

**UNIVERSITY OF DELHI**

**DEPARTMENT: MATHEMATICS**

**COURSE NAME: B.Sc. (Hons.) Mathematics**

**(SEMESTER - I)**  
based on

Undergraduate Curriculum Framework 2022 (UGCF)

(Effective from Academic Year 2022-23)



University of Delhi

Course name: **B.Sc. (Hons.) Mathematics (List of DSC Papers)**

Course Title	Nature of the Course	Total Credits	Components			Eligibility Criteria/ Prerequisite	Contents of the course and reference is in (of standing committee meeting)
			Lecture	Tutorial	Practical		
Algebra	DSC	4	3	1	0	12 <sup>th</sup> with Mathematics	Annexure-I
Elementary Real Analysis	DSC	4	3	1	0	12 <sup>th</sup> with Mathematics	Annexure-II
Probability and Statistics	DSC	4	3	0	1 (2 Hours)	12 <sup>th</sup> with Mathematics	Annexure-III

Course name: **Generic Elective (GE) for other than B.Sc. (Hons.) Mathematics**

Course Title	Nature of the Course	Total Credits	Components			Eligibility Criteria/ Prerequisite	Contents of the course and reference is in
			Lecture	Tutorial	Practical		
Fundamental of Calculus	GE (i)	4	3	1	0	12 <sup>th</sup> with Mathematics	Annexure-IV
Theory of Equations and Symmetries	GE (ii)	4	3	1	0	10 <sup>th</sup> with Mathematics	Annexure-V

## B.Sc. (Hons.) Mathematics (Sem I)

### DSC-1: Algebra

**Total Marks: 100** (Theory: 75, Internal Assessment: 25) **Examination: 3 Hrs.**

**Workload: 3 Lectures, 1 Tutorial (per week) Credits: 4**

**Course Objectives:** The primary objective of this course is to introduce the basic tools of theory of equations, number theory, and group theory. Symmetry group of a plane figure, basic concepts of cyclic groups and classification of subgroups of cyclic groups shall also be introduced.

**Course learning Outcomes:** This course will enable the students to:

- i) Determine number of positive/negative real roots of a real polynomial in one variable.
- ii) Solve cubic and quartic polynomial equations with special condition on roots and in general.
- iii) Employ De-Moivre's theorem in a number of applications to solve numerical problems.
- iv) Use modular arithmetic and basic properties of congruences.
- v) Recognize the algebraic structure, namely groups, and classify subgroups of cyclic groups.

#### Unit 1: Theory of Equations and Complex Numbers

General properties of polynomials and equations, Fundamental theorem of algebra, Relations between the roots and the coefficients, Upper bounds for the real roots; Theorems on imaginary, integral and rational roots; Newton's method for integral roots, Descartes' rule of signs; De-Moivre's theorem for integer and rational indices and their applications, The  $n$ th roots of unity, Cardan's solution of the cubic, Descartes' solution of the quartic equation.

#### Unit 2: Basic Number Theory

Division algorithm in  $\mathbb{Z}$ , Divisibility and the Euclidean algorithm, Fundamental theorem of arithmetic, Modular arithmetic and basic properties of congruences.

#### Unit 3: Basics of Group Theory

Groups, Basic properties, Symmetries of a square, Dihedral group, Order of a group, Order of an element, Subgroups, Center of a group, Centralizer of an element, Cyclic groups and properties, Generators of a cyclic group, Classification of subgroups of cyclic groups.

#### References:

1. Andreescu, Titu & Andrica, D. (2014). *Complex numbers from A to...Z*. (2nd ed.). Birkhäuser.
2. Dickson, Leonard Eugene (2009). *First Course in the Theory of Equations*. John Wiley & Sons, Inc. The Project Gutenberg eBook: <http://www.gutenberg.org/ebooks/29785>
3. Gallian, Joseph. A. (2017). *Contemporary Abstract Algebra* (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint 2021.
4. Goodaire, Edgar G., & Parmenter, Michael M. (2006). *Discrete Mathematics with Graph Theory* (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint 2018.

#### Additional Readings:

- i. Burnside, W.S., & Panton, A.W. (1979), *The Theory of Equations*, Vol. 1. Eleventh Edition, (Fourth Indian Reprint. S. Chand & Co. New Delhi), Dover Publications, Inc.
- ii. Burton, David M. (2011). *Elementary Number Theory* (7th ed.). McGraw-Hill Education Pvt. Ltd. Indian Reprint.
- iii. Rotman, Joseph J. (1995). *An Introduction to The Theory of Groups* (4th ed.). Springer-Verlag, New York.

## B.Sc. (Hons.) Mathematics (Sem I)

### DSC-2: Elementary Real Analysis

**Total Marks: 100** (Theory: 75, Internal Assessment: 25) **Examination: 3 Hrs.**

**Workload: 3 Lectures, 1 Tutorial (per week) Credits: 4**

**Course Objectives:** The course will develop a deep and rigorous understanding of real line  $\mathbb{R}$  with algebraic, order and completeness properties to prove the results about convergence and divergence of sequences and series of real numbers. These concepts have wide range of applications in real life.

**Course Learning Outcomes:** This course will enable the students to:

- i) Understand the fundamental properties of the real numbers, including completeness and Archimedean, and density property of rational numbers in  $\mathbb{R}$ .
- ii) Learn to define sequences in terms of functions from  $\mathbb{N}$  to a subset of  $\mathbb{R}$  and find the limit.
- iii) Recognize bounded, convergent, divergent, Cauchy and monotonic sequences and to calculate the limit superior and limit inferior of a bounded sequence.
- iv) Apply limit comparison, ratio, root, and alternating series tests for convergence and absolute convergence of infinite series of real numbers.

#### Unit 1: Real Number System

Algebraic and order properties of  $\mathbb{R}$ , Absolute value of a real number, Bounded above and bounded below sets, Supremum and infimum of a non-empty subset of  $\mathbb{R}$ , The completeness property of  $\mathbb{R}$ , Archimedean property, Density of rational numbers in  $\mathbb{R}$ .

#### Unit 2: Sequences

Sequences and their limits, Convergent sequence, Limit theorems, Monotone sequences, Monotone convergence theorem, Subsequences, Bolzano-Weierstrass theorem for sequences, Limit superior and limit inferior for bounded sequence, Cauchy sequence, Cauchy's convergence criterion.

#### Unit 3: Infinite Series

Convergence and divergence of infinite series of real numbers, Necessary condition for convergence, Cauchy criterion for convergence, Tests for convergence of positive term series, Integral test, Basic comparison test, Limit comparison test, D'Alembert's ratio test, Cauchy's  $n$ th root test, Raabe's test, Alternating series, Leibniz test, Absolute and conditional convergence.

#### References:

1. Bartle, Robert G., & Sherbert, Donald R. (2011). *Introduction to Real Analysis* (4th ed.). John Wiley & Sons. Wiley India Edition 2015.
2. Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). *An Introduction to Analysis* (2nd ed.). Jones and Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.
3. Denlinger, Charles G. (2011). *Elements of Real Analysis*. Jones and Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

#### Additional Readings:

- i. Aliprantis C. D., & Burkinshaw, O. (1998). *Principles of Real Analysis* (3rd ed.). Academic Press.
- ii. Ross, Kenneth A. (2013). *Elementary Analysis: The Theory of Calculus* (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian reprint.
- iii. Thomson, B. S., Bruckner, A. M., & Bruckner, J. B. (2001). *Elementary Real Analysis*. Prentice Hall.

## B.Sc. (Hons.) Mathematics (Sem I)

### DSC-3: Probability and Statistics

**Total Marks: 125** (Theory: 75, Internal Assessment: 25, and Practical: 25) **Examination:** 3 Hrs.

**Workload:** 3 Lectures, 1 Practical (2 Hours) (per week) **Credits:** 4

**Course Objectives:** To make the students familiar with the basic statistical concepts and tools which are needed to study situations involving uncertainty or randomness. The course intends to render the students to several examples and exercises that blend their everyday experiences with their scientific interests to form the basis of data science.

**Course Learning Outcomes:** This course will enable the students to:

- i) Understand some basic concepts and terminology—population, sample, descriptive and inferential statistics including stem-and-leaf plots, dotplots, histograms and boxplots.
- ii) Learn about probability density functions and various univariate distributions such as binomial, hypergeometric, negative binomial, Poisson, normal, exponential and lognormal.
- iii) Understand the remarkable fact that the empirical frequencies of so many natural populations, exhibit bell-shaped (i.e., normal) curves, using the Central Limit Theorem.
- iv) Measure the scale of association between two variables, and to establish a formulation helping to predict one variable in terms of the other, i.e., correlation and linear regression.

#### Unit 1: Descriptive Statistics, Probability, and Discrete Probability Distributions

Descriptive statistics: Populations, Samples, Stem-and-leaf displays, Dotplots, Histograms, Qualitative data, Measures of location, Measures of variability, Boxplots; Sample spaces and events, Probability axioms and properties, Conditional probability, Bayes' theorem and independent events; Discrete random variables and probability distributions, Expected values; Probability distributions: Binomial, geometric, hypergeometric, negative binomial, Poisson, and Poisson distribution as a limit.

#### Unit 2: Continuous Probability Distributions

Continuous random variables, Probability density functions, Uniform distribution, Cumulative distribution functions and expected values, The normal, exponential and lognormal distributions.

#### Unit 3: Central Limit Theorem and Regression Analysis

Sampling distribution and standard error of the sample mean, Central Limit Theorem and applications; Scatterplot of bivariate data, Regression line using principle of least squares, Estimation using the regression lines; Sample correlation coefficient and properties.

#### Reference:

1. Devore, Jay L. (2016). *Probability and Statistics for Engineering and the Sciences* (9th ed.). Cengage Learning India Private Limited. Delhi. Indian Reprint 2020.

#### Additional Reading:

- i. Mood, A. M., Graybill, F. A., & Boes, D. C. (1974). *Introduction to the Theory of Statistics* (3rd ed.). Tata McGraw-Hill Pub. Co. Ltd. Reprinted 2017.

**All Courses (other than B.Sc. (Hons.) Mathematics) (Sem I)****GE-1(i): Fundamentals of Calculus****Total Marks: 100** (Theory: 75, Internal Assessment: 25) **Examination: 3 Hrs.****Workload: 3 Lectures, 1 Tutorial (per week) Credits: 4**

**Course Objectives:** Calculus is referred as 'Mathematics of change' and is concerned with describing the precise way in which changes in one variable relate to the changes in another. Through this course, students can understand the quantitative change in the behaviour of the variables and apply them on the problems related to the environment.

**Course Learning Outcomes:** The students who take this course will be able to:

- i) Understand continuity and differentiability in terms of limits.
- ii) Describe asymptotic behavior in terms of limits involving infinity.
- iii) Understand the importance of mean value theorems and its applications.
- iv) Learn about Maclaurin's series expansion of elementary functions.
- v) Use derivatives to explore the behavior of a given function, locating and classifying its extrema, and graphing the polynomial and rational functions.

**Unit 1: Continuity and Differentiability of Functions**

Limits and continuity, Types of discontinuities; Differentiability of functions; Successive differentiation: Calculation of the  $n$ th derivatives, Leibnitz theorem; Partial differentiation, Euler's theorem on homogeneous functions.

**Unit 2: Mean Value Theorems and its Applications**

Rolle's theorem, Mean value theorems and applications to monotonic functions and inequalities; Expansion of functions: Taylor's theorem, Taylor's series, Maclaurin's series expansion of  $e^x$ ,  $\sin x$ ,  $\cos x$ ,  $\log(1+x)$  and  $(1+x)^m$ ; Indeterminate forms.

**Unit 3: Tracing of Curves**

Concavity and inflexion points, Asymptotes (parallel to axes and oblique), Relative extrema, Tracing graphs of polynomial functions, rational functions, and polar equations.

**References:**

1. Anton, Howard, Bivens, Irl, & Davis, Stephen (2013). *Calculus* (10th ed.). Wiley India Pvt. Ltd. New Delhi. International Student Version. Indian Reprint 2016.
2. Prasad, Gorakh (2016). *Differential Calculus* (19th ed.). Pothishala Pvt. Ltd. Allahabad.

**Additional Reading:**

- i. Thomas Jr., George B., Weir, Maurice D., & Hass, Joel (2014). *Thomas' Calculus* (13th ed.). Pearson Education, Delhi. Indian Reprint 2017.

**All Courses (other than B.Sc. (Hons.) Mathematics) (Sem I)**  
**GE-1(ii): Theory of Equations and Symmetries**

**Total Marks: 100** (Theory: 75, Internal Assessment: 25) **Examination: 3 Hrs.**

**Workload: 3 Lectures, 1 Tutorial (per week) Credits: 4**

**Course Objectives:** The goal of this paper is to acquaint students with certain ideas about integral roots, rational roots, an upper bound on number of positive or negative roots of a polynomial, and finding roots of cubic and quartic equations in special cases using elementary symmetric functions and in general using Cardon's and Descartes' methods, respectively.

**Course Learning Outcomes:** After completion of this paper, the students will be able to:

- i) Understand the nature of the roots of polynomial equations and their symmetries.
- ii) Solve cubic and quartic polynomial equations with special condition on roots and in general.
- iii) Find symmetric functions in terms of the elementary symmetric polynomials.

**Unit 1: Polynomial Equations and Properties**

General properties of polynomials and equations; Fundamental theorem of algebra and its consequences; Theorems on imaginary, integral and rational roots; Descartes' rule of signs for positive and negative roots; Relations between the roots and coefficients of equations, Applications to solution of equations when an additional relation among the roots is given; De Moivre's theorem for rational indices, the  $n$ th roots of unity and symmetries of the solutions.

**Unit 2: Cubic and Biquadratic (Quartic) Equations**

Transformation of equations (multiplication, reciprocal, increase/decrease in the roots by a given quantity), Removal of terms; Cardon's method of solving cubic and Descartes' method of solving biquadratic equations.

**Unit 3: Symmetric Functions**

Elementary symmetric functions and symmetric functions of the roots of an equation; Newton's theorem on sums of the like powers of the roots; Computation of symmetric functions such as

$\sum \alpha^2 \beta, \sum \alpha^2 \beta^2, \sum \alpha^2 \beta \gamma, \sum \frac{1}{\alpha^2 \beta \gamma}, \sum \alpha^{-3}, \sum (\beta + \gamma - \alpha)^2, \sum \frac{\alpha^2 + \beta \gamma}{\beta + \gamma}, \dots$  of polynomial equations;

Transformation of equations by symmetric functions and in general.

**References:**

1. Burnside, W.S., & Panton, A.W. (1979). *The Theory of Equations* (11th ed.). Vol. 1. Dover Publications, Inc. (4th Indian reprint. S. Chand & Co. New Delhi).
2. Dickson, Leonard Eugene (2009). *First Course in the Theory of Equations*. John Wiley & Sons, Inc. The Project Gutenberg eBook: <http://www.gutenberg.org/ebooks/29785>

**Additional Reading:**

- i. Prasad, Chandrika (2017). *Text Book of Algebra and Theory of Equations*. Pothishala Pvt Ltd.