's theorem, Schur's theorem and Burnside basis theorem.
- identify indecomposable spaces and to prove the Krull-Schmidt theorem.
- determine distinct presentations of a group.

SYLLABUS OF DSE-2 (i)

Unit – 1 **(18 hours)**
Normal series, Composition series, Zessenhaus lemma, Schreier's refinement theorem, Jordan-Hölder theorem, Solvable groups, Derived series.

Unit – 2 **(15 hours)**
Supersolvable groups, Minimal normal subgroup, Hall's theorem, Hall subgroup, p-complements, Central series, Schur's theorem. Nilpotent groups, Fitting subgroup, Jacobi identity.

Unit – 3 **(12 hours)**
Three subgroup lemma, Frattini subgroup, Burnside basis theorem, Indecomposable groups, Fitting's lemma, Krull-Schmidt theorem, Semidirect product, Free group, Generators and relations of a group.

Essential Readings
[1] J. J. Rotman, *An Introduction to the Theory of Groups*, Springer-Verlag, New York, 1995.

Suggested Readings
(i) T. W. Hungerford, *Algebra*, Springer-Verlag, New York, 1981.
(ii) D. J. S. Robinson, *A Course in the Theory of Groups*, Springer-Verlag, New York, 1996.
(iii) J. S. Rose, *A Course on Group Theory*, Dover Publication, New York, 1994.
(iv) M. Suzuki, *Group Theory-I*, Springer-Verlag, Berlin, 1982.

DISCIPLINE SPECIFIC ELECTIVE – 2 (ii): NONLINEAR OPTIMIZATION

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course		
		Lecture	Tutorial	Practical
DSE-2 (ii): Nonlinear Optimization	4	3	1	0

Learning Objectives
The primary objective of this course is to introduce:
- convex functions and separation theorems.
- optimality conditions for unconstrained and constrained nonlinear optimization problems.
- Lagrangian duals and study duality results.
- Wolfe's method for quadratic programming problems.

Learning Outcomes
This course will enable the students to:
- derive first and second order optimality conditions for unconstrained optimization problems.
- know the importance of Karush–Kuhn–Tucker necessary optimality conditions in constrained optimization.
- understand duality theory and saddle point theory in terms of Lagrangian function.
- investigate saddle point theory.

SYLLABUS OF DSE-2 (ii)

Unit – 1 **(18 hours)**
Existence theorems, First order optimality conditions for unconstrained optimization problems, Second order optimality conditions for unconstrained optimization problems, Convex functions, Differentiable convex functions.

Unit – 2 **(15 hours)**
Optimization on convex sets, Separation theorems, Fritz John optimality conditions for constrained optimization problems, Constraint qualifications, Karush−Kuhn−Tucker conditions for constrained optimization problems, Second order necessary and sufficient optimality conditions.

Unit – 3 **(12 hours)**
Langragian function, Lagrangian duality in nonlinear optimization, Strong duality in convex programming, Saddle points, Saddle point optimality, Duality for linear and quadratic problems, Wolfe's method for solving quadratic programming problems.

Essential Readings
[1] H. A. Eiselt & Carl-Louis Sandblom, *Nonlinear Optimization: Methods and Applications*, Springer, 2019.
[2] O. Güler, *Foundations of Optimization*, Springer, 2010.

Suggested Readings

(i) M. S. Bazaraa, H. D. Sherali & C. M. Shetty, *Nonlinear Programming: Theory and Algorithms*, Third Edition, John Wiley & Sons, 2013.
(ii) D. G. Luenberger & Y. Ye, *Linear and Nonlinear Programming*, Springer, 2008.
(iii) A. P. Ruszczyński, *Nonlinear Optimization*, Princeton University Press, 2006.

Two-Credit Course

COMMUNICATING MATHEMATICS

Learning Objectives

This course will train the students to read mathematics independently and to present it adeptly via posters as well as oral presentations.

Learning Outcomes

The course will enable the students to effectively communicate mathematical ideas, both verbally and in writing. It will also hone their analytical skills and their ability to think critically while encouraging them to work collaboratively.

Methodology

A group of 4-5 students will be assigned a mathematical article/ paper published in reputed journals/ periodicals, like *The American Mathematical Monthly* (Mathematical Association of America, Taylor and Francis, Link: https://www.tandfonline.com/journals/uamm20), *Mathematics Magazine* (Mathematical Association of America, Taylor and Francis, Link: https://www.tandfonline.com/journals/umma20), *The Mathematics Student* (Indian Mathematical Society, Link: https://www.indianmathsoc.org/MS.html) and *The Mathematical Intelligencer* (Springer Nature, Link: https://link.springer.com/journal/283). The designated groups will be required to comprehend these articles under the supervision of a faculty member. The poster and oral presentations will be conducted for these groups as a part of their assessment process.

Generic Elective (GE) Courses

GENERIC ELECTIVE – 1 (i): MATRIX ANALYSIS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical		
GE-1 (i): Matrix Analysis	4	3	1	0	Class XII pass with Mathematics	Knowledge of Basic Analysis and Linear Algebra

Learning Objectives

The primary objective of this course is to:

- create a bridge between undergraduate-level matrix theory and matrix theory used in applications and research.
- give exposure to some topics of linear algebra that find application in analysis.

Learning Outcomes

This course will enable the students to learn:

- various useful matrix decompositions.
- variety of matrix norms and useful facts about the spectral radius.
- positive definite matrices and matrix ordering.
- some useful products of matrices.
- powerful tool of non-negative matrices and the celebrated Perron's theorem which arises in many applications.

SYLLABUS OF GE-1 (i)

Unit – 1 **(18 hours)**

Schur decomposition, Spectral decomposition, Jordan decomposition, Singular value decomposition, QR factorization, LU factorization, Matrix norms, Gelfand formula for spectral radius, Matrix functions including matrix exponential.

Unit – 2 **(11 hours)**

Positive definite and semidefinite matrices and their properties, A pair of positive semidefinite matrices (matrix-ordering), Polar value decomposition, Fischer's inequality, Hadamard's inequality, Minkowski's inequality.

Unit – 3 **(16 hours)**

Schur and Kroneckar products and their properties, Schur and Kroneckar products of positive definite matrices, Gershgorin regions, Condition number of matrix, Eigenvalue perturbation theorem, Bauer-Fike's theorem, Permutation and Doubly Stochastic matrices, Birkhoff's theorem, Non-negative matrices and their properties, Perron's theorem.

Essential Readings

[1] R. A. Horn & C. R. Johnson, *Matrix Analysis*, Second Edition, Cambridge University Press,

2013.
[2] F. Zhang, *Matrix Theory: Basic Results and Techniques*, Second Edition, Springer, 2011.

Suggested Readings

(i) R. Bhatia, *Matrix Analysis*, Springer, 1997.
(ii) A. J. Laub, *Matrix Analysis for Scientists and Engineers*, SIAM, 2005.
(iii) C. D. Meyer, *Matrix Analysis and Applied Linear Algebra*, SIAM, 2000.

GENERIC ELECTIVE – 1 (ii): NONLINEAR OPTIMIZATION

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical		
GE-1 (ii): Nonlinear Optimization	**4**	**3**	**1**	**0**	**Class XII pass with Mathematics**	**Knowledge of Real Analysis, Two and Three Dimensional Geometry, Calculus**

Learning Objectives
The primary objective of this course is to introduce:
- convex functions and separation theorems.
- optimality conditions for unconstrained and constrained nonlinear optimization problems.
- Lagrangian duals and study duality results.
- Wolfe's method for quadratic programming problems.

Learning Outcomes
This course will enable the students to:
- derive first and second order optimality conditions for unconstrained optimization problems.
- know the importance of Karush–Kuhn–Tucker necessary optimality conditions in constrained optimization.
- understand duality theory and saddle point theory in terms of Lagrangian function.
- investigate saddle point theory.

SYLLABUS OF GE-1 (ii)

Unit – 1 **(18 hours)**
Existence theorems, First order optimality conditions for unconstrained optimization problems, Second order optimality conditions for unconstrained optimization problems, Convex functions, Differentiable convex functions.

Unit – 2 **(15 hours)**
Optimization on convex sets, Separation theorems, Fritz John optimality conditions for constrained optimization problems, Constraint qualifications, Karush−Kuhn−Tucker conditions for constrained optimization problems, Second order necessary and sufficient optimality conditions.

Unit – 3 **(12 hours)**
Langragian function, Lagrangian duality in nonlinear optimization, Strong duality in convex programming, Saddle points, Saddle point optimality, Duality for linear and quadratic problems, Wolfe's method for solving quadratic programming problems.

Essential Readings
[1] H. A. Eiselt & Carl-Louis Sandblom, *Nonlinear Optimization: Methods and Applications*, Springer, 2019.

[2] O. Güler, *Foundations of Optimization*, Springer, 2010.

Suggested Readings

(i) M. S. Bazaraa, H. D. Sherali & C. M. Shetty, *Nonlinear Programming: Theory and Algorithms*, Third Edition, John Wiley & Sons, 2013.
(ii) D. G. Luenberger & Y. Ye, *Linear and Nonlinear Programming*. Springer, 2008.
(iii) A. P. Ruszczyński, *Nonlinear Optimization*, Princeton University Press, 2006.

Semester-II

Discipline Specific Core (DSC) Courses

DISCIPLINE SPECIFIC CORE – 4: MODULE THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course		
		Lecture	Tutorial	Practical
DSC-4: Module Theory	4	3	1	0

Learning Objectives

The primary objective of this course is to:

- introduce a new algebraic structure, namely, module which is a generalization of a vector space when the underlying field is replaced by an arbitrary ring. The study of modules over a ring also provides an insight into the structure of the ring.
- study free modules, finitely generated modules, projective and injective modules.
- classify the finitely generated modules over a principal ideal domain (PID).

Learning Outcomes

This course will enable the students to:

- identify and construct examples of modules, and apply homomorphism theorems on the same.
- define and characterize Noetherian, Artinian module, and apply the structure theorem of finitely generated modules over PID.
- distinguish between projective, injective, free, and semi simple modules.
- prove universal property of tensor product of modules, and Hilbert basis theorem.

SYLLABUS OF DSC-4

Unit – 1 **(13 hours)**

Basic concepts of module theory, Fundamental theorems of homomorphism, Direct product and direct sum of modules, Exact sequences, Split exact sequences.

Unit – 2 **(15 hours)**

Free modules, Modules over PID's, Projective and injective modules, Dual basis lemma, Baer's criterion, Divisible modules.

Unit – 3 **(17 hours)**

Semi simple modules, Tensor product of modules, Chain conditions, Hilbert basis theorem.

Essential Readings

[1] M. F. Athiyah & I. G. Macdonald, *Introduction to Commutative Algebra*, Addison Wesley, 1969.

[2] P. M. Cohn, *Basic Algebra*, Springer International Edition, 2003.
[3] P. M. Cohn, *Classic Algebra*, John Wiley & Sons Ltd., 2000.

Suggested Readings

(i) D. S. Dummit & R. M. Foote, *Abstract Algebra*, Wiley India Pvt. Ltd., 2011.
(ii) N. Jacobson, *Basic Algebra*, Volume II, Dover Publications Inc., 2009.
(iii) T. W. Hungerford, *Algebra*, Springer-Verlag, 1981.

DISCIPLINE SPECIFIC CORE – 5: FUNCTIONAL ANALYSIS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course		
		Lecture	Tutorial	Practical
DSC-5: Functional Analysis	4	3	1	0

Learning Objectives
The primary objective of this course is to:
- familiarize students with the basic tools of Functional Analysis involving normed spaces, Banach spaces and Hilbert spaces.
- investigate various properties of continuous and linear transformations.
- introduce different types of bounded linear operators.

Learning Outcomes
This course will enable the students to:
- understand completeness with respect to a norm and the interplay between continuity and boundedness of linear operators.
- comprehend and apply Hahn-Banach theorem, Open mapping theorem, Closed graph theorem and the Uniform boundedness theorem.
- understand the decomposition of a Hilbert space in terms of orthogonal complements and representation of a bounded linear functional in terms of inner product.
- check the convergence of operators and functionals, and weak and strong convergence of sequences.

SYLLABUS OF DSC-5

Unit – 1 **(12 hours)**
Banach spaces, Linear and continuous operators, Normed spaces of operators, Dual spaces.

Unit – 2 **(18 hours)**
Hahn-Banach theorem, Consequences of the Hahn-Banach theorem, Natural imbedding map, Open mapping theorem, Closed graph theorem, Uniform boundedness theorem, Conjugate of an operator, Eigen values, spectrum and resolvent of an operator.

Unit – 3 **(15 hours)**
Hilbert spaces, Orthogonal complements, Orthonormal sets, Reflexivity of Hilbert spaces, Hilbert-adjoint of an operator, Self-adjoint operators, Normal and unitary operators, Projections.

Essential Readings
[1] G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill Education, 2017.

Suggested Readings
(i) G. Bachman & L. Narici, *Functional Analysis*, Dover Publications, 2000.

(ii) R. Bhatia, *Notes on Functional Analysis*, Hindustan Book Agency, India, 2009.
(iii) E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, India, 2006.
(iv) M. Schechter, *Principles of Functional Analysis*, Second Edition, American Mathematical Society, 2001.

DISCIPLINE SPECIFIC CORE – 6: COMPLEX ANALYSIS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course		
		Lecture	Tutorial	Practical
DSC-6: Complex Analysis	4	3	1	0

Learning Objectives
The primary objective of this course is to:
- gain insights of well-known classical results in the field of complex analysis.
- investigate various properties of analytic functions, conformal mappings and Möbius transformations.
- derive various forms of Cauchy's theorem, integral formulas and maximum principles.
- represent complex-valued functions as Taylor and Laurent series.

Learning Outcomes
This course will enable the students to:
- construct Möbius transformations using Symmetry and Orientation principles.
- foresee the usage of simply connected regions in the complex plane for the existence of primitives and branch of logarithm.
- understand the behavior of zeros of analytic functions and meromorphic functions through Argument principle and Rouche's theorem.
- evaluate the real integrals involving rational and trigonometric functions by contour integration using Residue theorem.
- apply Schwarz's lemma to characterize the conformal maps of the open unit disk onto itself.

SYLLABUS OF DSC-6

Unit – 1 **(12 hours)**
Extended plane and its spherical representation, Analytic functions, Branch of logarithm, Conformal mappings, Möbius transformations.

Unit – 2 **(18 hours)**
Line integrals, Fundamental theorem of Calculus for line integrals, Power series representation of analytic functions, Zeros of analytic functions, Liouville's theorem, Index of a closed curve, Cauchy's theorem and integral formula, Morera's Theorem, Homotopic version of Cauchy's theorem and simple connectivity, Counting Zeros, Open mapping theorem, Goursat's theorem.

Unit – 3 **(15 hours)**
Classification of singularities, Laurent series, Casorati-Weierstrass theorem, Residue theorem with applications, Argument principle, Rouche's theorem, Maximum principles, Schwarz lemma.

Essential Readings
[1] J. B. Conway, *Functions of One Complex Variable*, Second Edition, Narosa Publishing House,

New Delhi, 2002.

Suggested Readings

(i) L. V. Ahlfors, *Complex Analysis*, Mc Graw Hill Co., Indian Edition, 2017.
(ii) T. W. Gamelin, *Complex Analysis*, Springer New York, NY, 2001.
(iii) L. Hahn & B. Epstein, *Classical Complex Analysis*, Jones and Bartlett, 1996.
(iv) E. M. Stein & R. Shakarchi, *Complex Analysis*, Princeton University Press, 2003.

Discipline Specific Elective (DSE) Courses

DISCIPLINE SPECIFIC ELECTIVE – 3 (i): ALGEBRAIC NUMBER THEORY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course		
		Lecture	Tutorial	Practical
DSE-3 (i): Algebraic Number Theory	4	3	1	0

Learning Objectives

The primary objective of this course is to introduce:

- algebraic number fields and their ring of integers.
- factorization into irreducibles in the ring of integers.
- factorization of ideals of the ring of integers.
- lattices, geometric representation of algebraic numbers, class-group and class-number.

Learning Outcomes

This course will enable the students to:

- classify algebraic number fields, define algebraic integers, ring of integers and integral bases, and calculate norms and traces. It would be possible to determine the integral bases and ring of integers of quadratic and p-th cyclotomic fields.
- construct examples of non-unique factorization domains and apply the unique factorization of certain ring of integers of number fields to solve some Diophantine equations.
- prove uniqueness of factorization of ideal of ring of integers of a number field in terms of prime ideals. It also leads to deduction of Two-Squares and Four-Squares theorem using Minkowski’s theorem on convex sets.
- visualize ideals of the ring of integers as lattices and develop tools to prove the finiteness of class-group.

SYLLABUS OF DSE – 3 (i)

Unit – 1 **(14 hours)**

Algebraic numbers, Conjugates and discriminants, Algebraic integers, Integral bases, Norms and traces, Rings of algebraic integers, Quadratic and cyclotomic fields.

Unit – 2 **(17 hours)**

Trivial factorization, Factorization into irreducibles, Examples of non-unique factorization into irreducibles, Prime factorization, Euclidean domains, Euclidean quadratic fields, Consequence of unique factorization the Ramanujan-Nagell theorem, Prime factorization of ideals, Norm of an ideal.

Unit – 3 **(14 hours)**

Lattices, Quotient torus, Minkowski's theorem, Two-Squares theorem, Four-Squares theorem, The space L^{st}, Class-group and class-number, Finiteness of the class-group, Factorization of a rational prime, Minkowski's constants, Some class-number calculations.

Essential Readings
[1] I. Stewart & D. Tall, *Algebraic Number Theory and Fermat's Last Theorem*, Fourth Edition, CRC Press, Boca Raton, FL, 2016.

Suggested Readings
(i) Ş. Alaca & K. S. Williams, *Introductory Algebraic Number Theory*, Cambridge University Press, Cambridge, 2003.
(ii) K. Ireland & M. Rosen, *A Classical Introduction to Modern Number Theory*, Second Edition, GTM 84, Springer-Verlag, New York, 1990.
(iii) S. Lang, *Algebraic Number Theory*, Second Edition, GTM 110, Springer-Verlag, New York, 1994.
(iv) D. A. Marcus, *Number Fields*, Second Edition, Universitext, Springer, 2018.

DISCIPLINE SPECIFIC ELECTIVE – 3 (ii): GENERAL TOPOLOGY

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course		
		Lecture	Tutorial	Practical
DSE-3 (ii): General Topology	4	3	1	0

Learning Objectives
The primary objective of this course is to introduce:
- quotient spaces, local compactness, and one-point compactification.
- separation axioms, the Urysohn lemma, and the Tietze extension theorem.
- paracompactness, metrization theorems, and the partition of unity.

Additionally, the course aims to equip students with essential tools and foundational knowledge for conducting advanced research in topology and related fields.

Learning Outcomes
This course will enable the students to:
- explore notable examples of quotient spaces, such as cones and suspensions.
- determine the one-point compactification of spaces like the real line and the n-sphere.
- understand key results on complete regularity and the Stone-Čech compactification.
- study fundamental theorems, including Urysohn's lemma and Tietze extension theorem.
- learn about important metrization theorems, such as the Urysohn metrization theorem.
- gain insights into characterizations of paracompactness in regular spaces and the role of partition of unity.

SYLLABUS OF DSE – 3 (ii)

Unit – 1 **(16 hours)**
Quotient spaces, Identification maps, Local compactness, One-point compactification, Proper Maps and regularity.

Unit – 2 **(14 hours)**
Complete regularity, Stone-Čech compactification, Normality, Urysohn's lemma, Tietze extension theorem.

Unit – 3 **(15 hours)**
Urysohn metrization theorem, Paracompactness, Characterizations of paracompactness in regular spaces, Partition of unity.

Essential Readings
[1] J. R. Munkres, *Topology*, Updated Second Edition, Pearson, 2021.
[2] T. B. Singh, *Introduction to Topology*, Springer Nature, 2019.

Suggested Readings
(i) G. E. Bredon, *Topology and Geometry*, Springer, 2014.
(ii) J. Dugundji, *Topology*, Allyn and Bacon Inc., Boston, 1978.

(iii) J. L. Kelley, *General Topology*, Dover Publications, 2017.
(iv) G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill Education, 2017.
(v) L. A. Steen & J. A. Seebach, *Counterexamples in Topology*, Dover Publications, 2013.
(vi) S. Willard, *General Topology*, Dover Publications, 2004.

DISCIPLINE SPECIFIC ELECTIVE – 4 (i): FOURIER ANALYSIS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course		
		Lecture	Tutorial	Practical
DSE-4 (i): Fourier Analysis	4	3	1	0

Learning Objectives
The primary objective of this course is to introduce:
- basic tools related to Fourier series and Fourier multipliers.
- time-localized and frequency-localized signals.
- finite Fourier transforms and see their applications.
- time-frequency localized bases and filter banks.
- some types of summability kernels.

Learning Outcomes
This course will enable the students to:
- derive Fourier inversion formula for functions in finite dimensional spaces.
- calculate the finite Fourier transform including Parseval's identity.
- comprehend the translation invariance of operations including the convolution product.
- realize the role of Fourier multipliers in signal analysis.
- explore time and frequency-localized signals.
- analyze discrete signals in terms of time-frequency localized bases.
- understand summability kernels and Fourier coefficients.

SYLLABUS OF DSE-4 (i)

Unit – 1 **(13 hours)**
Properties of the finite Fourier transform: The Fourier inversion formula, Parseval's identity, Computation of the finite Fourier transform, Finite Fourier transform and translation-invariant linear operator, Circulant matrices.

Unit – 2 **(16 hours)**
Basic properties of the convolution operator, Translation invariance of the convolution product, Fourier multipliers, Relation between the convolution operator and Fourier multipliers, Time and frequency-localized signals, Involution, Time-frequency localized bases.

Unit – 3 **(16 hours)**
Cesàro summation of series, Riemann Lebesgue lemma, Fourier series, Dirichlet's and Fejér's kernels, Uniqueness theorem, Fourier coefficients of derivatives, Pointwise convergence.

Essential Readings
[1] A. Vretblad, *Fourier Analysis and its Application*, Springer-Verlag, New York, 2003.
[2] M. W. Wong, *Discrete Fourier Analysis*, Birkhäuser, 2011.

Suggested Readings
(i) S. A. Broughton & K. Bryan, *Discrete Fourier Analysis and Wavelets*, Second Edition, John

Wiley & Sons, Inc., 2018.
(ii) V. Serov, *Fourier Series, Fourier Transform and their Applications to Mathematical Physics*, Springer, 2017.

DISCIPLINE SPECIFIC ELECTIVE – 4 (ii): INTEGRAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course		
		Lecture	Tutorial	Practical
DSE-4 (ii): Integral Equations	4	3	1	0

Learning Objectives
The main objective of this course is to introduce the learner to:
- the concepts of integral and integro-differential equations.
- methods for solving Volterra and Fredholm integral equations.
- study of non-linear and singular integral equations.
- solutions of integro-differential equations and system of integral equations.

Learning Outcomes
This course will enable the students to:
- compute solutions to Volterra integral equations by different methods.
- solve the system of integral equations and integro-differential equations.
- determine the solutions of Fredholm integral equations and derivation of the Hilbert-Schmidt theorem.
- solve non-linear and singular integral equations.
- relate the integral equations with Green's function.

SYLLABUS OF DSE-4 (ii)

Unit – 1 **(12 hours)**
Types of integral equations, Introduction and relation with linear differential equation, Volterra integral equations and its solutions, Method of resolvent kernels, Method of successive approximations, Decomposition method, Convolution type of equation, Method of Laplace transform.

Unit – 2 **(16 hours)**
System of Volterra integral equations, Solutions of Integro-differential equation, Abel's integral equation and its generalizations, Non-linear integral equations, Fredholm integral equations and its solutions, Method of resolvent kernels, Method of successive approximations, Integral equations with degenerate kernels.

Unit – 3 **(17 hours)**
Solutions of Fredholm integral equations using characteristic numbers and eigenfunctions with their properties, Hilbert-Schmidt theorem, Non-homogeneous Fredholm integral equation with symmetric kernel, Fredholm alternatives, Applications of Green's function for solution of the boundary value problems, Singular integral equations. Applications of integral equations: Volterra's population growth model.

Essential Readings
[1] W. Hackbusch, *Integral Equations: Theory and Numerical Treatment*, Birkhäuser, 1995.
[2] M. L. Krasnov, A. I. Kiselev & G. I. Makarenko, *Problems and Exercises in Integral Equations*,

Mir Publication Moscow, 1971.
[3] A. C. Pipkin, *A Course on Integral Equations*, Springer, 1991.
[4] A. M. Wazwaz, *Linear and Non-linear Integral Equations*, Springer 2011.

Suggested Readings

(i) S. G. Georgiev, *Integral Equations on Time Scales*, Atlantis Press, 2016.
(ii) F. B. Hildebrand, *Methods of Applied Mathematics*, Dover Publications, 1992.
(iii) J. D. Logan, *Applied Mathematics*, Fourth Edition, John Wiley & Sons, 2013.

Two-Credit Course

APPRECIATING MATHEMATICS VIA WORKSHOPS AND SEMINARS

Learning Objectives
In this course, students will be trained to grasp mathematical ideas and techniques disseminated in workshops and seminars.

Learning Outcomes
This course will enable the students to quickly distill the central ideas in any mathematical discourse, particularly outside the classroom setting, and write a short summary encompassing these ideas.

Methodology
The students will attend a requisite number of workshops and seminars organized by the Department through the semester. They will be expected to prepare and submit summaries of a few of these (as mandated by the Department) for assessment.

Generic Elective (GE) Courses

GENERIC ELECTIVE – 2 (i): FOURIER ANALYSIS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical		
GE-2 (i): Fourier Analysis	4	3	1	0	Class XII pass with Mathematics	Knowledge of Real Analysis including Riemann Integration

Learning Objectives

The primary objective of this course is to introduce:

- basic tools related to Fourier series and Fourier multipliers.
- time-localized and frequency-localized signals.
- finite Fourier transforms and see their applications.
- time-frequency localized bases and filter banks.
- some types of summability kernels.

Learning Outcomes

This course will enable the students to:

- derive Fourier inversion formula for functions in finite dimensional spaces.
- calculate the finite Fourier transform including Parseval's identity.
- comprehend the translation invariance of operations including the convolution product.
- realize the role of Fourier multipliers in signal analysis.
- explore time and frequency-localized signals.
- analyze discrete signals in terms of time-frequency localized bases.
- understand summability kernels and Fourier coefficients.

SYLLABUS OF GE-2 (i)

Unit – 1 **(13 hours)**

Properties of the finite Fourier transform: The Fourier inversion formula, Parseval's identity, Computation of the finite Fourier transform, Finite Fourier transform and translation-invariant linear operator, Circulant matrices.

Unit – 2 **(16 hours)**

Basic properties of the convolution operator, Translation invariance of the convolution product, Fourier multipliers, Relation between the convolution operator and Fourier multipliers, Time and frequency-localized signals, Involution, Time-frequency localized bases.

Unit – 3 **(16 hours)**

Cesàro summation of series, Riemann Lebesgue lemma, Fourier series, Dirichlet's and Fejér's kernels, Uniqueness theorem, Fourier coefficients of derivatives, Pointwise convergence.

Essential Readings

[1] A. Vretblad, *Fourier Analysis and Its Application*, Springer-Verlag, New York, 2003.
[2] M. W. Wong, *Discrete Fourier Analysis*, Birkhäuser, 2011.

Suggested Readings

(i) S. A. Broughton & K. Bryan, *Discrete Fourier Analysis and Wavelets,* Second Edition, John Wiley & Sons, Inc., 2018.
(ii) V. Serov, *Fourier Series, Fourier Transform and their Applications to Mathematical Physics*, Springer, 2017.

GENERIC ELECTIVE – 2 (ii): INTEGRAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course			Eligibility Criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical		
GE-2 (ii): Integral Equations	4	3	1	0	Class XII pass with Mathematics	Knowledge of Calculus and Differential Equations

Learning Objectives

The main objective of this course is to introduce the learner to:

- the concepts of integral and integro-differential equations.
- methods for solving Volterra and Fredholm integral equations.
- study of non-linear and singular integral equations.
- solutions of integro-differential equations and system of integral equations.

Learning Outcomes

This course will enable the students to:

- compute solutions to Volterra integral equations by different methods.
- solve the system of integral equations and integro-differential equations.
- determine the solutions of Fredholm integral equations and derivation of the Hilbert-Schmidt theorem.
- solve non-linear and singular integral equations.
- relate the integral equations with Green's function.

SYLLABUS OF GE-2 (ii)

Unit – 1 **(12 hours)**

Types of Integral equations, Introduction and relation with linear differential equation, Volterra integral equations and its solutions, Method of resolvent kernels, Method of successive approximations, Convolution type of equation, Method of Laplace Transform.

Unit – 2 **(16 hours)**

System of Volterra integral equations, Solutions of Integro-differential equation, Abel's integral equation and its generalizations, Non-linear integral equations, Fredholm integral equations and its solutions, Method of resolvent kernels, Method of successive approximations, Integral equations with degenerate kernels.

Unit – 3 **(17 hours)**

Solutions of Fredholm integral equations using characteristic numbers and eigenfunctions with their properties, Hilbert-Schmidt theorem, Non-homogeneous Fredholm integral equation with symmetric kernel, Fredholm alternatives, Applications of Green's function for solution of the boundary value problems, Singular integral equations.

Essential Readings

[1] W. Hackbusch, *Integral Equations: Theory and Numerical Treatment*, Birkhäuser, 1995.
[2] M. L. Krasnov, A. I. Kiselev & G. I. Makarenko, *Problems and Exercises in Integral Equations*, Mir Publication Moscow, 1971.
[3] A. C. Pipkin, *A Course on Integral Equations*, Springer, 1991.
[4] A. M. Wazwaz, *Linear and Non-linear Integral Equations*, Springer 2011.

Suggested Readings

(i) S. G. Georgiev, *Integral Equations on Time Scales*, Atlantis Press, 2016.
(ii) F. B. Hildebrand, *Methods of Applied Mathematics*, Dover Publications, 1992.
(iii) J. D. Logan, *Applied Mathematics*, Fourth Edition, John Wiley & Sons, 2013.

Semester-III

Discipline Specific Core (DSC) Courses

DISCIPLINE SPECIFIC CORE – 7: FLUID DYNAMICS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course		
		Lecture	Tutorial	Practical
DSC-7: Fluid Dynamics	4	3	1	0

Learning Objectives

The objective of this course is to:

- prepare a mathematical foundation to study the motion of fluids.
- develop concept, models, and techniques that enable to solve the problems of fluid flow.
- develop the ability to conduct advanced studies and research in the broad field of fluid dynamics.

Learning Outcomes

After studying this course, the student will be able to:

- understand the concept of fluids, their classification, flow lines, models and approaches to study fluid flow.
- formulate mass and momentum conservation principles and obtain their solution for non- viscous flow.
- know potential flow, Bernoulli's equation, Kelvin's minimum energy and circulation theorems.
- understand two- and three-dimensional motion, complex potential, circle theorem, Blasius theorem, Weiss's and Butler's sphere theorems.
- apply the concept of stress and strain in viscous flow to derive Navier–Stokes equation of motion and energy equation.

SYLLABUS OF DSC-7

Unit – 1 **(12 hours)**

Classification of fluids, Continuum model, Eulerian and Lagrangian approach of description, Differentiation following the fluid motion, Flow lines, vorticity and circulation, Conservation of mass: Equation of Continuity, Boundary surface, Forces in fluid motion, Conservation of momentum: Euler's equation of motion.

Unit – 2 **(18 hours)**

Theory of irrotational motion: Integration of Euler's equation under different conditions, Bernoulli's equation, Impulsive motion, Kelvin's minimum energy and circulation theorems, Potential theorem, Two-dimensional motion: Complex potential, Line sources, sinks, doublets and vortices, Two-dimensional image system, Milne–Thomson circle theorem, Images with respect to a plane and cylinder, Blasius theorem. Three-dimensional flows, Weiss's sphere theorem, Images with respect to sphere, Axi-symmetric flow, Stokes stream function, Butler's sphere theorem.

Unit – 3 **(15 hours)**

Stress and strain analysis, Newton's law of viscosity, Laminar flow, Navier-Stokes equation of motion, Steady flow between parallel planes and Poiseuille flow, Constitutive equation, Energy equation, Flow past spheres and cylinders.

Essential Readings

[1] F. Chorlton, *Text Book of Fluid Dynamics*, CBS Publisher, 2005.
[2] R. W. Fox, P. J. Pritchard & A. T. McDonald, *Introduction to Fluid Mechanics*, Seventh Edition, John Wiley & Sons, 2009.
[3] P. K. Kundu, I. M. Cohen & D. R. Dowling, *Fluid Mechanics*, Sixth Edition, Academic Press, 2016.

Suggested Readings

(i) L. M. Milne-Thomson, *Theoretical Hydrodynamics*, The Macmillan company, USA, 1969.
(ii) D. E. Rutherford, *Fluid Dynamics*, Oliver and Boyd Ltd., 1978.

DISCIPLINE SPECIFIC CORE – 8: MEASURE AND INTEGRATION

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course		
		Lecture	Tutorial	Practical
DSC-8: Measure and Integration	4	3	1	0

Learning Objectives

The primary objective of this course is to:

- extend the notion of length of an interval with the introduction of the concept of Lebesgue outer measure for any subset of real line.
- investigate the properties of Lebesgue measurable sets and functions.
- familiarize students with the Lebesgue integration of functions and its comparison with Riemann integration.
- generalize the concepts of measure and integration to an abstract space.

Learning Outcomes

This course will enable the students to:

- verify whether a given subset of $\mathbb{R}$ or a real valued function is measurable.
- understand the requirement and the concept of the Lebesgue integral (a generalization of the Riemann integration) along with its properties.
- understand the statements and proofs of the fundamental integral convergence theorems and demonstrate their applications.
- carry out a comprehensive study of functions of bounded variation and their utility in understanding differentiation and integration.
- apply Hölder and Minkowski inequalities in L^p-spaces and understand completeness of L^p-spaces.

SYLLABUS OF DSC-8

Unit – 1 **(18 hours)**

Lebesgue outer measure, Measurable sets, Lebesgue measure, Borel sets, Regularity, Measurable functions, Borel and Lebesgue measurability, Non-measurable sets.

Unit – 2 **(15 hours)**

Integration of nonnegative functions, General integral, Integration of series, Riemann and Lebesgue integrals.

Unit – 3 **(12 hours)**

Functions of bounded variation, Lebesgue's differentiation theorem, Differentiation and integration, Absolute continuity of functions, Measures and outer measures, Measure spaces, Integration with respect to a measure, L^p-spaces, Hölder's and Minkowski's inequalities, Completeness of L^p-spaces.

Essential Readings

[1] G. de Barra, *Measure Theory and Integration*, Ellis Horwood Ltd., Chichester, John Wiley & Sons, Inc., New York, 1981 (Indian Reprint, 2014).

Suggested Readings

(i) M. Capinski & P. E. Kopp, *Measure, Integral and Probability*, Springer, 2005.
(ii) E. Hewitt & K. Stromberg, *Real and Abstract Analysis: A Modern Treatment of the Theory of Functions of a Real Variable*, Springer, Berlin, 1975.
(iii) H. L. Royden & P.M. Fitzpatrick, *Real Analysis*, Fourth Edition, Pearson, 2015.

Semester-IV

Discipline Specific Core (DSC) Courses

DISCIPLINE SPECIFIC CORE – 9: PARTIAL DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course		
		Lecture	Tutorial	Practical
DSC-9: Partial Differential Equations	4	3	1	0

Learning Objectives

The main objective of this course is to introduce:

- well-posedness, fundamental solutions, existence and uniqueness of solutions for Laplace equation, Poisson equation and Heat equation.
- solution for wave equation by spherical means.
- characteristics, complete integrals, envelopes and conservation laws for first-order nonlinear partial differential equations.
- classical solution techniques such as Green's function, similarity solutions and transform methods.

Learning Outcomes

This course will enable the students to:

- understand Laplace equation, Poisson equation, and Heat equation, their fundamental solutions, uniqueness principles, mean value properties, and Green's function.
- apply the method of spherical means to solve homogeneous and nonhomogeneous wave equations.
- use characteristics to solve nonlinear partial differential equations, construct complete integrals and envelopes, and understand conservation laws.
- implement various techniques such as similarity solutions and transform methods to derive solutions of different types of partial differential equations.

SYLLABUS OF DSC-8

Unit – 1 **(18 hours)**

Well-posed problems, Classical solution, Laplace equation, Poisson equation, Fundamental solution, Strong maximum principle and uniqueness of solution, Mean value formulas, Representation formula, Green's function, Poisson's formula.

Heat equation, Fundamental solution for homogeneous and nonhomogeneous initial-value problems, Mean value formula, Strong maximum principle and uniqueness of solution, Local estimates for the solution.

Unit – 2 **(15 hours)**
Wave equation: Solution of homogeneous and nonhomogeneous problems by spherical means.

Nonlinear first order partial differential equations: Complete integrals and envelopes, Characteristics, Introduction to conservation laws.

Unit – 3 **(12 hours)**
Other solution methods: Similarity solutions, Fourier transform and Laplace transform, Cole-Hopf transformation, Potential function, Hodograph and Legendre transform.

Essential Readings

[1] L. C. Evans, *Partial Differential Equations*, American Mathematical Society, Providence, RI, 1998.
[2] F. John, *Partial Differential Equations*, Fourth Edition, Springer-Verlag, New York, 1982.

Suggested Readings

(i) P. R. Garabedian, *Partial Differential Equations,* John Wiley & Sons, Inc., New York- London-Sydney, 1964.
(ii) A. K. Nandakumaran & P. S. Datti, *Partial Differential Equations: Classical Theory with a Modern Touch*, Cambridge University Press, 2020.

DISCIPLINE SPECIFIC CORE – 10: ANALYSIS OF SEVERAL VARIABLES

CREDIT DISTRIBUTION OF THE COURSE

Course Title & Code	Credits	Credit Distribution of the Course		
		Lecture	Tutorial	Practical
DSC-10: Analysis of Several Variables	4	3	1	0

Learning Objectives
The primary objective of this course is to:
- introduce differentiation of vector valued functions on $\mathbb{R}^n$ and their properties.
- familiarize students with integration of functions over rectangles and bounded sets in $\mathbb{R}^n$.
- extend integration of functions to unbounded sets in $\mathbb{R}^n$.
- study change of variables and its applications.

Learning Outcomes
This course will enable the students to:
- check differentiability of vector valued functions on $\mathbb{R}^n$, understand the relation between directional derivative and differentiability, apply chain rule, mean value theorems, inverse and implicit function theorems.
- understand higher order derivatives and be able to apply Taylor's formulas with integral remainder, Lagrange's remainder and Peano's remainder.
- master the concepts of integration over rectangles and bounded sets in $\mathbb{R}^n$.
- generalize the integration theory to unbounded sets in $\mathbb{R}^n$.
- grasp the effect of change of variables in integration.

SYLLABUS OF DSC-10

Unit– 1 **(18 hours)**
The differentiability of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$, Partial derivatives and differentiability, Directional derivatives and differentiability, Chain rule, Mean value theorems, Derivatives of higher order, Taylor's formulas with integral remainder, Lagrange's remainder and Peano's remainder, Inverse function theorem and Implicit function theorem.

Unit– 2 **(15 hours)**
The integral over a rectangle, Existence of the integral, Evaluation of the integral, Fubini's theorem, The integral over a bounded set.

Unit– 3 **(12 hours)**
Rectifiable sets, Improper integrals**,** The change of variable theorem, Applications of change of variables.

Essential Readings
[1] M. Giaquinta & G. Modica, *Mathematical Analysis: An Introduction to Functions of Several Variables*, Birkhäuser, 2009.
[2] J. R. Munkres, *Analysis on Manifolds*, CRC Press, Taylor & Francis, 2018.

Suggested Readings

(i) W. Rudin, *Principles of Mathematical Analysis*, Third Edition, Mc Graw Hill, 1986.

(ii) M. Spivak, *Calculus on Manifolds: A Modern Approach to Classical Theorems of Advanced Calculus*, Taylor & Francis, 2018.